LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO- T070304-00-R *LIGO* 18th December 2007

Vibration Transmission to OMC Bench via Cabling

Norna A Robertson

Distribution of this document:
LIGO Science Collaboration

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project – MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW17-161
175 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

1 Introduction

The output modecleaner (OMC) for Enhanced and Advanced LIGO consists of a silica bench with optics and electronics attached, suspended as the lower mass of a double pendulum. The original baseline design for the electronic wiring from the bench to the outside world was to route the cabling up the suspension chain, attaching it to the top mass of the double pendulum before taking it to the OMC support structure and hence to the vacuum chamber feedthroughs. See figure 1. However transfer function measurements and alignment tests showed that attaching the cabling to the top mass was significantly affecting the dynamics by introducing cross-coupling and was affecting the DC alignment of the bench. It was also significantly damping the motion. See entries 93, 99, 102 and 105 in the SUS elog at

http://dziban.ligo.caltech.edu:40/SUS_Lab

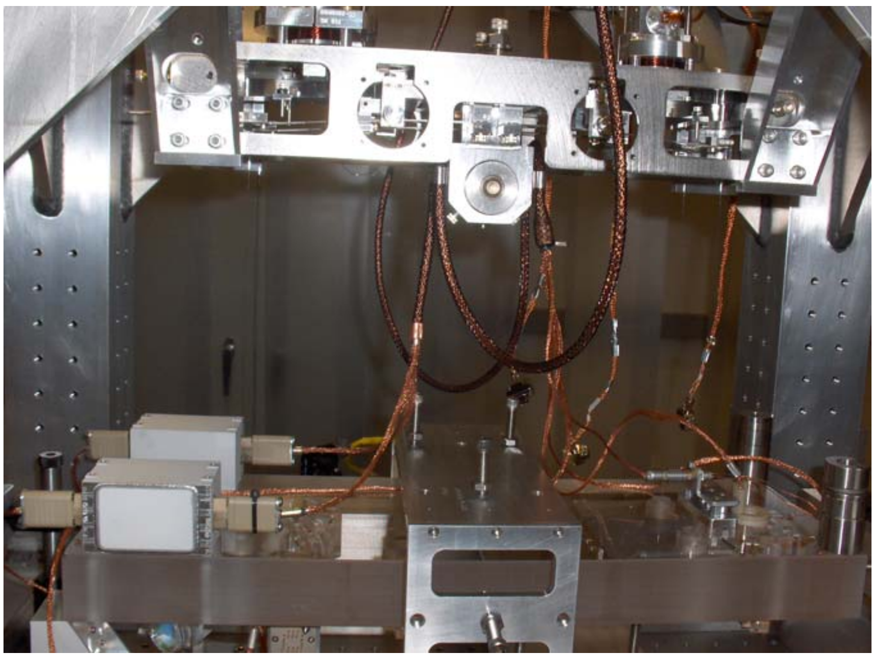

Figure 1 Original concept for wiring of OMC bench. Note the black peek shielding in the upper sections of the two cabling bundles, one bundle from the two preamp boxes on the left and the other from the QPDs etc on the right side.

It was therefore decided to change the baseline in two ways.

1) remove the relatively stiff black peek insulating shield from the cabling,

2) take the wiring directly from the bench to the top of the structure.

Figure 2 shows a mock-up of one set of cabling in the new configuration.

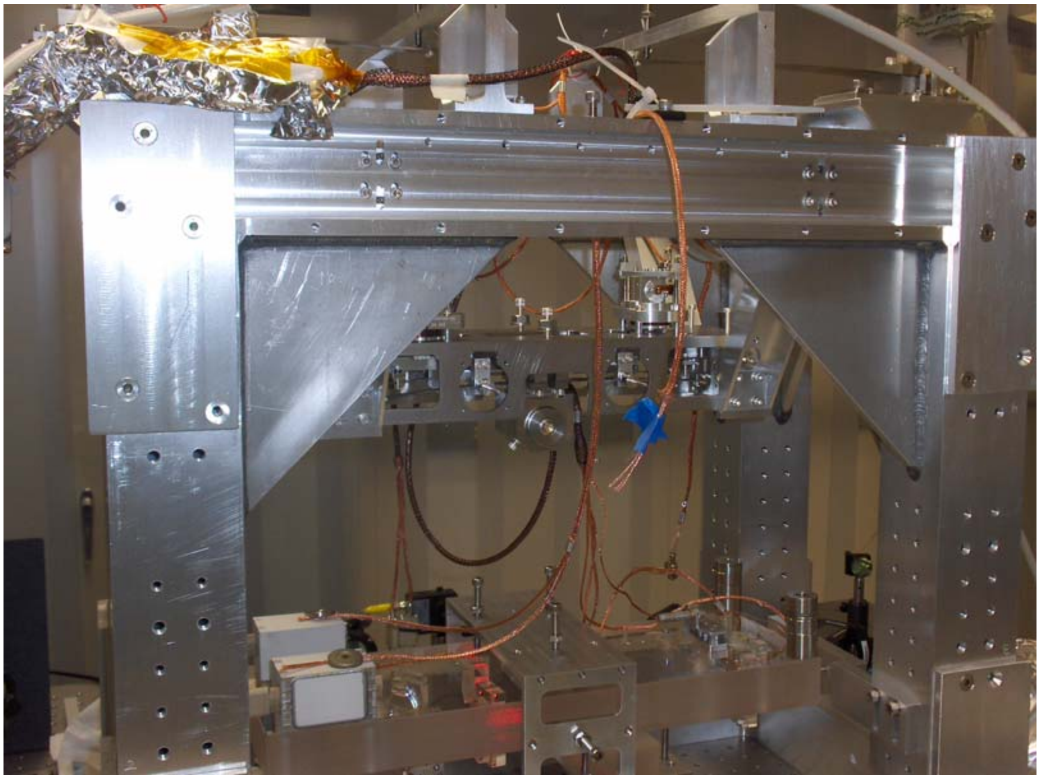

Figure 2. Concept for revised wiring – see cables shown on left hand side. The top end is attached to the top crossbar of the structure and the bottom ends are weighted down on the preamp boxes to simulate the electronic connection while mechanical transfer functions measurements were made. The other cable was removed from the top mass and laid on the optical bench when measurements were made.

2 Effect of revised cabling on Isolation

Dennis Coyne carried out a preliminary assessment of the effect of the stiffness of cabling on the isolation based on the modeling done for acoustic coupling in HAM-SAS in T060038-00-D and some experimental values for the bending stiffness of a sample of cable taken by Bob Taylor, see

http://ilog.ligo-wa.caltech.edu:7285/advligo/OMC_Suspension

at the end of the design section. In this analysis it was shown that assuming the Q of the cable is low, the isolation of the OMC double pendulum does not appear to be compromised below ~ 30 Hz, above which frequency the isolation would fall off more slowly than for a double pendulum. This looked acceptable. However the modeling assumed there were 2 cables in total, each with one twisted pair of wires in a Cu shield. In fact there are significantly more wires in the current OMC design – a total of 36 which will be bundled in two cables, one from each end of the optics bench. It thus seems prudent to carry out further investigations of the possible mechanical shorting effect of the cabling.

3 Experiments with a simple pendulum

3.1 Experimental set-up

A simple set-up was investigated as a preliminary experiment to see if the cabling effect on isolation could be directly measured. See Figure 1. A simple pendulum was made using a plastic egg filled with coins (~ 80 gm) secured with silly putty. The suspension wire was trapped in the egg and suspended at the top using a large spring clip. The periods in the x and y directions were measured by timing 30 swings, where the x direction was in plane of the figure and y direction into the plane. A mock-up piece of cabling was then added as shown in Figure 1. This cable contained 8 teflon coated wires surrounded by 1/8 inch (inner diam.) Cu braid. One end was attached to the mass and the other was trapped at the edge of a drawer. The periods were remeasured in both directions. By using simple theory the equivalent spring constant of the wire was found for x and y directions. The Q factor of the pendulum with the cabling in place was also estimated by timing the decay of the amplitude to 1/e of its initial value, using a piece of paper on the floor beneath the pendulum as a guide. All measurements were taken several times to get an idea of the repeatability and magnitude of errors.

Figure 1. Experimental set-up for simple pendulum investigations

3.2 Simple pendulum theory

We model the system as shown in figure 2. The equation of motion for horizontal motion in the plane of the page is

$$M\ddot{x}_1 = -\frac{Mg}{L}\left(x_1 - x_0\right) - \frac{k_c}{M}\left(x_1 - x_0\right) - \frac{b}{M}\left(\dot{x}_1 - \dot{x}_0\right) \qquad \text{Equation 1}$$

where x_1 is the motion of the mass and x_0 the motion of the ground.

We have ignored the damping due to air. The transfer function x_1/x_0 is given by

$$\frac{x_1}{x_0} = \frac{\frac{bs}{M} + \left(\omega_0^2 + \frac{k_c}{M}\right)}{s^2 + \frac{bs}{M} + \left(\omega_0^2 + \frac{k_c}{M}\right)} \qquad \text{Equation 2}$$

where $\omega_0^2 = g/L$.

This represents a damped simple pendulum, whose angular frequency, ω_{new} is given by

$$\omega_{new} = \left(\omega_0^2 + \frac{k_c}{M}\right)^{1/2} \qquad \text{Equation 3}$$

And whose quality factor is given by

$$Q = \frac{\omega_{new} M}{b} \qquad \text{Equation 4}$$

The amplitude of the pendulum will decay exponentially with a decay time, τ, to 1/e given by

$$\tau = \frac{2Q}{\omega_{new}} = \frac{2M}{b} \qquad \text{Equation 5}$$

By making measurements of the frequency of the pendulum with and without the cabling attached, the value of the spring constant k_c can be found using equation 3. By making measurements of the decay time of the pendulum with the cabling attached (and assuming the damping due to the cabling is much larger than the damping due to air) the damping constant b can be found using equation 5.

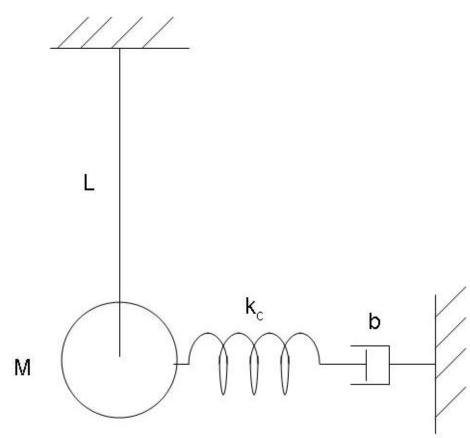

Figure 2. Model of simple pendulum, mass M, length L, with cable attached, where cable consists of spring k_c plus damper b.

3.3 Results

See table 1.

Data taken 7th and 10th Dec 2007 NAR

single wire x	no. swings	time (s)		period, T (s)	w^2 = (2pi/T)^2	L (calc)	Kc/M	Kc (N/m)	decay time	Q	b (kg/s)
	30	36.38					(=w^2-g/L)		(periods)		
	30	36.4									
	30	36.22									
	30	36.2									
		36.3	average	1.210	26.964	0.364					
		35.7	10th Dec	1.190	27.878	0.352					
single wire y	30	36.22									
	30	36.44									
		36.25									
		36.13									
		36.26	average	1.209	27.024	0.363					
		35.8	10th Dec	1.190	27.878	0.352					
with cable x	15	17.28									
	15	17.41									
	15	17.44									
	15	17.41									
		17.385	average	1.159	29.390		2.425	0.203	9	28	0.01603
with cable x	10	10.19									
repeat with more (7)	10	10.22									
wires trapped	10	10.22									
Dec 10th		10.21	average	1.021	37.871		9.993	0.835	4	13	0.04094
with cable y	30	36.25									
	30	36.06									
		36.1									
		36.13									
		36.135	average	1.205	27.211		0.187	0.016	70	220	0.00198
repeat with more	30	35.13									
wires Dec 10th	30	35.34									
y direction	30	35.19									
		35.22	average	1.174	28.643		0.765	0.064	41	129	0.00347

M w/cable 0.0912 kg
cable 0.0076
M 0.0836

M 4 quarters, 6 dimes, 3 nickels 3 pennies
plus silly putty in plastic egg

pendulum L approx (not incl egg) 33.7 cm (7th Dec numbers)
cable L approx 36 cm (7th Dec numbers)

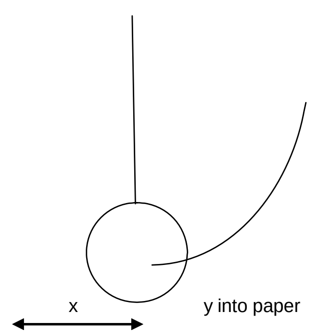

Table 1: results from measurements of simple pendulum

We can see from the results that the value of k_c in the y direction is considerable less (~ one order of magnitude) than in the x direction. The *b* value is similarly an order of magnitude less. This can be attributed to the way the cabling is "dressed' such that it is lying in the x-z plane. The experiment was done in two ways, firstly with one Teflon wire attached to the pendulum mass then with 7 (out of the 8) attached. As expected, with more wires attached there was more affect on the frequency and damping, with the values for k_c and b increasing by factor of ~4. Note that we don't necessarily expect a linear effect since only the attachment at the mass was changed and not the attachment at the other end of the cable. We wish to extrapolate these results firstly to having 9 wires attached to the mass (which represents one quarter of the total wiring in the OMC). A conservative extrapolation to 9 wires would be 9/7 * result for 7 wires. Similarly the b value changed more slowly than linearly with number of teflon wires attached. Again taking 9/7 of the result for 7 teflon wires is conservative. Doing this we arrive at the following estimates:

x direction: Cable bundle with 9 teflon wires in 1/8 inch Cu braid: k_c = 1.1 N/m, b = 0.051 kg/s

y direction: Cable bundle with 9 telfon wires in 1/8 inch Cu braid: k_c = 0.082 N/m, b = 0.0045 kg/s

To scale these results to 4 such bundles, as will be used in the OMC, we multiply by factor of 4.

x direction: OMC cabling: K_c = 4.4 N/m, B = 0.20 kg/s

y direction: OMC cabling: K_c = 0.33 N/m, B = 0.018 kg/s

4 Modelling of OMC double pendulum with electronic cabling

4.1 Theory

We have put together a simple model of the OMC with electronic cabling as shown in figure 3.

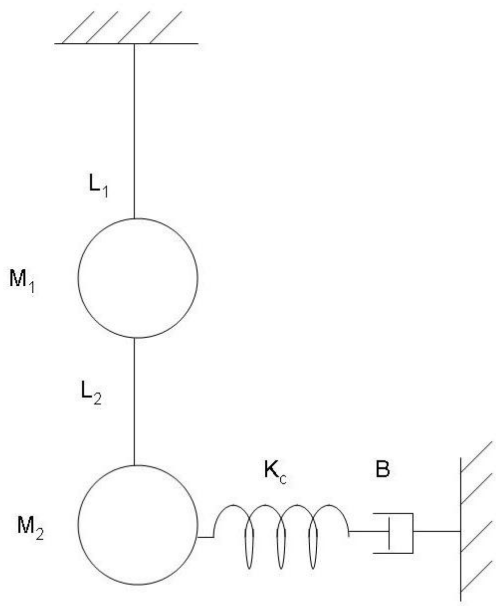

Figure 3 Model of OMC double pendulum with electronic cabling attached.

The values for M_1, M_2, L_1 and L_2 are taken as 2.8 kg, 7 kg, 0.25 m and 0.25 m respectively, which represent the current values for the OMC design.

The equations of motion for masses M_1 and M_2 are as follows:

$$M_1 \ddot{x}_1 = -\frac{(M_1 + M_2)g}{L_1}(x_1 - x_0) + \frac{M_2 g}{L_2}(x_2 - x_1) \qquad \text{Equation 6}$$

$$M_2\ddot{x}_2 = -\frac{M_2 g}{L_2}(x_2 - x_1) + K_c(x_2 - x_0) - B(\dot{x}_2 - \dot{x}_0) \qquad \text{Equation 7}$$

From these equations we can calculate the transfer function x_2/x_0. We have written a MATLAB m-file to plot the magnitude of x_2/x_0 as a function of frequency – see Appendix A. The form of the transfer function, TF is as follows:

TF = num/den where num has s^3, s^2, s^1 and s^0 terms and den has s^4, s^3, s^2, s^1 and s^0 terms, and TF is given by the following MATLAB code:

```
num=[B/M2,Kc/M2,(B/M2)*((1+M2/M1)*(g/L1)+(M2/M1)*(g/L2)),s0]
```
Equation 8

```
den=[1,B/M2,((1+M2/M1)*(g/L1)+(1+M2/M1)*(g/L2)+Kc/M2),(B/M2)*((1+M2/M1)*(g/L1)+
(M2/M1)*(g/L2)),s0]
```
Equation 9

where s0 is given by

```
s0 =(Kc/M2)*((1+M2/M1)*(g/L1)+(M2/M1)*(g/L2))+(1+M2/M1)*(g/L1)*(g/L2
```
Equation 10

When Kc and B are zero, the numerator only has an s^0 term, and the denominator leading term is s^4, as expected for a double pendulum with two resonant frequencies and no natural damping. Above the two resonant frequencies the isolation falls as $1/s^4$ i.e. as $1/f^4$.

Consider firstly the addition of the spring constant Kc, but no damping. The addition of the spring constant Kc introduces an s^2 term in the numerator. This means that there is a zero in the transfer function at a certain frequency, which can be understood as the frequency at which the forces on M_2 from the suspension and from the cabling are equal and opposite in magnitude. Below that frequency the isolation falls off as $1/f^4$. Above that frequency the isolation falls off at $1/f^2$. With the further addition of damping from the cabling (non-zero B), terms in s^3 and s are added to the numerator. Thus above a certain frequency the TF falls only as s^3/s^4, i.e. as 1/f.

4.2 Application of theory using estimated values for Kc and B

We now apply this to the results from section 3. Firstly consider the x direction. The resultant transfer functions are shown in figure 4. Three curves are shown. The solid curve is the case with no cabling. The dashed curve is for the values K_c = 4.4 N/m, B = 0.20 kg/s, as deduced in section 3. The dotted curve shows the effect of making B negligible, so that the position of the zero can be seen. We see that the isolation starts to be compromised around 10 Hz. Further we see that the damping is in fact the stronger effect, in that the frequency at which the isolation becomes 1/f occurs *below* the zero seen in the dotted curve.

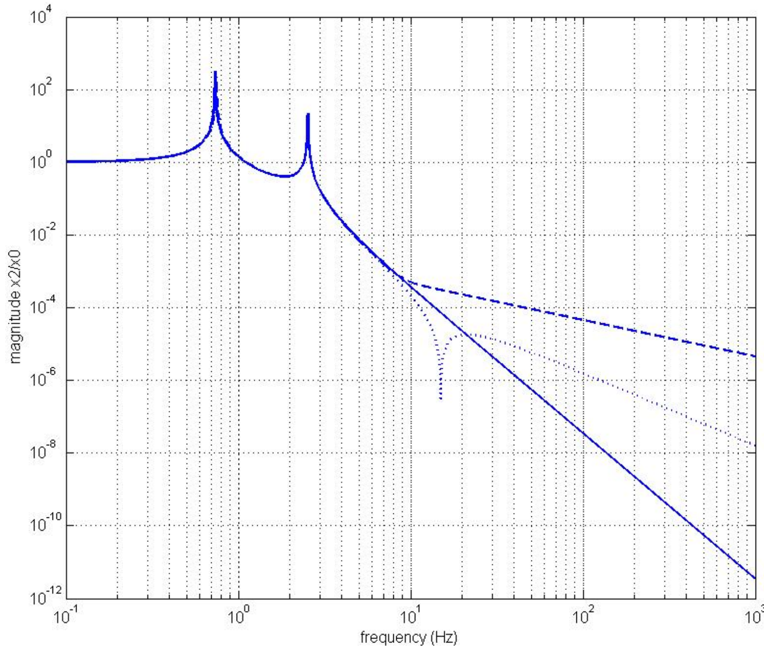

Figure 5. Transfer function for double pendulum with and without electronic cabling attached, for x direction (in plane of cabling). Solid line – without cabling, dotted line – with cabling (spring constant only), dashed line – with cabling (both spring constant and damping included).

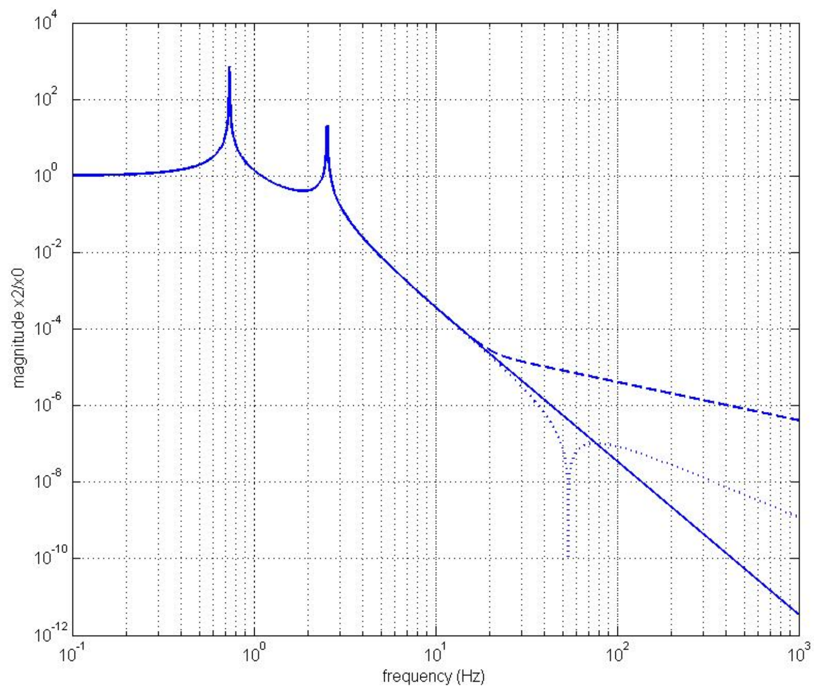

Figure 5. Transfer function for double pendulum with and without electronic cabling attached, for y direction (out of plane of cabling). Solid line – without cabling, dotted line – with cabling (spring constant only), dashed line – with cabling (both spring constant and damping included).

Now consider the y direction. See figure 5. The dashed curve is for values K_c = 0.33 N/m, B = 0.018 kg/s. For this direction the isolation starts to be compromised around 20 Hz, and again it is the effect of the damping which is more significant.

We note that the frequency at which the damping starts to affect the transfer function occurs when the s^3 term (equal to s^3*b/M2) in the numerator starts to dominate over the s^0 term. This frequency varies as the inverse cube root of the damping constant B. Thus, for example, if the damping were 8 times larger, the frequency at which the isolation starts to be compromised is only a factor of 2 lower.

4.3 Isolation values and comparison to requirements

4.3.1 x direction

At 10 Hz without cabling TF = 3.7×10^{-4} (isolation 2.7×10^{3}); with cabling TF = 5.0×10^{-4} (isolation 2.0×10^{3})

At 70 Hz without cabling TF = 1.4×10^{-7} (isolation 7.1×10^{6}); with cabling TF = 6.5×10^{-5} (isolation 1.5e $\times 10^{4}$)

4.3.2 y direction

At 10 Hz without cabling TF = 3.7×10^{-4} (isolation 2.7×10^{3}); with cabling TF = 3.6×10^{-4} (isolation 2.8×10^{3})

At 70 Hz without cabling TF = 1.4×10^{-7} (isolation 7.1×10^{6}); with cabling TF = 5.8×10^{-6} (isolation 1.7×10^{5})

We can compare these numbers to an estimate of the required isolation – ref e-mail from Dennis Coyne (see Appendix B). Note that estimating the isolation requirement for the OMC is not straightforward, and so this should only be taken as a guide. The number quoted there for 70 Hz is an isolation of 150,000, i.e. 1.5×10^{5}. We see that this is achieved in the y direction but not in the x direction by an order of magnitude. However we note that the most sensitive direction of motion for the OMC is that of the input beam to the OMC – which is perpendicular to the long axis of the optics bench. Fortunately the dressing of the cables is likely to make them lie predominantly in the plane parallel to the long axis. Thus the "y' direction for the OMC is the sensitive direction and this is the one for which the isolation is less affected by the cable and for which the isolation meets the requirement.

5 Conclusions

We have taken data from a simple pendulum experiment and used it to estimate the effect of the electronic cabling on the isolation of the OMC using a simple model of the dynamics. This analysis suggests that for the sensitive direction the isolation still meets requirements. We note however that several approximations have been made to arrive at this conclusion. One factor for example which has not been taken into account was that the cable used in the above experiments was approximately half the length of the ones which will be used in the OMC. In addition the OMC cables are not simply four separate bundles, each of 9 teflon covered wires inside 1/8 inch Cu braid, as assumed at the end of section 3. In fact from each end of the OMC bench two such 9-wire bundles are brought together approximately half way along the total length of that cable to form one 18 wire bundle, shielded by ¼ inch inner diameter Cu braid. The ¼ inch braid is not as much as twice as heavy as the 1/8 inch braid* so that the total mass is not doubled for double the length. It is not clear how to scale Kc and B with length and mass to use in the simplified analysis above. For a simple spring, a longer spring would in fact have a smaller Kc, but this is not a simple spring. The damping is likely to be increased with a longer cable but the relationship may not be linear. If we conservatively assume an increase in B by a factor of 2, we note that will only decrease the frequency at which the isolation starts to be compromised by the cube root of 2 (1.3). In conclusion it would be prudent to repeat the experiment using a better engineered experimental set-up and a cable more closely representative of the ones which will be used in the OMC to increase confidence in the results presented here.

*(10 ft of 1/8 inch braid weighs 9.8 gm and 10ft of ¼ inch braid weighs 13.2 gm.)

Appendix A

double_pend_TFs.m

```
clear;
%look at effect of electronic cabling on isolation of OMC
%compare "simple" double pendulum in one degree of freedom with and without
extra spring
%and damper between bottom mass and the ground
% NAR 3rd December 2007

% set up parameters as for OMC
L1= 0.25; %top length
L2 = 0.25; %bottom length
g=9.81;
M1 = 2.8; %top mass
M2 = 7; %bottom mass

freq =logspace(-1,3,2000);
w=2*pi*freq;

%"simple" double pendulum with no damping

num1 = [(1+M2/M1)*(g/L1)*(g/L2)];
den1 = [1,0,((1+M2/M1)*(g/L1)+(1+M2/M1)*(g/L2)),0,(1+M2/M1)*(g/L1)*(g/L2)];

spring1tf = tf(num1,den1);
Y1= freqresp(spring1tf,w);

loglog(freq,abs(squeeze(Y1)));
hold on;


%"simple" double pendulum with no damping plus extra spring and damper to
%ground to simulate electronic cabling

%extra parameters needed
Kc = 0.33; %cable spring constant. Set the value as required
B = 0.018; %cable damping constant. Set the value as required


%s0 term
s0 =(Kc/M2)*((1+M2/M1)*(g/L1)+(M2/M1)*(g/L2))+(1+M2/M1)*(g/L1)*(g/L2);

num2=[B/M2,Kc/M2,(B/M2)*((1+M2/M1)*(g/L1)+(M2/M1)*(g/L2)),s0];
den2=[1,B/M2,((1+M2/M1)*(g/L1)+(1+M2/M1)*(g/L2)+Kc/M2),(B/M2)*((1+M2/M1)*(g/L1)
+ (M2/M1)*(g/L2)),s0];


spring2tf = tf(num2,den2);
Y2= freqresp(spring2tf,w);
```

```
loglog(freq,abs(squeeze(Y2)),'--');
grid;

% redo with different damping constant
B = B/100000; % see the effect with and without damping

%s0 term
s0 =(Kc/M2)*((1+M2/M1)*(g/L1)+(M2/M1)*(g/L2))+(1+M2/M1)*(g/L1)*(g/L2);

num3=[B/M2,Kc/M2,(B/M2)*((1+M2/M1)*(g/L1)+(M2/M1)*(g/L2)),s0];
den3=[1,B/M2,((1+M2/M1)*(g/L1)+(1+M2/M1)*(g/L2)+Kc/M2),(B/M2)*((1+M2/M1)*(g/L1)
+ (M2/M1)*(g/L2)),s0];



spring3tf = tf(num3,den3);
Y3= freqresp(spring3tf,w);

loglog(freq,abs(squeeze(Y3)),':');

hold off
```

Appendix B

E-mail from Dennis Coyne

X-Sieve: CMU Sieve 2.2
Delivered-To: nornar@stanford.edu
X-Sender: coyne@acrux.ligo.caltech.edu
X-Mailer: QUALCOMM Windows Eudora Version 6.1.0.6
Date: Thu, 27 Jul 2006 14:30:47 -0700
To: Peter Fritschel <pf@ligo.mit.edu>, Norna Robertson <nornar@stanford.edu>
From: Dennis Coyne <coyne@ligo.caltech.edu>
Subject: OMC isolation requirements
X-Spam-Score: undef - Sender Whitelisted (coyne@ligo.caltech.edu: Mail from user authenticated via SMTP AUTH allowed always)
X-Canit-Stats-ID: 4030446 - e4a0a2c1bc29
X-Scanned-By: CanIt (www . roaringpenguin . com) on 131.215.115.14

Peter & Norna,
Daniel has suggested that calculation of the isolation requirements for the OMC is difficult (due to the need to calculate the effect of vibration/jitter on the propagating mode structure of the OMC). He suggests that we simply scale our requirements from the test results with the GEO OMC used on H1 (see for example, Keita's presentation, G040326-00). While we suspect (know?) that most of the OMC noise is due to acoustic coupling to the OMC assembly through the air, we should assume that acoustic, vibration transmission through the supporting structure might be roughly equal in amplitude. Under the (likely conservative) assumption that ground vibration dominated in the OMC H1 test, we can set the vibration isolation requirements for the OMC in-vacuum as follows:

M = 200, the measured OMC noise floor ratio to the InL SRD at the minimum (150 Hz) (see pg 6 of G040326-00)
E = 2, the Enhanced LIGO factor of sensitivity improvement (at 150 Hz) compared to InL SRD (see pg. 29 of the Enhanced LIGO proposal, T060156-01)
A > 10, say 15, the Advanced LIGO factor of sensitivity improvement, broadly from ~70 Hz to 300 Hz, assuming one can reduce the coating thermal noise) compared to the InL SRD (see pg. 7 of the AdL Reference Design Document, M060056-06 and pg 14 of the NSF technical review talk, G060226-00)
T = 10, technical noise factor of safety (single noise source contribution should be at 1/10 amplitude of desired noise floor)

F = M*A*T = 30k, the required isolation factor at ~70Hz for the AdL OMC
Of course this also assumes that the OMC tested in H1 was exposed to the ground motion (i.e. no isolation or amplification of the ground motion spectrum by the support structure and optical table).

In order to determine the OMC suspension isolation requirements, consider the isolation afforded by the single stage HAM SEI system. From figure 3, pg 5 of the HAM revised (single stage) isolation requirements, T060075-00, the required isolation is ~50 at 10 Hz. Figure 1 of the "LIGO

Observatory Environment" (T010074-03), and Figures 6-1 through 6-5 of "Ambient Ground Vibration Measurements at the Livingston LIGO Site" (C961022-A), indicate that the ground noise continues to fall beyond 10 Hz at ~1/f^2. One expects that the passive component of the single stage SEI system gives a similar 1/f^2 roll-off in isolation performance. Hence the isolation for a *rigid* SEI platform, at 70 - 300 Hz, should be the same as at 10 Hz, i.e. an isolation factor of 50. However, the SEI structure will have elastic modes with moderately high Qs starting at ~100 Hz which will spoil the isolation. I think that this is (in part) the reason the SEI team proposed a flat performance above 30 Hz at 1.3e-11 m/rHz, on pg 4 of "Single Stage HAM for Advanced LIGO: Performance Modeling" (G060190-00). This flat performance level implies (or allows) an amplification of ground noise by the SEI system in-band near the AdL noise spectrum minimum (70 - 200 Hz). At ~70 Hz, the isolation by the HAM SEI might be Fhsei = 0.2 (i.e. amplification by a factor of 5).

Consequently the isolation required by the AdL OMC SUS is
Fsus = F/Fhsei = 150k at 70 Hz
If we use a single pendulum suspension with ~1 Hz frequencies, then the isolation will be ~(70/1)^2 = 5k, or inadequate. A double suspension should give an isolation of ~(70/1)^4 = 24e6, or more than adequate. Alignment control is also easier using a double suspension (marionette). Note that I have not accounted for the effect of the blade spring modes (which are likely to start ~100 Hz) in reducing the vertical isolation performance. However, a horizontal ring cavity should be less sensitive to vertical motion.

If we were to employ SAS technology, with a single 30 mHz inverted pendulum and a single *layer* of 30 mHz GAS filters, the isolation should be ~80 dB, or ~10k and not adequate. One could add a pendulum stage, but then the mechanical & control complexity is roughly comparable to the double suspension.

BTW, for Enhanced LIGO, F = M*E*T where T might be set to 5 instead of 10, then F = 2k. If there were no HAM seismic isolation (or amplification), then a single pendulum (including vertical spring isolation) or a SAS (single IP & 3 GAS filters) could adequately isolate the OMC optical bench. This begs the question as to whether components in HAM6, other than the OMC readout chain, need some isolation.

Anything grossly wrong with my arguments?
Dennis