

# LIGO Laboratory / LIGO Scientific Collaboration

LIGO- T040027-04-R

LIGO

12 Jan 2009

# Conceptual Design of Beamsplitter Suspension for Advanced LIGO

Norna A Robertson and Mark Barton

Distribution of this document: LIGO Science Collaboration

This is an internal working note of the LIGO Project.

California Institute of Technology LIGO Project – MS 18-34 1200 E. California Blvd. Pasadena, CA 91125 Phone (626) 395-2129 Fax (626) 304-9834 E-mail: info@ligo.caltech.edu

LIGO Hanford Observatory P.O. Box 1970 Mail Stop S9-02 Richland WA 99352 Phone 509-372-8106 Fax 509-372-8137 Massachusetts Institute of Technology LIGO Project – NW17-161 175 Albany St Cambridge, MA 02139 Phone (617) 253-4824 Fax (617) 253-7014 E-mail: info@ligo.mit.edu

LIGO Livingston Observatory P.O. Box 940 Livingston, LA 70754 Phone 225-686-3100 Fax 225-686-7189

http://www.ligo.caltech.edu/

## 1 Introduction

This is Rev-04 of the beamsplitter conceptual design document.

## History

Rev-00: 9<sup>th</sup> February 2004

This version, entitled "Design of Beamsplitter Suspension for Advanced LIGO" presented the case for making the beamsplitter suspension a triple pendulum rather than a quadruple pendulum as used for the ETMs and ITMs. A conceptual design based on the size of beamsplitter at that time (350 mm diameter by 60 mm thick) was presented and curves for predicted seismic isolation performance and thermal noise were given. It was shown that these met the noise requirements for the beamsplitter. The thermal noise curve was produced assuming that the beamsplitter was suspended by four silica fibres of circular cross-section.

Rev-01: 19<sup>th</sup> November 2007

This version reflected the fact that several factors had changed since the original document was written.

a) The beamsplitter (BS) size has been increased to 370 mm diameter x 60 mm thick. At this time it was expected to have a wedge angle of  $0.9^{\circ}$ . This diameter has been shown to have sufficient free aperture to give an acceptable level of optical loss with or without flats on the side– see G070471-00-E for information on losses with flats.

b) A reassessment of the need for silica fibres has taken place. The baseline is now to use steel wires.

c) The decision has been taken that the design of the BS and folding mirror (FM) suspensions should be the same.

Rev-02: 16 January 2008

The document has been modified to include transfer functions (from the symmetric MATLAB model) and thermal noise curves (from the Mathematica model) produced using the *same* parameter set (the current set at the time of writing) rather than slightly differing sets which had evolved over the previous few months. The thermal noise curves are presented with more easily read axes. The detailed listing of the Mathematica parameters has been replaced with a reference. The section on choice of parameters has been reduced with details moved into Appendix C. The thermal noise section has been edited.

Rev-03: April 2008

Section 4.2 added – discussion of phi value used for steel wire.

Appendix E added – diagrams and descriptions giving identification of parameters used in the MATLAB model, as listed in appendix A.

Current prototype design rendering has been included (figure 7).

Section 7 added re requirement for a reaction chain. Conclusions section renumbered as 8.

Rev-04: Jan 2009

The requirements for the beamsplitter have been revised as presented in T080192-01-D "Displacement Noise in Advanced LIGO Triple Suspensions", (M. Evans and P. Fritschel), and subsequently given in the updated Cavity Optics Suspension Subsystem Design Requirements Document, T010007-04 M. (Barton et al.). Basically the changes come from the reduction in the finesse to be used in Adv LIGO, which leads to a tighter requirement on the beamsplitter displacement noise. Section 2 is revised to reflect this, and comparisons of data to requirements are updated.

It has been discovered that the thermal noise curves presented in rev -03 were not correct. The thermoelastic noise was incorrectly estimated due to half a line of code accidentally being deleted, resulting in an overestimation of the expected noise around 10 Hz. This has been corrected for generating the revised thermal noise curve.

The parameter set has been updated to reflect several changes. The most significant, as detailed in T080267-00-R is to the blade parameters. Specifically their thicknesses have been reduced to gain more vertical isolation following a recommendation in T080192-01-D. Other changes include asbuilt masses and moments of inertia, change of wedge angle for the optic, and a change to one of the "d' values following RODA M080134-00-Y. The revised parameters are given in Appendix A. All graphs have been updated using the new parameter list.

Section 4.2 on choice of phi for producing the thermal noise curves has been updated with information from measurements made at MIT on LIGO 1 style suspensions.

Section 5.4 had been added, commenting on the need or otherwise for damping the internal modes of the blades.

Appendices A B and D have been revised with updated parameters.

## 2 Beamsplitter Requirements

The revised requirements as per T080192-01-D are as follows:

Combined longitudinal and vertical noise from all sources, assuming a coupling factor of no larger than 0.001, should be  $6.4 \times 10^{-18}$  m/ $\sqrt{\text{Hz}}$  at 10 Hz, falling to  $2 \times 10^{-19}$  m/ $\sqrt{\text{Hz}}$  at 40 Hz except for a bounce mode peak ( the highest vertical mode of the suspension). These noise requirements are incorporated in the revised Cavity Optics Design Requirements Document, T010007-05. One further point to note is that as per RODA M040006-00, the beamsplitter and folding mirror optics are identical, and we will use the same suspension design for these optics.

## 3 Choice of Parameters

The original working design which was investigated was of a triple suspension with approximately equal masses (12.7 kg for the original size of BS) and equal wire lengths of 60 cm at each stage. The choice of equal masses and equal wire lengths as a baseline has come from experience with previous designs and leads to good coupling of modes. In addition using three equal lengths gives the best isolation for a given overall length. For various reasons (available length, change in size of the optic, consequences of changing from silica fibres to steel wires) this original design has been modified. The current parameter list is given in appendix A, and details on the history and reasons for changes are given in Appendix C.

## 4 Suspension Thermal Noise

4.1 Thermal noise estimate using steel wire and wedged optic

In the 2004 design it was shown that a final stage of the suspension consisting of 4 silica fibres of circular cross-section, 140 micron radius (stress ~500 MPa) and 60 cm length comfortably met the noise requirement (see rev-00 for more details). Silica was chosen as the baseline design. However this decision has since been revisited. There are compelling reasons to use steel wire if it gives acceptable performance: its use gives a significant reduction in complexity of design and construction.. It is found that with the use of steel wires and a coupling factor of 0.001 from vertical to longitudinal motion, the thermal noise estimate just meets the noise requirements at 10 Hz and above, except for the highest vertical peak which is at 17.5 Hz. At 10 Hz the value of the total thermal noise is  $4.9 \times 10^{-18}$  m/ $\sqrt{\text{Hz}}$  compared to the requirement of  $6.4 \times 10^{-18}$  m/ $\sqrt{\text{Hz}}$ , and at 40 Hz the total thermal noise  $1.9 \times 10^{-19}$  m/ $\sqrt{\text{Hz}}$  compared to requirement of  $2 \times 10^{-19}$  m/ $\sqrt{\text{Hz}}$ . The longitudinal noise dominates except at the highest vertical peak. See figure 1 below. The main parameters which affect the noise level are the wire loss, taken as  $2 \times 10^{-4}$ , the bottom wire diameter 250 µm and the bottom wire length of 0.50 m. These graphs have been produced assuming a horizontal wedge of value 0.05 degrees, the current value at time of writing (Jan 09). However the presence or absence of a wedge this small has very little effect on the noise level. These graphs have been produced using Mark Barton's Mathematica model of the beamsplitter, see Appendix B for further details.



Figure 1. Thermal noise for BS on steel wires, parameters as referenced in Appendix B. Longitudinal and vertical/1000 noise estimates are shown separately and summed quadratically. Also shown is the overall noise requirement as per T080192-01-D.

#### 4.2 Value of phi for steel wire suspension

The value of the intrinsic loss (phi) in the wire assumed for these curves is  $2x10^{-4}$  (ref G Cagnoli et al Phys Lett A 255, p 230, 1999). Recent work by Penn, Harry, Evans, Weiss et al has shown that whereas the intrinsic phi for steel music wire may be even better than this at ~ $6x10^{-5}$  (see G080108-00-Z), the design of break-off bars at the mirror as used in LIGO1 gives higher loss and variability. More repeatable and better results have been obtained using a double prism design of break-off, or a clamp, see refs T080270, P080083. We are advocating that we pursue the double prism technique (sapphire prism with laser ablated groove and smaller steel prism below) for these suspensions. This is more fully described in T080266-03. Rai Weiss (e-mail to NAR 24 June 2008) has estimated a  $\phi$  value from the MIT experiments, and deduced a value of 5.7 x 10<sup>-4</sup> with the double prism approach. He believes that since the clamp and the double prism gave almost identical loss at 330 Hz, it is most likely not the loss in the wire but rather an additional loss in the setup which is the reason for the discrepancy between this number and measurements of the intrinsic  $\phi$  for the steel wire. Our conclusion is that we cannot guarantee that a suspension using the double prism technique will yield a  $\phi$  of 2x10<sup>-4</sup>, but that using this technique is the best approach.

## 5 Seismic Isolation, Mode Frequencies and Damping

#### **5.1 Transfer Functions**

The longitudinal and vertical transfer functions derived from the MATLAB model of the beamsplitter for the parameter set given in appendix A are shown in figures 2 and 3. The mode frequencies are also given in the appendix. The damping has been chosen to give a decay time to 1/e of approximately 10 secs in each direction. The damping control function (to be found with the MATLAB model) is a simplified version of that used in the GEO suspensions, and consists of a low pass, a high pass and two transitional differentiators.

#### 5.2 Residual Seismic Noise.

In figure 4 we show the expected residual seismic noise using information on the requirements for the BSC\_ISI in the MATLAB file bsc\_seismic.m posted on the seismic wiki page at

http://ilog.ligo-wa.caltech.edu:7285/advligo/BSC\_Noise\_Curves

We have assumed the optical layout is such that the vertical to longitudinal coupling is no larger than 0.001. It can be seen that apart from the highest vertical peak the residual seismic noise lies well below the beamsplitter noise requirement shown in green in the figure. This means that if we combine the seismic and thermal noise the thermal noise dominates and essentially gives the limiting noise performance.

#### 5.3 Other Noise Sources

Using the MATLAB model we can also estimate the magnitude of pitch and yaw contributions. The larger of these transfer functions at 10 Hz is for yaw, at ~6 x 10<sup>-6</sup>. Assuming an angular input at the platform of around 3 x 10<sup>-13</sup> rad/ $\sqrt{\text{Hz}}$  (guesstimate based on the longitudinal requirement of 3 x 10<sup>-13</sup> m/ $\sqrt{\text{Hz}}$  over a ~1m baseline) and a 1mm beam offset we find a longitudinal noise level of ~1.8 x 10<sup>-21</sup> m/ $\sqrt{\text{Hz}}$  at 10 Hz, negligible compared to the requirement.

A further consideration is that of noise introduced by local control. A combination of steep electronic filtering and some eddy current damping (ECD) should yield a workable solution. In fact ECD could comfortably be used without any active control for some modes, and ECD is being

incorporated into the design. It has been checked that the thermal noise associated with using ECD is below the noise requirement for the beamsplitter – see Appendix D.



Figure 2. Longitudinal transfer function for beamsplitter triple suspension.



Figure 3. Vertical transfer function for beamsplitter triple suspension.

### 5.4 Blade Internal Noise

It has been checked to see if the transmissibility at the peaks of the internal modes of the blades is low enough that those modes do not require to be damped. The conclusion is that they do not need to be damped. See T080229-00-R, which used input from FEA analysis by Justin Greenhalgh (ref. T060295-00-K). We note however that these calculations were done for the blade parameters prior to making them thinner to increase vertical isolation. These documents will be updated in due course. However the conclusion is not expected to change.

## 6 Consideration of Requirement for a Reaction Chain

It was originally assumed that the beamsplitter and folding mirror suspensions would require a reaction chain down to the level of the penultimate mass (also called the intermediate mass in a triple pendulum) to allow low-noise feedback. However if the reaction chain is not needed there is obvious saving on design effort. Ken Strain has carried out estimates of the noise introduced by the motion of the actuators assumed attached rigidly to the active platform; see T060157-01-K. The actuator motion is coupled into force noise acting on the intermediate mass and hence into displacement of the optic. It is shown that using actuators consisting of LIGO1 style coils with double-length magnets (2 mm diam x 6 mm long), which would give 10mN rms force, that there is a safety margin of at least 120. If a larger actuation force is required the Birmingham design of actuator could be used. For 40mN rms force and assuming a larger offset from the sweet spot, the coupling is 4 times smaller than the allowed maximum. Further details can be found in T060157-01-K. In conclusion it appears that a reaction chain is not required and the baseline design does not include one. More details on the electronics requirements and design are given in the next section. In particular it ahs been concluded that the Birmingham actuators will be used at the intermediate mass.

## 7 Electronics Design

The responsibility for designing the electronics for the beamsplitter suspensions is shared between the UK and the US, where the UK are responsible for the analogue sensing and actuation sections of the electronics and the US for the digital electronics and the anti-alias, anti-image, whitening and dewhitening functions. In addition the UK is responsible for the design and production of BOSEMs – Birmingham OSEMs used at the top mass and intermediate mass of the beamsplitter. The electronics requirements are spelt out in T080065-E-C, and are the result of modeling done by Peter Fritschel and Matt Evans documented at:

http://ilog.ligo-wa.caltech.edu:7285/advligo/TripleSuspensionActuation

For the beamsplitter, the actuation (local control) at the top mass uses 10mm diam. x 10mm thick magnets, whereas at the middle mass smaller magnets 10m diam. by 5mm thick will be used.





Figure 4. Residual seismic noise at beamsplitter, obtained by combining BSC ISI noise requirement as referenced in text and MATLAB transfer functions for the beamsplitter.

#### 8 Conclusions

We have investigated the use of a triple pendulum suspension for the beamsplitter and conclude that it appears to satisfy the noise requirements. The use of steel wires instead of silica fibres has been studied with respect to suspension thermal noise considerations and it is concluded that using steel wires in conjunction with a vertical to longitudinal coupling of 0.001 gives acceptable performance.

The latest parameter set at the time of producing rev 04 (Jan 2009) is given in Appendix A.

A solidworks rendering of the design of the triple pendulum within its support structure as currently being developed at the Rutherford Appleton Laboratory (RAL) is shown in figure 5 (courtesy of Joe O'Dell). This depicts an all-metal prototype. The yellow struts are stiffeners to increase the resonant frequencies of the support structure. The magenta piece supports the BOSEMs for global alignment and control at the intermediate mass. A picture of the all metal prototype constructed at RAL is shown in figure 6.



Figure 5. Solidworks rendering of beamsplitter prototype triple suspension.



Figure 6. Picture of beamsplitter prototype triple suspension at RAL.

## Appendix A

A.1 Summary of parameters used in the MATLAB code to generate figures 2, 3 and 4.

A zipped file of the suite of MATLAB files used to generate the figures can be found on the BS/FM wiki at

http://ilog.ligo-wa.caltech.edu:7285/advligo/BS\_Suspension#preview

called BSmodel Nov08.zip

All numbers are in SI units.

See Appendix E for full explanation of parameter names

m1: 12.6210 material1: 'steel' I1x: 0.1659 I1y: 0.0247 I1z: 0.1643 m2: 13.5750 ix: 0.0571 ir: 0.1850 I2x: 0.2592 I2y: 0.1298 I2z: 0.1359 m3: 14.1678 material3: 'silica' tx: 0.0598 tr: 0.1850 I3x: 0.2424 I3y: 0.1255 I3z: 0.1255 11: 0.6120 12: 0.6015 13: 0.5000 nw1:2 nw2:4 nw3:4 r1: 3.1250e-004 r2: 2.0000e-004 r3: 1.2500e-004 Y1: 2.1190e+011 Y2: 2.1190e+011 Y3: 2.1190e+011 11b: 0.2500 a1b: 0.0625

h1b: 0.0022 ufc1: 2.4200 12b: 0.1400 a2b: 0.0258 h2b: 0.0015 ufc2: 2.8400 su: 0 si: 0.0150 sl: 0.0050 n0: 0.0770 n1: 0.1300 n2: 0.0600 n3: 0.1915 n4: 0.1865 n5: 0.1865 stage2: 1 d0: -0.0018 d1: -9.0695e-004 d2: 0.0081 d3: -8.1371e-005 d4: -8.1371e-005 tl1: 0.6079 tl2: 0.5941 tl3: 0.4998 1 cofm: 1.7019 1 total: 1.8869 ribbon: 0 db: 0 g: 9.8100 kc1: 1.4590e+003 kc2: 2.1613e+003 1 suspoint to centreofoptic: 1.7019 1 suspoint to bottomofoptic: 1.8869 flex1: 0.0028 flex2: 0.0019 flex3: 0.0011 flex3tr: 0.0011 longpitch1: [0.4197 0.4875 1.0418] longpitch2: [1.0574 1.3873 1.6926] yaw: [0.4893 1.3737 2.1329] transroll1: [0.4229 1.0501 1.5706] transroll2: [2.2647 3.5001 24.3392] vertical: [1.1496 4.0733 17.5565]

These frequencies can be compared to those in the Mathematica model given in appendix B. The agreement is good to 4 sig. figs.

#### Notes

1) The "d" values shown above are the actual positions of the break-off points to get an "effective" "d" value of 1 mm in general, taking into account the flexure lengths of the wires. The exception is d2, which has been changed to have an effective value of 10 mm as per RODA M080134-00.

2) The transverse compliance of the blades has not been included in the model. An FEA model by Justin Greenhalgh (see T080133-01-K) shows that the compliance at the lower blades reduces the effective d1 value by  $\sim 0.5$  mm. This has no significant change on the overall behaviour of the suspension, which is dominated in pitch by the large value of d2, as shown in appendix 2 of T080133-01-K.

## Appendix B

Information on Mark Barton's Mathematica models used to generate the thermal noise curves can be found at <u>http://www.ligo.caltech.edu/%7ee2e/SUSmodels/</u>

under the sidebar -follow the link to the Triple Xtra-Lite model page.

Further details can be obtained from Mark.

```
Ν
   f
                       type
1
   0.4197064546468998
                       x3 x2
2
   0.42291451734783164 y3 y2
3
   0.487544928995774
                      pitch3
   0.4892626712349226 yaw3
4
                              yaw2
5
   1.041766207820019
                      pitch2
                              pitch1 x2 pitch3
б
   1.0501394645456619
                      y2 y3
7
   1.0573672808267283 x2 x1
                              x3 pitch2
   1.1495570387919678 z3 z2
8
9
   1.373689215442516
                       yaw1
10 1.3873454645496484 pitch1 pitch2
11 1.5705901528468342
                      y1 y2
                       x1 x2
12
   1.6926304156287817
13
   2.1328930868735827
                       yaw2
                              yaw3
14 2.264669656786472
                              roll2
                       roll1
                                      roll3
                      roll1
                              roll3 roll2
15 3.5001315975043594
16 4.073351237984953
                       z1
17 17.556625377542254 z2 z3
18 24.33930022521283
                       roll3
```

Note that the coordinates ("type") in the listing come from a crude mode ID function that ranks the coefficients in the eigenvector in descending order and prints coefficient names until half the total squared amplitude in the mode has been accounted for.

## Appendix C: History of modifications to parameter set.

### C.1 Design of beamsplitter mass

The details of diameter, thickness and wedge for the beamsplitter have evolved since the original conceptual design document was written. At the time of finalising Rev-01 (19<sup>th</sup> November 2007) RODA M070120-02 has been produced giving the design as follows: 370 mm diameter, horizontal

symmetrical wedge with full wedge angle 0.9 degrees, thick end of wedge 60 mm thick, giving a mass of 13.5 kg. The mass was represented in the MATLAB model by assuming a thickness of the beamsplitter which is the average of the thin end and thick end of the wedge (Note that the MATLAB model assumes symmetry in the mass shapes). Note that the wedge at time of writing this (04) revison is now 0.05 degrees.

#### C.2 Violin Mode Frequencies and Length of Wires

The SUS group was asked by Peter F to consider shortening the length of the final stage of the suspension so that its violin mode frequency is higher than what would be obtained with the 600 mm length originally proposed. By shortening to 500 mm and allowing a stress level of  $\sim$  710 MPa (slightly more than the working value assumed for other Adv LIGO wire suspensions of 670 MPa) the frequency is raised from  $\sim$  240 Hz to 300 Hz. Note that the use of steel rather than silica has reduced the expected violin mode frequency due to steel's higher density.

#### C.3 Overall Length of Suspension

The original overall length was chosen to satisfy the available length for a beamsplitter suspension in a BSC (noting that this was at that time expected to be 70 mm longer than for an ETM) *prior* to considerations to reduce the overall length of BSC suspension structures as summarized in T040028-00. Since then the recommendations on length in T040028 have been adopted, and the decision to make the FM the same design as the BS has been taken. Since the FM must necessarily be very close to the same length as an ITM (they are adjacent to each other and the laser beam is close to horizontal), this implies that for a common BS/FM design, the choice for the length of the BS or FM is now such that the BS, FM and ITM mirror centres are the same distance from the optics table. Note that this doesnt imply that the suspension lengths will necessarily be the same. The distance between the top suspension point and the optics table above need not be the same.

Ian W at RAL has indicated that a longer pendulum length for the beamsplitter or folding mirror could be incorporated within the same overall structure length by changing the way the top blade assembly is fixed within the structure compared to how this is done in the quad. The overall length of the pendulum could be increased by 66mm. Since this in principal gives a little more isolation, it has been used in the latest parameter set. The details on length are as follows

As per the following document, the optic table to optic CL (CL = centre line) for the ETM quad suspension is 1742 mm

http://www.eng-external.rl.ac.uk/advligo/Reviews/PDR3/documents/overview/t060142-00-k.pdf.

For the quad the length from tip of top blade to centre of optics is 1636 mm. Thus this allows 1742-1636 = 106 mm as space to fit in the blade supports and mount to the table in the quad. For the beamsplitter Ian is proposing that we can mount the blade tips closer to the table by 66 mm, so that they are now only 40 mm from the table. This means that we can make the overall length of the splitter from blade tip to centre of optic be 1636 + 66 = 1702 mm.

## Appendix D: Use of Eddy Current Damping

In the current detailed design for the top mass ECD units similar to those being used in the ETM/ITM noise prototype are being incorporated. These units are arranged in clusters of 4 magnets (nominally 10 mm diam x 10 mm thick) with 4 such clusters acting in each of the longitudinal and vertical directions, arranged so that they also provide pitch, roll and yaw damping. Four clusters of four such magnets will give a damping constant of  $b \sim 27$  kg/s whe the magnets are fully positoned within the Cu blovck (ref P060013-00-R). The decay time to 1/e = 10.2 secs for longitudinal and 4.6 secs for vertical. We may choose to reduce magnet strength for this direction-see below.

We can estimate the thermal noise due to this damping. The noise force at the top mass where the damping is applied is given by  $F^2 = 4kTb$ , where k = Boltzmann's constant and T = temperature (K).

For b = 27 kg/s,  $F = 6.7 \text{ x } 10^{-10} \text{ N/rt Hz}$ .

From the MATLAB model we find the following:

a) Longitudinal TF at 10 Hz for force at top mass to displacement of mirror, TF(long) =  $9.0 \times 10^{-10} \text{ m/N}$ .

Hence longitudinal motion due to thermal noise =  $F \times TF(long) = 6.0 \times 10^{-19} \text{ m/rtHz}.$ 

b) Vertical TF at 10 Hz for force at top mass to displacement of mirror,  $TF(vert) = 1.4 \times 10^{-6} \text{ m/N}$ .

Hence vertical motion due to thermal noise =  $F \ge TF(vert) = 9.4 \ge 10^{-16} \text{ m/rtHz}$ . Assuming coupling of 0.1%, this gives longitudinal motion of 9.4 x 10^-19 m/rt Hz.

These values of longitudinal motion should be compared to the *technical* noise requirement for the beamsplitter of 6.4 x  $10^{-19}$  m/rtHz at 10 Hz. The longitudinal damping noise is below the requirement. The vertical is above this requirement if we use b = 27 kg/s. However, as seen above, this damping gives a decay time of less than 5 seconds. The technical noise requirement can be met with a b value approximately 12.5, which gives a decay time of ~ 9.4 secs. Such a b value could be achieved by moving the magnets back from the full recessed position within the Cu block, or simply by using smaller magnets.

#### LIG0

## Appendix E

E.1 The parameters of a triple pendulum (side on view)





E.2 The parameters for a triple pendulum (face on view)

### E.3 Other parameters listed in appendix A.

m1, m2, m3: masses from top to bottom

Iix, Iiy, Iiz where i = 1,2,3 from top to bottom mass = moments of inertia as followsIix: moment of inertia (transverse roll)Iiy: moment of inertia (longitudinal pitch)Iiz: moment of inertia (yaw)

nwi = number of suspension wires at each stage from top to bottom

ri = wire radius from top to bottom

Yi = Young's modulus of wire/fibre from top to bottom

11b, a1b, h1b: length, width at root, thickness of top blades

ufc1, st1, intmode\_1: uncoupled frequency of top blade with mass immediately below it, stress in blade and estiamted first internal mode frequency ( all data returned from opt.m m-file routine)

12b etc – same as above for lower blades

```
stage 2 = 1
```

**LIGO** 

If pend.stage2 is defined and non-zero, d0-d4 are interpreted as raw values, i.e., as actual wire breakoff vertical positions

tl1, tl2, tl3: centre to centre vertical separations at each stage - from top suspension point to centre of top mass, centre of top mass to centre of intermediate mass, and centre of intermediate mass to centre of beamsplitter optic respectively

ribbon = 0: round wires/fibres are used (i.e not ribbons)

db = 0: no natural damping included

g: accel. due to gravity

kc1, kc2: blade stiffness (top and bottom respectively)

l\_suspoint\_to\_centreofoptic: length from top suspension point to centre of topic = tl1+tl2+tl3

l\_suspoint\_to\_bottomofoptic: length from top suspension point to bottom of optic

flex1, flex2, flex3: flexure length for wire (top to bottom respectively)

flextr – flexure length for ribbon in transverse/roll direction (same as flex3 if round fibre used)