LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T0810007-v2 *LIGO* November 27, 2008

Effect of ITM thermal lens in beam reflection

Hiro Yamamoto

Distribution of this document:
LIGO Science Collaboration

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project – MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW17-161
175 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

1 Introduction

For the tuning of the eLIGO OMC, a field reflected by a cold ITMX is used. The effect of the thermal lens in the ITM on the beam reflection from the AR side is studied. It is shown that the curvature of the reflected field by ITM HR coating depends on the thermal state of the ITM, and the curvature of the reflected field by a cold ITM can be half of that reflected by an optimally heated ITM.

2 Lens effect in vacuum

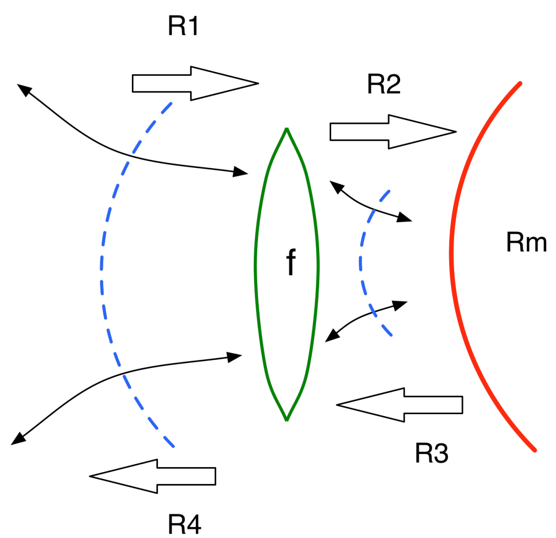

Figure 1 Reflection with a lens in the path

In the simple optical system in vacuum shown above, when a converging beam with ROC R1 (<0) goes through a lens with f (>0 for convergence lens), reflected by a mirror with curvature Rm (>0), the curvature of the reflected field R4 (>0) is given by the following equation.

$$
\begin{aligned}
\frac{1}{R2} &= \frac{1}{R1} - \frac{1}{f} \\
\frac{1}{R3} &= \frac{1}{R2} + \frac{2}{Rm} \\
\frac{1}{R4} &= \frac{1}{R3} - \frac{1}{f} \\
\Rightarrow \frac{1}{R4} &= \frac{1}{R1} - \frac{2}{f} + \frac{2}{Rm}
\end{aligned} \tag{1}
$$

The beam curvature convention is that the curvature is positive for the diverging beam. When f = ∞ and R1 = -Rm, R4 = Rm. I.e., the converging beam is reflected to become a diverging beam.

As this formula shows, the beam curvature is affected by the lens placed in front of a mirror.

3 iLIOG and eLIGO optimal thermal state

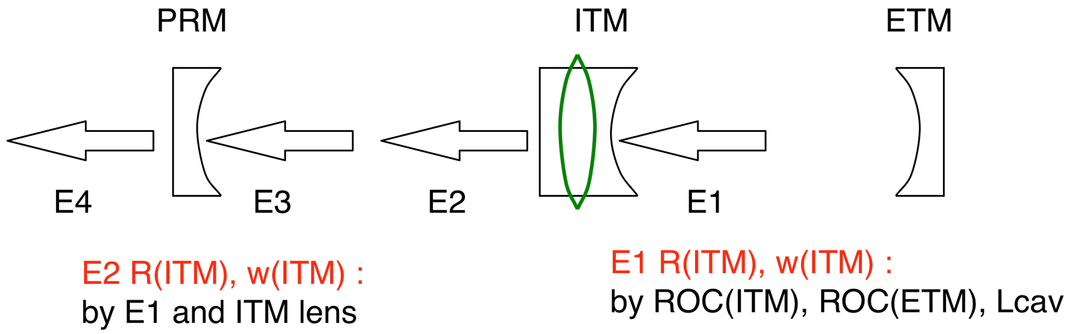

Figure 2 Mode matching

The curvatures of the initial LIGO core optics, hence the same for eLIGO, were chosen

(1) without assuming any dynamical thermal compensation system, and

(2) assuming certain power absorptions in ITM.

The curvatures of PRM, ITM and ETM cannot be changed. Only adjustable quantity is the thermal lens in ITM. The optimal mode matching means the following.

(1) the arm defines a mode defined by ROC(ITM) and ROC(ETM)

(2) the field with this mode, E1, goes out of ITM, E2, and the curvature of this field on HR side of PRM, E3, should match with ROC(PRM). ITM thermal lens should be adjusted as such.

(3) E3 goes out of PRM to E4. This defines the mode of the input beam.

The lens effect of ITM is characterized by the f number of ITM:

$$\frac{1}{f_{ITM}} = -\frac{n-1}{R_{ITM}} + \frac{1}{f_{TL}} \qquad (2)$$

The first part is the intrinsic lens effect of a mirror with a concave surface, and the second one represents the extra lens effect dominated by thermal deformation.

Because ROC(PRM) is almost identical to ROC(ITM), ROC(E1) and ROC(E3) should be almost the same for the optimally mode matched system. That means f_{ITM} should be very large, or the optimal lens effect f_{TL} should be

$$\frac{1}{f_{TL-optimal}} = \frac{n-1}{R_{ITM}} \qquad (3)$$

4 ITM thermal effect and the reflected field curvature

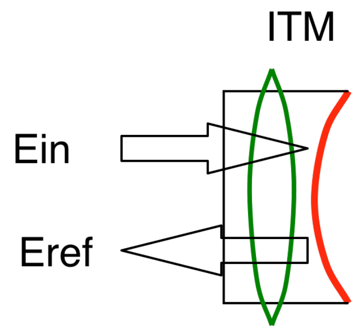

Figure 3 Reflection by ITM with thermal lens

The curvature of the reflected field is given as follows.

$$\frac{1}{R_{ref}} = \frac{1}{R_{in}} - \frac{2}{f_{TL}} + \frac{2n}{R_{ITM}} \qquad (4)$$

For the specific case that R_{in}=-R_{ITM}, which should be the case that the input beam is properly setup to match with the optimally heated IFO, this becomes

$$\begin{aligned}\frac{1}{R_{ref}} &= \frac{2n-1}{R_{ITM}} - 2\Delta\frac{1}{f_{TL-optimal}} \\ &= \frac{1}{R_{ITM}}(2n-1-2\Delta(n-1)) \qquad (5) \\ &= \begin{cases} \frac{1}{R_{ITM}} & (\Delta = 1) \\ \frac{2n-1}{R_{ITM}} & (\Delta = 0) \end{cases}\end{aligned}$$

Here, Δ is the level of the thermal lens effect. Δ=0 corresponds to no thermal lens and Δ=1 to optimal thermal state. When optimally heated, Δ=1, the curvature of the reflected field matches with the curvature of PRM. Here after, this optimally hearted state is called hot state.

Without the thermal effect, Δ=0, the curvature becomes R_{ITM} / 1.9 using n = 1.45, almost half of the curvature with optimally heated ITM.

5 CR vs SB or the role of the resonating arm

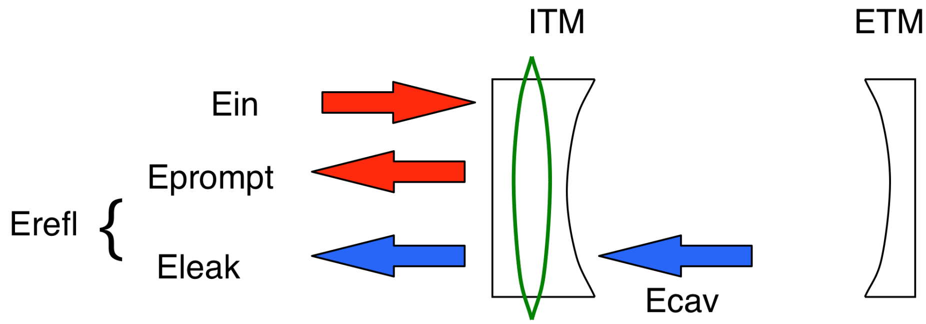

Figure 4 Reflection by a cavity

The reflected field by a cavity is consisted of two components. The incident field, Ein, is reflected by the HR side of ITM and reflected back, marked Eprompt in the figure above. Part of the incident field goes into the cavity. This field, Ecav, leaks out of the cavity and becomes the part of the reflected field, Eleak in the figure above.

When the field is resonating in the cavity, Eleak ≈ -2 Ein, while Eprompt ≈ Ein. So the reflected field is Erefl ≈ - Ein. When the field is not resonating in the arm, Eleak << Ein and hence Erefl ≈ Ein. This is a result based on a scalar field approximation.

The curvature of Eleak is give by the same formula as the 3rd relation in Eq(1).

$$\begin{aligned}\frac{1}{R_{leak}} &= \frac{1}{R_{cav}} - \frac{1}{f_{ITM}} \\ &= \frac{1}{R_{ITM}} + \frac{n-1}{R_{ITM}} - \frac{1}{f_{TL}} \\ &= \frac{n}{R_{ITM}} - \frac{1}{f_{TL}}\end{aligned} \quad (6)$$

For the resonating field, the reflected field can be written as:

$$\begin{aligned}&E_{ref} = E_{prompt} + E_{leak} \\ &\approx E_{in}(\exp(-i\frac{kr^2}{2R_{prompt}}) - 2\exp(-i\frac{kr^2}{2R_{leak}})) \\ &\approx E_{in}(-1 - i\frac{kr^2}{2}(\frac{1}{R_{prompt}} - \frac{2}{R_{leak}})) \\ &= E_{in}(-1 - i\frac{kr^2}{2}((\frac{1}{R_{in}} - \frac{2}{f_{TL}} + \frac{2n}{R_{ITM}}) - 2(\frac{n}{R_{ITM}} - \frac{1}{f_{TL}}))) \\ &= E_{in}(-1 - i\frac{kr^2}{2}\frac{1}{R_{in}}) \\ &\approx -E_{in}\exp(+i\frac{kr^2}{2R_{in}})\end{aligned} \quad (7)$$

This shows that the curvature of the reflected field is the same as that of the input field, only with the sign being changed. I.e., the converging beam becomes diverging beam with the same curvature independent of the lens effect of ITM. This is an argument keeping only the leading order effect, and the loss due to the mode mismatch is neglected. Contrary to the resonating field, the curvature of the promptly reflected field alone, or the field which does not resonant in the arm, does depend on the lens effect of ITM as is shown in eq.(4) and (5).

A similar result can be derived by using a modal model, which contains extra information. The coupling of two mode bases, one with the mode base parameter (radius of curvature, beam size) = (R1, w) and the other (R2, w) is dependent on the following parameter.

$$\alpha = \frac{kw^2}{4}\left(\frac{1}{R2} - \frac{1}{R1}\right) \qquad (8)$$

For the incoming field, Ein, and the cavity mode, this value on ITM becomes

$$\begin{aligned} \alpha &= \frac{kw^2}{4}\left(\frac{1}{R_{in}} + \frac{n-1}{R_{ITM}} - \frac{1}{f_{TL}} + \frac{1}{R_{ITM}}\right) \\ &= \frac{kw^2}{4R_{ITM}}(-\beta + n - 1 - \Delta(n-1) + 1) \\ &= \frac{kw^2}{4R_{ITM}}(n_{eff} - \beta) \\ &= 0.15(n_{eff} - \beta) \\ R_{in} &\Rightarrow -R_{ITM}/\beta,\ f_{TL} \Rightarrow f_{TL-optimal}/\Delta, \\ n_{eff} &= n - (n-1)\Delta \end{aligned} \qquad (9)$$

LLO cavity parameters are used to calculate the first factor as an example of eLIGO value. N_{eff} is the effective refractive index of ITM, and it is n (=1.45) at cold state ($\Delta = 0$) and it is 1 at hot state ($\Delta = 1$).

The input beam curvature matches with the ITM curvature when $\beta = n_{eff}$.

Using this parameter α, the mixing of two modes can be written as follows.

$$\begin{aligned} M_{00\leftrightarrow 00}(\alpha) &= \frac{1}{1+i\alpha} \\ M_{00\leftrightarrow 02}(\alpha) &= \frac{-i\alpha}{\sqrt{2}(1+i\alpha)^2} \end{aligned} \qquad (10)$$

The power loss of the TEM00 mode due to the curvature mismatch is $O(\alpha^2)$, i.e., O(1%). As an example, the following figure shows the cavity power loss in % due to the curvature mismatch, calculated using SIS simulation packaged based on FFT calculation.

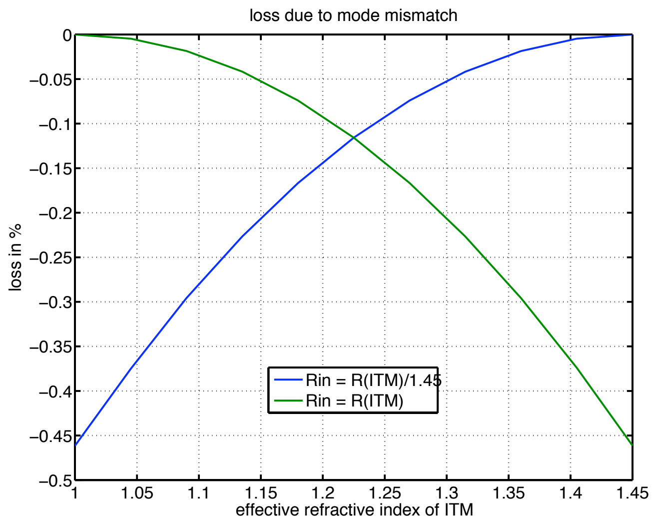

Figure 5 Cavity power loss vs mode mismatch

The blue line is the case that the input beam curvature is R_{ITM}, i.e., $\beta = 1$, and the green one curvature $\beta = 1.45$, and the horizontal axis is the effective refractive index of ITM, n_{eff}. Each case has the maximal power when $n_{eff} = \beta$. As can be seen from this plot, the loss due to the curvature mismatch is ~0.5% when the ITM state changes from cold to hot, or when the input beam curvature changes from R_{ITM} to R_{ITM} / 1.45.

When a field of TEM00 mode with mode parameter (R_{in}, w), where w is the beam size of the cavity mode on ITM, goes to a FP cavity, the reflected fields are written as follows.

$$\begin{aligned} E_{non-resonant} &= \frac{1}{1+i\alpha} E_{00} - \frac{i\alpha}{\sqrt{2}(1+i\alpha)^3}(E_{20}+E_{02}) + O(\alpha^2) \\ E_{resonant} &= -\frac{1}{1+i\alpha} E_{00} + O(\alpha^2) \end{aligned} \tag{12}$$

Here, E00, E20 and E02 are TEMnm modes with ROC = -Rin and the beam size on ITM is w, i.e., reflected mode of the input beam. The reflected field has the same curvature as the input field if the field is resonating in the cavity, while the field has a different curvature because it has additional TEM02 and TEM20 mode with complex amplitude.

An important thing is that the reflection amplitude of the resonating field is complex, and the phase is O(α), i.e., O(10%). Because of that, it may be easy to achieve the almost full power in each arm, the locking of the entire system with a high recycling gain of the entire system could be achieved only by making two α's for each arm to very small.

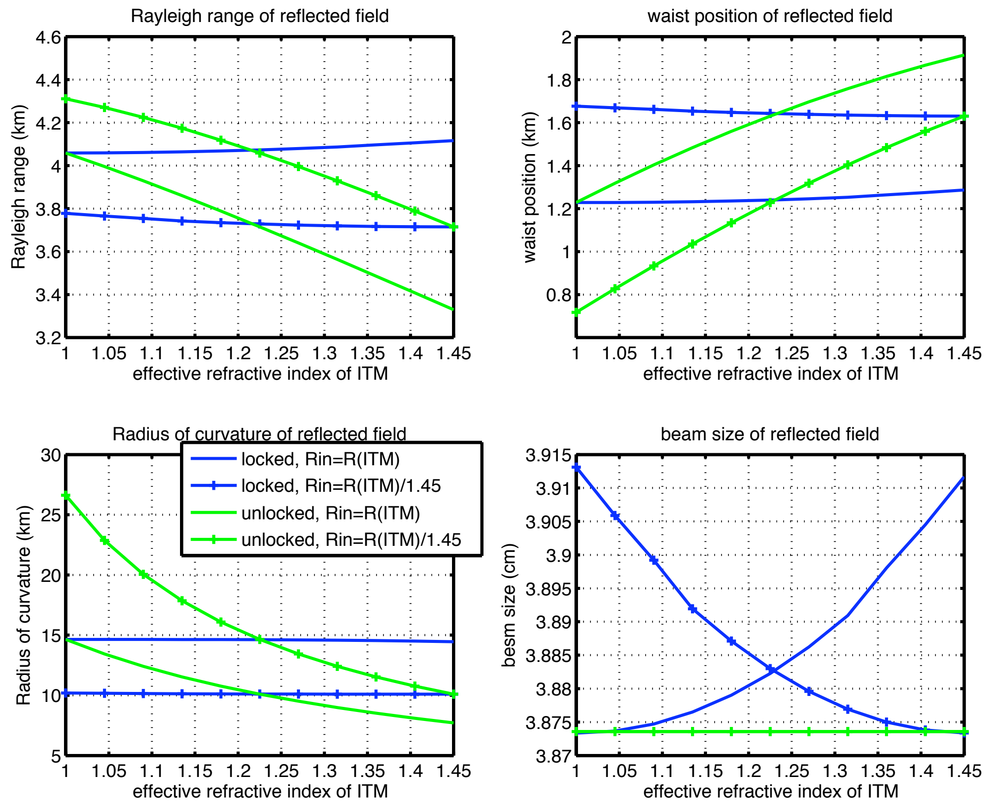

Figure 6 Reflected field profile and n(ITM)

Figure 6 shows profiles of the reflected field, (1) Rayleigh range, (2) waist position, (3) radius of curvature and (4) beam size, all on the AR side of ITM, as a function of n_{eff}, or different thermal lens state.

Blue lines are the ones when the FP cavity is locked, and green lines are the ones only reflected by ITM. Solid lines are the ones with input beam ROC is R_{ITM}, or $\beta = 1$, and lines with + marks are the ones with R_{ITM} / 1.45, or β = 1.45. When the field is resonating in the cavity (blue lines), all quantities vary only 1% for n_{eff}=1~1.45, while the quantities of the field just reflected by ITM (green lines) varies by a factor.

From this plot, one can see that the beam parameters of the reflected field by the resonating arm is independent on the thermal state, or n_{eff}, it depends on the input beam curvature. To have the correct beam profile representing the optimally mode matched state, one needs to setup an appropriate input beam.