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LIGO- T040027-03-R

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12th April 2008

Conceptual Design of Beamsplitter Suspension for
Advanced LIGO

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LIGO Science Collaboration

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1 Introduction

This is Rev-03 of the beamsplitter conceptual design document.

History

Rev-00: 9th February 2004

This version, entitled “Design of Beamsplitter Suspension for Advanced LIGO” presented the case for making the beamsplitter suspension a triple pendulum rather than a quadruple pendulum as used for the ETMs and ITMs. A conceptual design based on the size of beamsplitter at that time (350 mm diameter by 60 mm thick) was presented and curves for predicted seismic isolation performance and thermal noise were given. It was shown that these met the noise requirements for the beamsplitter. The thermal noise curve was produced assuming that the beamsplitter was suspended by four silica fibres of circular cross-section.

Rev-01: 19th November 2007

This version reflected the fact that several factors had changed since the original document was written.

- a) The beamsplitter (BS) size has been increased to 370 mm diameter x 60 mm thick. Currently it is expected to have a wedge angle of 0.9° . This diameter has been shown to have sufficient free aperture to give an acceptable level of optical loss with or without flats on the side— see G070471-00-E for information on losses with flats.
- b) A reassessment of the need for silica fibres has taken place. The baseline is now to use steel wires.
- c) The decision has been taken that the design of the BS and folding mirror (FM) suspensions should be the same.

Rev-02: 16 January 2008

The document has been modified to include transfer functions (from the symmetric MATLAB model) and thermal noise curves (from the Mathematica model) produced using the *same* parameter set (the current set at the time of writing) rather than slightly differing sets which had evolved over the previous few months. The thermal noise curves are presented with more easily read axes. The detailed listing of the Mathematica parameters has been replaced with a reference. The section on choice of parameters has been reduced with details moved into Appendix C. The thermal noise section has been edited.

Rev-03: April 2008

Section 4.2 added – discussion of phi value used for steel wire.

Appendix E added – diagrams and descriptions giving identification of parameters used in the MATLAB model, as listed in appendix A.

Current prototype design rendering has been included (figure 7).

Section 7 added re requirement for a reaction chain. Conclusions section renumbered as 8.

2 Beamsplitter Requirements

Currently the noise requirement at 10 Hz from the sum (suitably added) of BS optics axis motion and vertical motion is $2e-17$ m/rt Hz (ref. the cavity optics noise requirements document T010007-02 and ref P Fritschel, e-mail 27 Jan 2004). See Rev-00 of the conceptual design document for fuller discussion. Technical noise sources should be 1/10 of the fundamental noise requirement.

3 Choice of Parameters

The original working design which was investigated was of a triple suspension with approximately equal masses (12.7 kg for the original size of BS) and equal wire lengths of 60 cm at each stage. The choice of equal masses and equal wire lengths as a baseline has come from experience with previous designs and leads to good coupling of modes. In addition using three equal lengths gives the best isolation for a given overall length. For various reasons (available length, change in size of the optic, consequences of changing from silica fibres to steel wires) this original design has been modified. The current parameter list is given in appendix A, and details on the history and reasons for changes are given in Appendix C.

4 Suspension Thermal Noise

4.1 Thermal noise estimate using steel wire and wedged optic

In the 2004 design it was shown that a final stage of the suspension consisting of 4 silica fibres of circular cross-section, 140 micron radius (stress ~ 500 MPa) and 60 cm length comfortably met the noise requirement (see rev-00 for more details). Silica was chosen as the baseline design. However this decision has since been revisited. There are compelling reasons to use steel wire if it gives acceptable performance: its use gives a significant reduction in complexity of design and construction. One key issue is how much vertical thermal noise is coupled into the longitudinal direction. It is found that with the use of steel wires and a coupling factor of 0.001 from vertical to longitudinal motion, the thermal noise estimate meets the noise requirement at 10 Hz. See figures 1 and 2 below, which are produced assuming an unwedged optic. These estimates have been carried out using Mark Barton's Mathematica model of the beamsplitter, see Appendix B for more details.

The baseline has now been changed to the use of steel wires in conjunction with a coupling factor of ≤ 0.001 to be met by the suspension design and the optical layout. From consideration of the optical layout, the practical consequence of requiring the coupling to be 0.001 or less is that the orientation of the wedge on the beamsplitter should be horizontal (rather than vertical) and this is now the baseline configuration for the optic. RODA M070120-02 captures the salient details of the current baseline design: beamsplitter optic size, geometry, wedge orientation and suspension wire material.

Using a horizontal wedge makes very little difference to the overall thermal noise performance expect for the introduction of some coupling of roll into vertical motion, introducing an extra peak at the highest roll mode of 25 Hz. This peak can be removed by moving the spot on the optic from the centre to the sweet spot (the new centre of mass) which for the current parameter set is 2.1 mm from the centre. See figure 3. However even without moving the spot, the roll peak is hardly visible in the sum of longitudinal and vertical with 0.001 coupling, see figure 4.

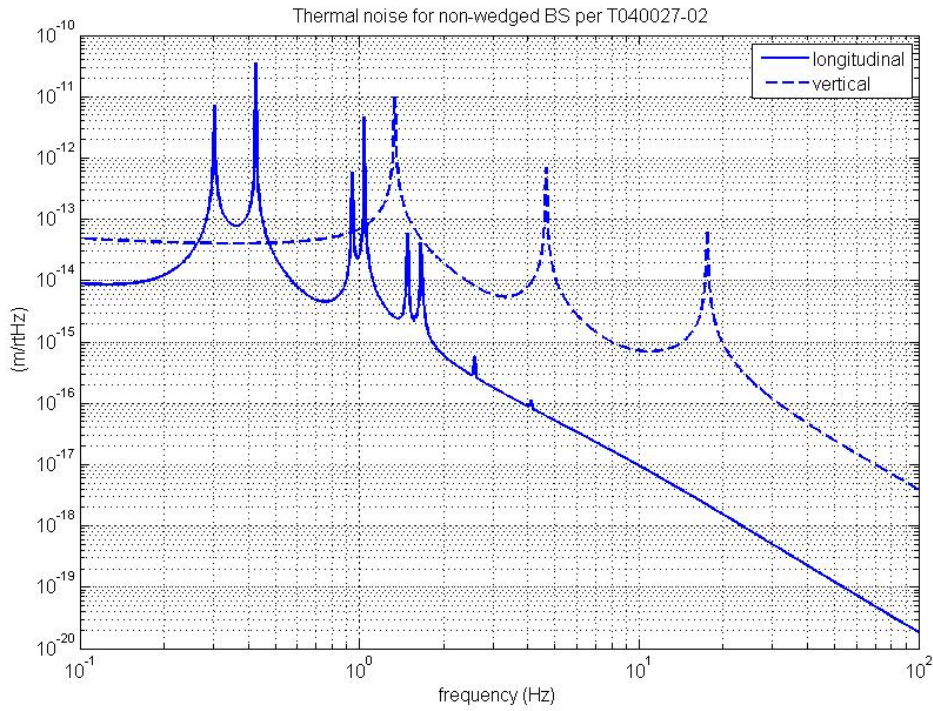


Figure 1. Thermal noise for BS on steel wires, parameters as referenced in Appendix B (unwedged case). Longitudinal and vertical noise estimates are shown separately. Note that at frequencies above the highest vertical mode at 17.5 Hz the vertical thermal noise curve lies more than a factor of 100 above the longitudinal curve.

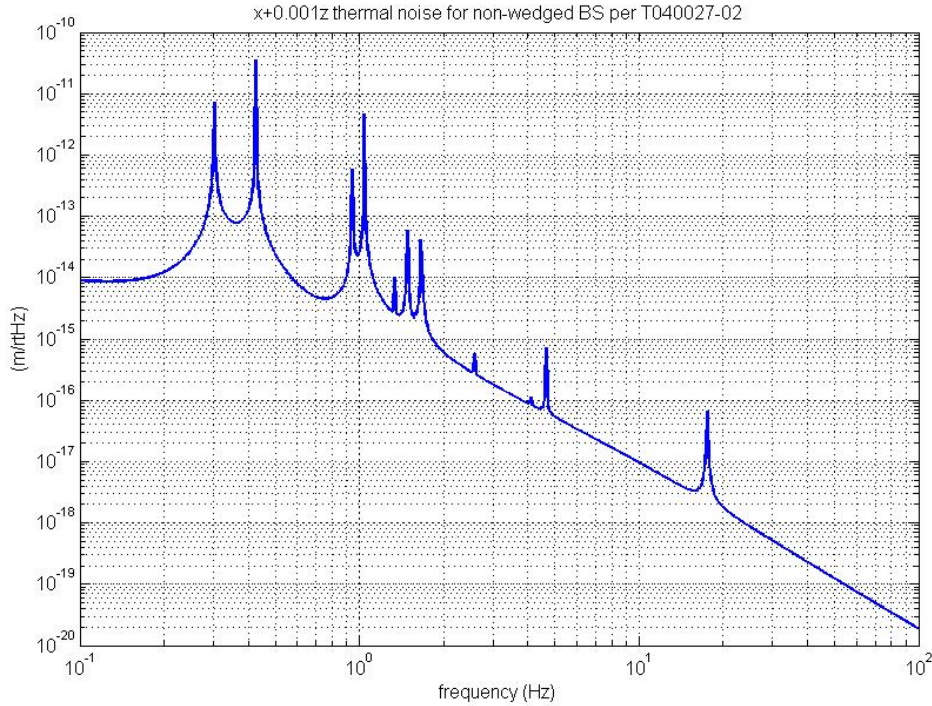


Figure 2. Thermal noise for BS on steel wires (unwedged case) with longitudinal and 0.001 times vertical summed quadratically. The noise is dominated by the longitudinal contribution except at the vertical mode peaks including the highest vertical mode at 17.5 Hz. The level at 10 Hz is 10^{-17} m/rt Hz, below the requirement of 2×10^{-17} m/rt Hz.

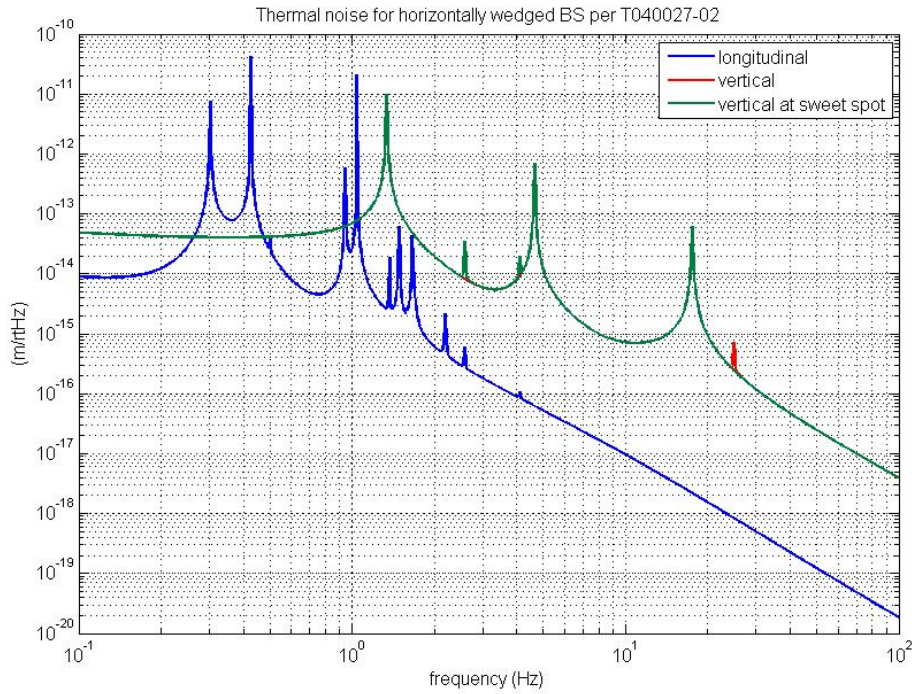


Figure 3. Thermal noise for BS on steel wires, parameters as referenced in Appendix B (horizontal wedged case). Longitudinal and vertical noise estimates are shown separately. The highest roll mode peak can be seen in the vertical plot at 25 Hz. The peak disappears when the beam spot is moved from the centre of the mirror face to the centre of mass (sweet spot),

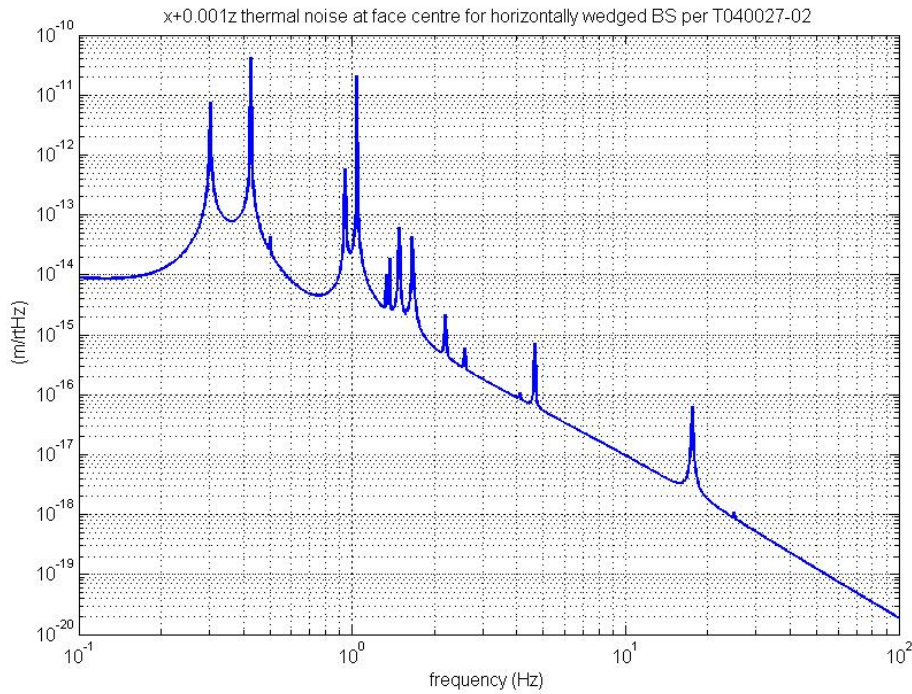


Figure 4. Thermal noise for BS on steel wires (horizontally wedged case) with longitudinal and 0.001 times vertical summed quadratically, for the beam hitting centre of face of optic. The residual roll peak can just be seen at 25 Hz.

4.2 Value of phi for steel wire suspension

The value of the intrinsic loss (ϕ) in the wire assumed for these curves is 2×10^{-4} (ref G Cagnoli et al Phys Lett A 255, p 230, 1999). Recent work by Penn, Harry, Evans, Weiss et al has shown that whereas the intrinsic ϕ for steel music wire may be even better than this at $\sim 6 \times 10^{-5}$ (see G080108-00-Z), the design of break-off bars at the mirror as used in LIGO1 gives higher loss and variability. Evidence for this comes both from studying the violin mode Qs seen in LIGO 1 and from bench-top experiments. More repeatable results have been obtained using a grooved prism design of break-off, or a clamp. This is an ongoing area of R and D, and any lessons learnt on good design will be taken into account. It is of interest to note that a value of intrinsic $\phi \sim$ four times larger than that assumed for the figures above, i.e. a value of 8×10^{-4} , would raise the noise level at 10 Hz by a factor of 2, still just meeting the noise requirement. Note that the thermoelastic peak is well above 10 Hz (~ 460 Hz for the current wire diameter) and so does not significantly affect this argument.

5 Seismic Isolation, Mode Frequencies and Damping

The longitudinal and vertical transfer functions derived from the MATLAB model of the beamsplitter for the parameter set given in appendix A are shown in figures 5 and 6. The mode frequencies are also given in the appendix. The longitudinal transfer function using eddy current damping (with damping time ~ 10 secs) has a magnitude at 10 Hz of $\sim 1.6 \times 10^{-6}$. This, combined with active platform noise level of 2×10^{-13} m/ $\sqrt{\text{Hz}}$ at 10 Hz, gives a noise level at the optic of $\sim 3.2 \times 10^{-19}$ m/ $\sqrt{\text{Hz}}$ at 10 Hz. The vertical transfer function is $\sim 8.3 \times 10^{-3}$ at 10 Hz (with damping time ~ 5 secs), giving vertical noise level at optic of $\sim 1.7 \times 10^{-15}$ m/ $\sqrt{\text{Hz}}$. Including a 10^{-3} coupling factor gives a residual noise level in the horizontal due to vertical motion of 1.7×10^{-18} m/ $\sqrt{\text{Hz}}$. Taking the quadratic sum of these numbers yields a total essentially the same as the noise due to vertical alone, and lying well below the requirement.

6 Other Noise Sources

Using the MATLAB model we can also estimate the magnitude of pitch and yaw contributions. The larger of these transfer functions at 10 Hz is for yaw, at $\sim 7 \times 10^{-6}$. Assuming an angular input at the platform of around 2×10^{-13} rad/ $\sqrt{\text{Hz}}$ and a 1mm beam offset we find a horizontal noise level of $\sim 1.4 \times 10^{-21}$ m/ $\sqrt{\text{Hz}}$ at 10 Hz, negligible compared to the requirement.

A further consideration is that of noise introduced by local control. A combination of steep electronic filtering and some eddy current damping (ECD) should yield a workable solution. In fact ECD could comfortably be used without any active control for some modes, and ECD is being incorporated into the design. It has been checked that the thermal noise associated with using ECD is below the noise requirement for the beamsplitter – see Appendix C.

7 Consideration of Requirement for a Reaction Chain

It was originally assumed that the beamsplitter and folding mirror suspensions would require a reaction chain down to the level of the penultimate mass (also called the intermediate mass in a triple pendulum) to allow low-noise feedback. However if the reaction chain is not needed there is obvious saving on design effort. Ken Strain has carried out estimates of the noise introduced by the motion of the actuators assumed attached rigidly to the active platform; see T060157-01-K. The actuator motion is coupled into force noise acting on the intermediate mass and hence into

displacement of the optic. It is shown that using actuators consisting of LIGO1 style coils with double-length magnets (2 mm diam x 6 mm long), which would give 10mN rms force, that there is a safety margin of at least 120. If a larger actuation force is required the Birmingham design of actuator could be used. For 40mN rms force and assuming a larger offset from the sweet spot, the coupling is 4 times smaller than the allowed maximum. Further details can be found in T060157-01-K. In conclusion it appears that a reaction chain is not required and the baseline design does not include one.

8 Conclusions

We have investigated the use of a triple pendulum suspension for the beamsplitter and conclude that it appears to satisfy the noise requirements. The use of steel wires instead of silica fibres has been studied with respect to suspension thermal noise considerations and it is concluded that using steel wires in conjunction with a vertical to longitudinal coupling of 0.001 gives acceptable performance.

The latest parameter set at the time of finishing revision (02) of the document (January 2008) is given in Appendix A. However it should be noted that this is still a conceptual design. Detailed design is currently being carried out and these parameters should only be taken as a guide to the likely final set.

A solidworks rendering of the design of the triple pendulum within its support structure as currently being developed at the Rutherford Appleton Laboratory is shown in figure 7 (courtesy of Joe O'Dell). This depicts an all-metal prototype. The yellow struts are stiffeners to increase the resonant frequencies of the support structure. The magenta piece supports the OSEMs for global alignment and control at the intermediate mass.

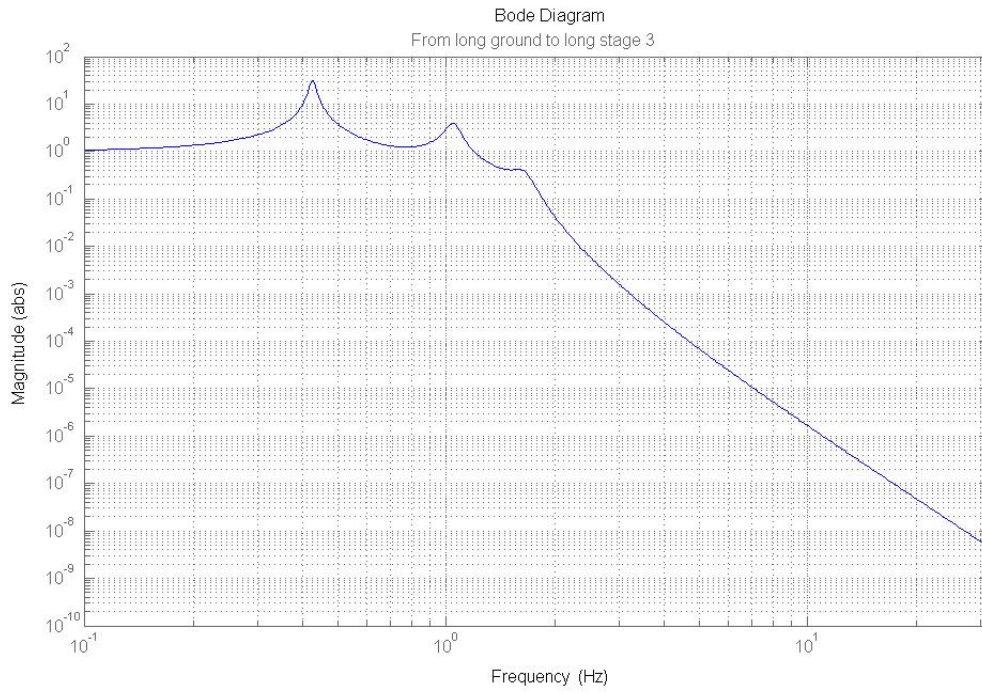


Figure 5. Horizontal (longitudinal) transfer function for beamsplitter triple suspension (with eddy current damping).

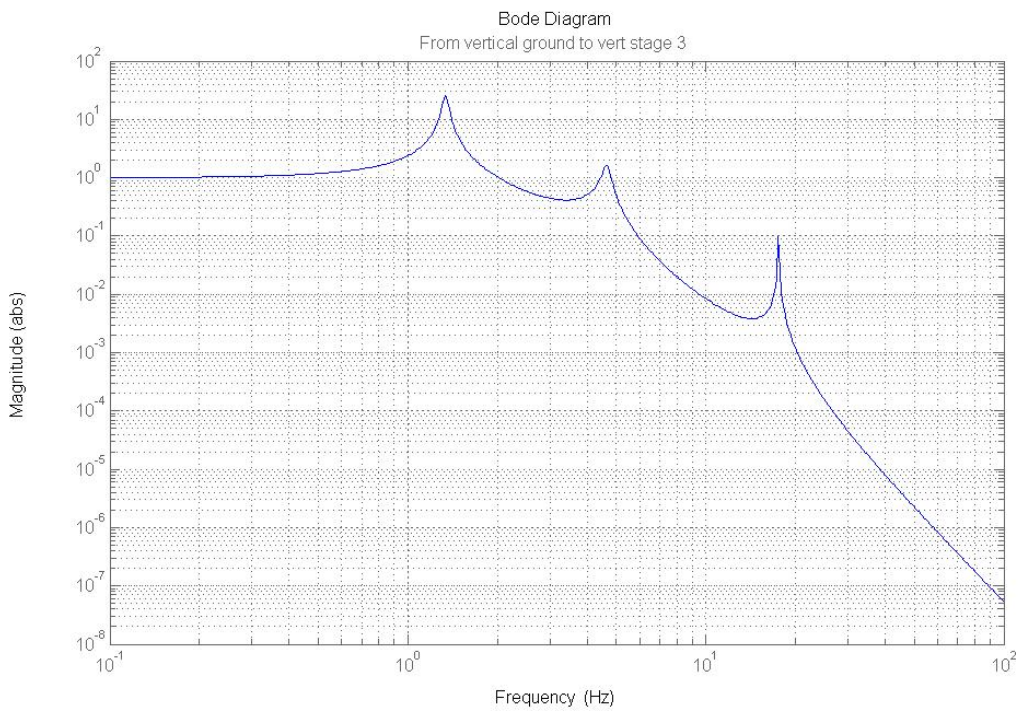


Figure 6. Vertical transfer function for beamsplitter triple suspension (with eddy current damping).



Figure 7. Solidworks rendering of beamsplitter prototype triple suspension.

Appendix A

A.1 Summary of parameters used in the MATLAB code to generate figures 5 and 6.

All numbers are in SI units.

See Appendix E for full explanation of parameter names

```

                m1: 1.2627e+001
material1: 'steel'
    I1x: 1.6350e-001
    I1y: 2.4230e-002
    I1z: 1.6190e-001
    m2: 1.3517e+001
    ix: 5.7090e-002
    ir: 1.8500e-001
    I2x: 2.3130e-001
    I2y: 1.1932e-001
    I2z: 1.1932e-001
    m3: 1.3517e+001
material3: 'silica'
    tx: 5.7090e-002
    tr: 1.8500e-001
    I3x: 2.3130e-001
    I3y: 1.1932e-001
    I3z: 1.1932e-001
    l1: 6.1200e-001
    l2: 6.1000e-001
    l3: 5.0000e-001
    nw1: 2
    nw2: 4
    nw3: 4
    r1: 3.1500e-004
    r2: 1.8700e-004
    r3: 1.2300e-004
    Y1: 2.1190e+011
    Y2: 2.1190e+011
    Y3: 2.1190e+011
    l1b: 2.5000e-001
    a1b: 6.5000e-002
    h1b: 2.4000e-003
    ufc1: 2.8087e+000
    st1: 7.7938e+008
intmode_1: 1.4457e+002
    l2b: 1.4000e-001
    a2b: 2.7556e-002
```

```

h2b: 1.6000e-003
ufc2: 3.2469e+000
st2: 7.8948e+008
intmode_2: 3.0733e+002
su: 0
si: 1.5000e-002
sl: 5.0000e-003
n0: 7.7000e-002
n1: 1.3000e-001
n2: 6.0000e-002
n3: 1.9150e-001
n4: 1.8650e-001
n5: 1.8650e-001
stage2: 1
d0: -1.8859e-003
d1: -6.9059e-004
d2: -6.9059e-004
d3: -7.1965e-005
d4: -7.1965e-005
tl1: 6.0781e-001
tl2: 5.9428e-001
tl3: 4.9986e-001
l_cofm: 1.7019e+000
l_total: 1.8869e+000
ribbon: 0
db: 0
g: 9.8100e+000
kc1: 1.9663e+003
kc2: 2.8127e+003
l_suspoint_to_centreofptic: 1.7019e+000
l_suspoint_to_bottomofptic: 1.8869e+000
flex1: 2.8859e-003
flex2: 1.6906e-003
flex3: 1.0720e-003
flex3tr: 1.0720e-003
longpitch1: [3.0258e-001 4.2554e-001 9.4183e-001]
longpitch2: [1.0410e+000 1.4855e+000 1.6624e+000]
yaw: [5.0122e-001 1.3672e+000 2.1890e+000]
transroll1: [4.2414e-001 1.0394e+000 1.5825e+000]
transroll2: [2.5848e+000 4.1031e+000 2.4986e+001]
vertical: [1.3368e+000 4.6494e+000 1.7531e+001]

```

These frequencies can be compared to those in the Mathematica model given in appendix B. The agreement is good to 4 sig. figs.

Notes

- 1) The “d” values shown above are the actual positions of the break-off points to get an “effective” “d” value of 1 mm, taking into account the flexure lengths of the wires. The longitudinal compliance of the blades has not been included in this calculation. FEA models will be required to estimate the compliance and hence the revised positioning of the wire break-off points to compensate for that compliance.
- 2) There has been discussion of increasing the first pitch mode (currently 0.3 Hz) to decrease the overall rms motion of the optic. This can be done by increasing some of the “d” values and correspondingly decreasing wire lengths (to keep the overall height the same). This is ongoing work and is not reflected in the above parameter set.
- 3) The blade parameters and vertical compliance are modeled using the opt.m routine. These should only be taken as representative and not the final design.
- 4) The top mass is represented by the parameters from the Solidworks design for this mass which has been put together by the RAL team. Ref e-mail from Joe O’Dell 17 Oct 2007.
- 5) The penultimate mass and optic are represented in the MATLAB model as being identical in mass, size and moments of inertia. In addition they are represented as symmetric cylinders. In practice
 - i) the penultimate mass will be made of metal
 - ii) the optics will be horizontally wedged as described in Appendix C.1 below. It is proposed that the penultimate mass is similarly wedged, and hung such that the wedge is oppositely oriented so that the overall loading on the blades above is symmetric.

Appendix B

Mark Barton’s Mathematica models used to generate the thermal noise curves can be found at <http://www.ligo.caltech.edu/%7ee2e/SUSmodels/> under the sidebar –follow the link to the Triple Xtra-Lite model page.

For the unwedged case the resulting mode frequencies in Hz are

"N	f	type	
1	0.3025809648892492	pitch3	pitch2
2	0.424137285679903	y3	y2
3	0.42553476024458786	x3	x2
4	0.5012213586831055	yaw3	yaw2
5	0.9417991878837583	pitch2	pitch3
6	1.0393741072077616	y2	y3
7	1.0409607596847752	x2	x3
8	1.3367465497338584	z3	z2

9	1.3672350227875962	yaw1	
10	1.4855813775299265	pitch1	
11	1.5825732377865354	y1	y2
12	1.6624534003792206	x1	x2
13	2.188980912315431	yaw2	yaw3
14	2.5848004448313247	roll1	roll3
15	4.1031558811539375	roll1	roll3
16	4.649514275151824	z1	
17	17.530495104930278	z2	z3
18	24.985132724495962	roll2	roll3"

For the horizontally wedged beamsplitter with optic parameters as described in Appendix C.1 the resulting mode frequencies in Hz are

N	f	type	
1	0.3025713527967201	pitch3	pitch2
2	0.4241368839053088	y3	y2
3	0.4255231004518705	x3	x2
4	0.5010788376001445	yaw3	yaw2
5	0.9417746185630207	pitch2	pitch3
6	1.0393734894344189	y2	y3
7	1.041104703254956	x2	x3
8	1.3367460041342083	z3	z2
9	1.367205502017105	yaw1	
10	1.4855740334487244	pitch1	
11	1.5825364214780144	y1	y2
12	1.6623505850841167	x1	x2
13	2.189237845349943	yaw2	yaw3
14	2.5846090953208374	roll1	roll3
15	4.102872628595596	roll1	roll3
16	4.649513116237302	z1	
17	17.533422108920426	z2	z3
18	24.985170357403824	roll2	roll3

The wedged case assumes a symmetric horizontal wedge on the BS, an opposite wedge on the penultimate mass, adjustments to the wires such that everything hangs straight, rotation of the structure by half the wedge angle so that the HR face is realigned with +x, and in the case of the noise at the sweet spot, alignment of the beam on the sweet spot in front of the COM.

Note that the coordinates (“type”) in the listing come from a crude mode ID function that ranks the coefficients in the eigenvector in descending order and prints coefficient names until half the total squared amplitude in the mode has been accounted for.

Appendix C: History of modifications to parameter set.

C.1 Design of beamsplitter mass

The details of diameter, thickness and wedge for the beamsplitter have evolved since the original conceptual design document was written. At the time of finalising Rev-01 (19th November 2007) RODA M070120-02 has been produced giving the design as follows: 370 mm diameter, horizontal symmetrical wedge with full wedge angle 0.9 degrees, thick end of wedge 60 mm thick, giving a mass of 13.5 kg. The mass is represented in the MATLAB model by assuming a thickness of the beamsplitter which is the average of the thin end and thick end of the wedge (Note that the MATLAB model assumes symmetry in the mass shapes). The penultimate mass has been modeled to be identical in mass and size, but will be made of metal with suitable holes to give the correct mass.

C.2 Violin Mode Frequencies and Length of Wires

The SUS group was asked by Peter F to consider shortening the length of the final stage of the suspension so that its violin mode frequency is higher than what would be obtained with the 600 mm length originally proposed. By shortening to 500 mm and allowing a stress level of ~ 710 MPa (slightly more than the working value assumed for other Adv LIGO wire suspensions of 670 MPa) the frequency is raised from ~ 240 Hz to 300 Hz. Note that the use of steel rather than silica has reduced the expected violin mode frequency due to steel's higher density.

C.3 Overall Length of Suspension

The original overall length was chosen to satisfy the available length for a beamsplitter suspension in a BSC (noting that this was at that time expected to be 70 mm longer than for an ETM) *prior* to considerations to reduce the overall length of BSC suspension structures as summarized in T040028-00. Since then the recommendations on length in T040028 have been adopted, and the decision to make the FM the same design as the BS has been taken. Since the FM must necessarily be very close to the same length as an ITM (they are adjacent to each other and the laser beam is close to horizontal), this implies that for a common BS/FM design, the choice for the length of the BS or FM is now such that the BS, FM and ITM mirror centres are the same distance from the optics table. Note that this doesn't imply that the suspension lengths will necessarily be the same. The distance between the top suspension point and the optics table above need not be the same.

Ian W at RAL has indicated that a longer pendulum length for the beamsplitter or folding mirror could be incorporated within the same overall structure length by changing the way the top blade assembly is fixed within the structure compared to how this is done in the quad. The overall length of the pendulum could be increased by 66mm. Since this in principal gives a little more isolation, it has been used in the latest parameter set. The details on length are as follows

As per the following document, the optic table to optic CL (CL = centre line) for the ETM quad suspension is 1742 mm

<http://www.eng-external.rl.ac.uk/advligo/Reviews/PDR3/documents/overview/t060142-00-k.pdf>

For the quad the length from tip of top blade to centre of optics is 1636 mm. Thus this allows $1742 - 1636 = 106$ mm as space to fit in the blade supports and mount to the table in the quad. For the beamsplitter Ian is proposing that we can mount the blade tips closer to the table by 66 mm, so that they are now only 40 mm from the table. This means that we can make the overall length of the splitter from blade tip to centre of optic be $1636 + 66 = 1702$ mm.

Appendix D: Use of Eddy Current Damping

In the current detailed design for the top mass ECD units similar to those being used in the ETM/ITM noise prototype are being incorporated. These units are arranged in clusters of 4 magnets (nominally 10 mm diam x 10 mm thick) with 4 such clusters acting in each of the longitudinal and vertical directions, arranged so that they also provide pitch, roll and yaw damping. Four clusters of four such magnets will give a damping constant of $b \sim 27$ kg/s (ref P060013-00-R). The transfer functions shown in figures 5 and 6 assume this value of b in longitudinal and vertical directions. The decay time to $1/e = 10.1$ secs for longitudinal and 4.7 secs for vertical (we may choose to reduce magnet strength for this direction).

We can estimate the thermal noise due to this damping and show that it is acceptable. The noise force at the top mass where the damping is applied is given by $F^2 = 4kTb$, where k = Boltzmann's constant and T = temperature (K).

For $b = 27$ kg/s, $F = 6.7 \times 10^{-10}$ N/rt Hz.

From the MATLAB model we find the following:

a) Longitudinal TF at 10 Hz for force at top mass to displacement of mirror, $TF(\text{long}) = 8.7 \times 10^{-10}$ m/N.

Hence longitudinal motion due to thermal noise = $F \times TF(\text{long}) = 5.8 \times 10^{-19}$ m/rtHz.

b) Vertical TF at 10 Hz for force at top mass to displacement of mirror, $TF(\text{vert}) = 1.9 \times 10^{-6}$ m/N.

Hence vertical motion due to thermal noise = $F \times TF(\text{vert}) = 1.3 \times 10^{-15}$ m/rtHz.

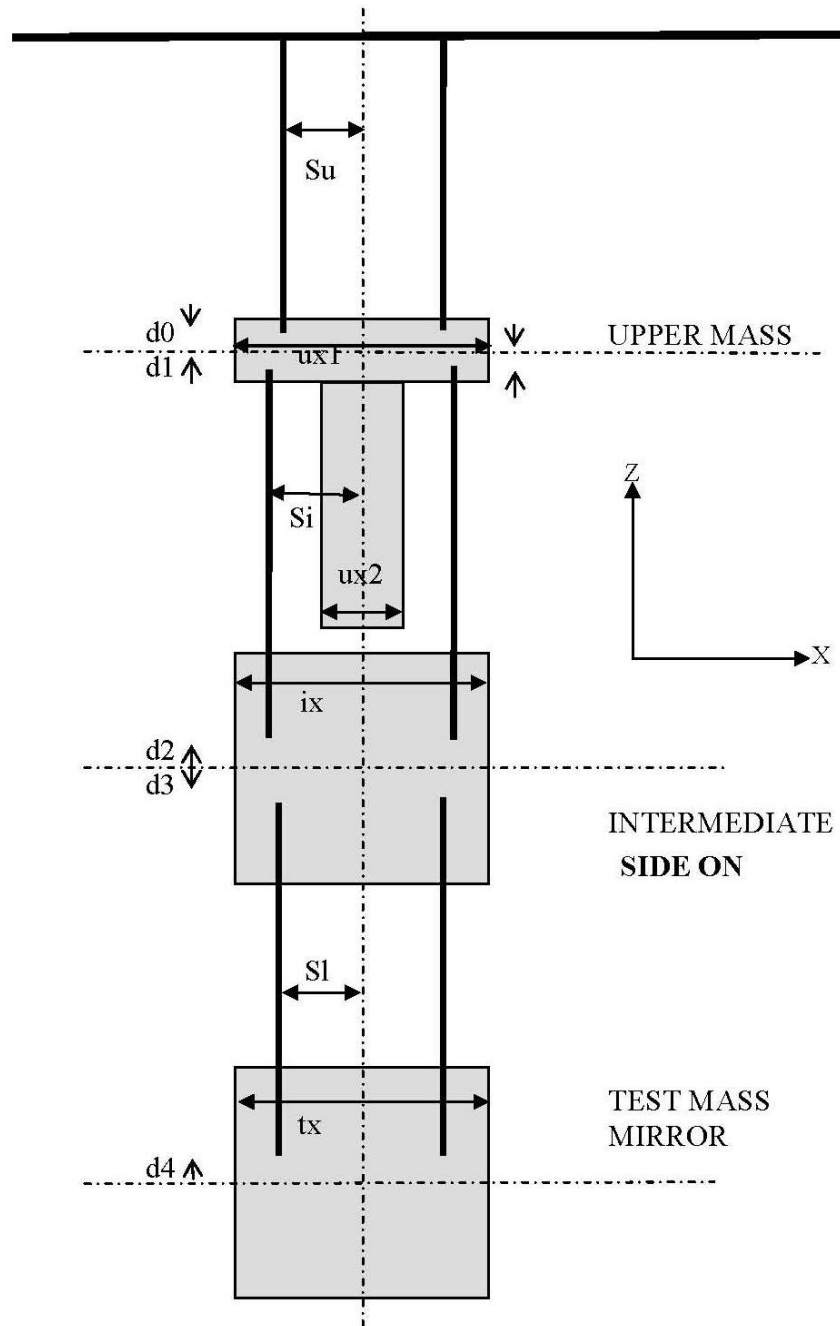
Assuming coupling of 0.1%, this gives longitudinal motion of

1.2×10^{-18} m/rt Hz.

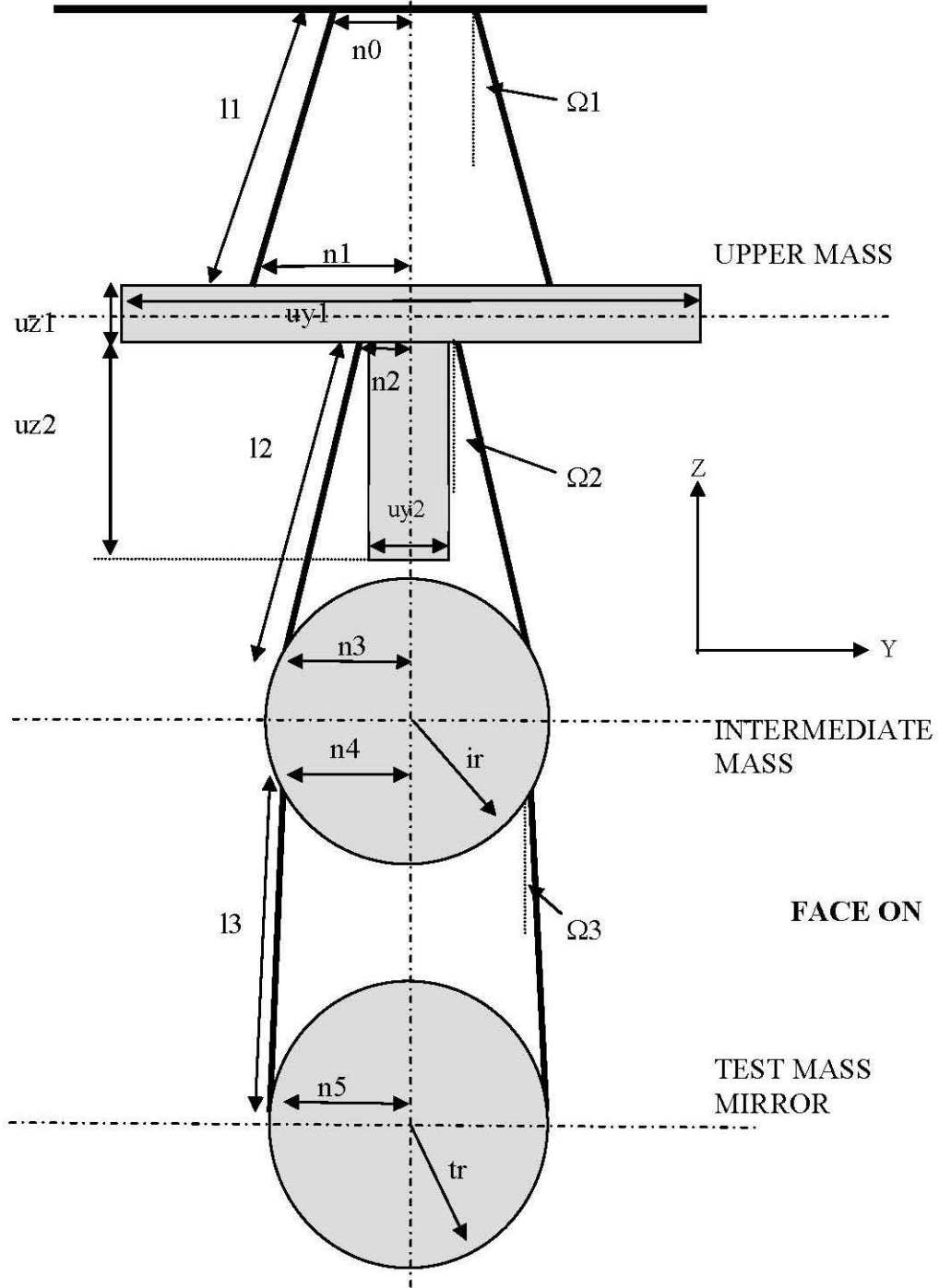
These values of longitudinal motion should be compared to the *technical* noise requirement for the beamsplitter of 2×10^{-18} m/rtHz at 10 Hz. They are both below that value.

Appendix E

E.1 The parameters of a triple pendulum (side on view)



E.2 The parameters for a triple pendulum (face on view)



E.3 Other parameters listed in appendix A.

m_1, m_2, m_3 : masses from top to bottom

I_{ix}, I_{iy}, I_{iz} where $i = 1, 2, 3$ from top to bottom mass = moments of inertia as follows

I_{ix} : moment of inertia (transverse roll)

I_{iy} : moment of inertia (longitudinal pitch)

I_{iz} : moment of inertia (yaw)

n_{wi} = number of suspension wires at each stage from top to bottom

r_i = wire radius from top to bottom

Y_i = Young's modulus of wire/fibre from top to bottom

l_{1b}, a_{1b}, h_{1b} : length, width at root, thickness of top blades

$ufc_1, st_1, intmode_1$: uncoupled frequency of top blade with mass immediately below it, stress in blade and estimated first internal mode frequency (all data returned from opt.m m-file routine)

l_{2b} etc – same as above for lower blades

stage 2 = 1

If `pend.stage2` is defined and non-zero, d_0 - d_4 are interpreted as raw values, i.e., as actual wire breakoff vertical positions

t_{l1}, t_{l2}, t_{l3} : centre to centre vertical separations at each stage - from top suspension point to centre of top mass, centre of top mass to centre of intermediate mass, and centre of intermediate mass to centre of beamsplitter optic respectively

$ribbon = 0$: round wires/fibres are used (i.e not ribbons)

$db = 0$: no natural damping included

g : accel. due to gravity

kc_1, kc_2 : blade stiffness (top and bottom respectively)

$l_suspoint_to_centrefoptic$: length from top suspension point to centre of optic = $t_{l1}+t_{l2}+t_{l3}$

$l_suspoint_to_bottomofoptic$: length from top suspension point to bottom of optic

$flex_1, flex_2, flex_3$: flexure length for wire (top to bottom respectively)

$flex_{tr}$ – flexure length for ribbon in transverse/roll direction (same as $flex_3$ if round fibre used)