LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T0900043-v3 *ADVANCED LIGO* 02/26/09

Optical Layout and Parameters for the Advanced LIGO Cavities

Muzammil A. Arain and Guido Mueller

Distribution of this document:
LIGO Science Collaboration

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project – MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW17-161
175 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

Table of Contents

1 Introduction

1.1 Purpose and Scope

This document describes the optical parameters of the various cavities for Advanced LIGO. The lengths between various optical elements, ROC values of the mirrors, and their tolerances are listed. The various cavity parameters like Finesse, linewidths, and transversal mode spacing are calculated. Included also are the higher order modes offsets from resonances. The recycling cavity parameters are picked assuming that the TCS keeps the IFO same for both the cold as well as full power operation. The ROCs of the recycling cavity mirrors can be changed a little such that the recycling cavity matches the arm cavity mode at reduced power without engaging TCS.

1.2 Definitions

Finesse: Measure of the selectiveness/build-up of the cavity given by $F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$

Free Spectral Range: FSR is given by FSR = $\frac{c}{2L}$ where c is the speed of light while L is the length of the cavity. The units we use are Hz.

Linewidth: The point at which the normalized transmission through a cavity becomes 1/2. This is calculated as Linewidth = $\frac{0.5 * FSR}{F}$ = Half-Width-Half-Max (HWHM).

Transversal Mode Spacing: Transversal mode spacing in the frequency difference between two Gaussian modes. For example, this is the frequency difference between TEM_{00} mode and TEM_{01}. For any higher order TEMnm mode, the difference between TEMoo and TEMnm mode is given by $\frac{(n+m) FSR * a\cos(\pm\sqrt{g})}{\pi}$ where g is the G-factor of the cavity. Note that we will use Hz as the units of transversal mode spacing.

Sagitta or Sag: For a beam with 1/e^2 beam size of w incident on a mirror if ROC R, the sag is given by $\frac{w^2}{2R}$ between the center of the beam and the beam radius.

1.3 Acronyms

ROC: Radius of Curvature

PRC: Power Recycling Cavity

SRC: Signal recycling Cavity

1.3.1 LIGO Documents

1. Michael Smith and Dennis Coyne, “Stable Recycling Cavity Mirror Coordinates and Recycling Cavity Lengths,” LIGO-T080078-06-D.
2. Muzammil A. Arain and Guido Mueller, "Design of the Advanced LIGO recycling cavities," Opt. Express 16, 10018-10032 (2008) http://www.opticsinfobase.org/abstract.cfm?URI=oe-16-14-10018.
3. R. Abbott et al., “Advanced LIGO Interferometer Sensing and Control Conceptual Design,” LIGO-T070247-00-I.

2 Optical Configuration

The optical configuration of the Advanced LIGO cavities is given in Fig. 1 where we include both recycling cavities and the arm cavities.

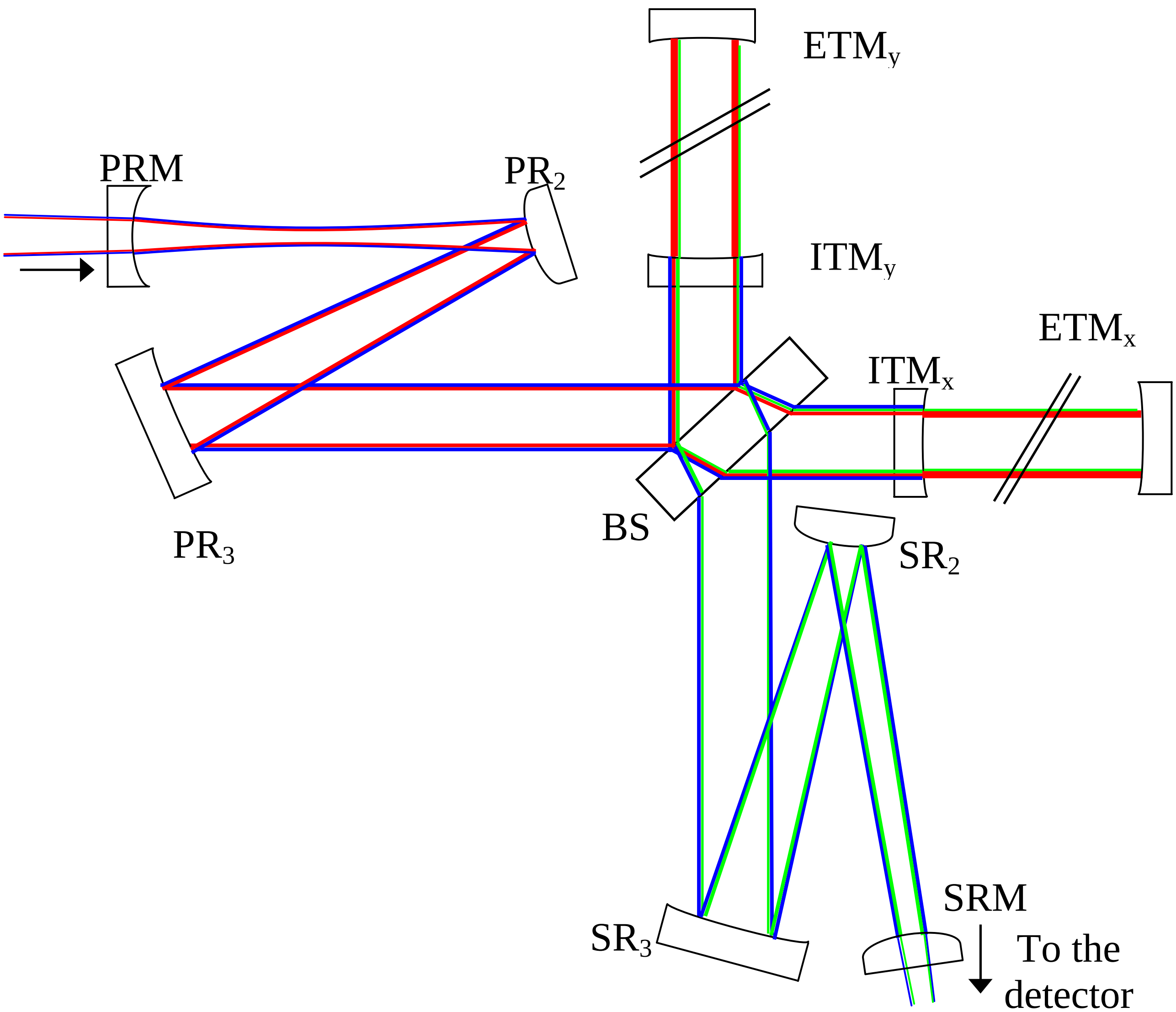


Fig. 1: Optical layout of Advanced LIGO cavities.

The various distances involved between optical elements are taken from Ref. 1 and 2. These are shown in Table 1. Various lengths have been taken from Ref. 1. The criterion of designing the recycling cavities and the related Gouy phase selection has been discussed in Ref. 2.

Table 1: Optical Parameters and Distances in Advanced LIGO Cavities

Definition	***Unit***	***PRC***		***SRC***	
		Straight	Folded	Straight	Folded
P(S)RM radius of curvature	m	9.35	9.40	-11.77	-136.7
Distance b/w P(S)RM and P(S)R_2	m	16.6329	15.7972	15.727	15.9407
P(S)R_2 ROC	m	-2.27	-1.92	-3.77	-2.48
Distance b/w P(S)R_2 and P(S)R_3	m	16.1709	15.2	15.471	16.0016
P(S)R_3 ROC	m	34	31.8	34.0	34
Distance b/w P(S)R_3 and BS	m	19.511	19.4139	19.374	20.0991
BS Effective thickness	mm	0	0	131.5	132
Distance b/w BS and CP	m	4.8526	9.4652	4.8073	9.4455
Distance b/w CP and ITM	mm	5	5	5	5
ITM ROC	m	1934	1934	1934	1934
Reqd. beam waist size in arm	mm	12.0	12.01	12.0	12.01
Spot Size at ITM	cm	5.30	5.31	5.30	5.31
Beam waist location from ITM	m	1884.4	1885	1884.4	1885
Arm Cavity Length	m	3994.75	3996.0	3994.75	3996.0
ETM ROC	m	2245	2245	2245	2245
Angle of Incidence at P(S)R_2	degree	0.748	0.963	0.892	0.878
Angle of incidence at P(S)R_3	degree	0.608	1.144	0.774	0.916

The associated component specifications for the recycling cavity mirrors are given in Table 2 and 3.

Table 2: Component Parameters for Recycling Cavity Mirrors

Optics	ROC (m)		Beam Size (mm)		Sag (µm)		ROC Tolerance in % and mm			Tol. Sag (nm)	
	Straight	Folded	Straight	Folded	Straight	Folded	Both (%)	Straight (mm)	Folded (mm)	Straight	Folded
PRM	9.35	9.40	1.9	2.1	0.20	0.23	1	93.5	94.0	2.0	2.3
PR2	-2.27	-1.92	3.3	3.0	-2.36	-2.35	1	-22.7	-19.2	-23.4	-23.3
PR3	34.00	31.80	54.0	54.5	42.87	46.63	0.5	170.0	159.0	213.3	232.0
SRM	-11.77	-136.71	2.1	3.1	-0.18	-0.03	1	-117.7	-1367.1	-1.8	-0.3
SR2	-3.77	-2.48	5.5	3.8	-3.97	-2.97	1	-37.7	-24.8	-39.3	-29.4
SR3	34.00	34.00	54.0	54.5	42.86	43.68	0.5	170.0	170.0	213.2	217.3

Note: Here 'Sag' is the sagitta change due to ROC while 'Tol Sag' is the change in sagitta between the nominal ROC value and when the ROC is at the end of the tolerance. For example, for PRM, 'Tol. Sag' = (Beam size)^2/(2*10.43) - (Beam size)^2/{2*(10.43+0.1)}

Note that the tolerances of $P(S)R_3$ are based upon our ability to correct any manufacturing tolerance by repositioning $P(S)R_2$. From layout standpoint, we can reposition $P(S)R_2$ by ± 10 cm requiring P(S)RM be moved by ± 20 cm. Thus we had to select 0.5% tolerance for $P(S)R_3$. Tolerance of $P(S)R_2$ and P(S)RM is loosely based upon what we can get from the manufacturers easily. Any error in ROC of $P(S)R_2$ and P(S)RM can also be corrected by repositioning the mirrors but the range of motion required for these mirrors is very small.

2.1 BS and Schnupp Asymmetry

Note that we have used the HR side of the BS for designing the PRC while for the SRC, the beam passing through the BS AR side is chosen. Thus for the straight cavity, PRC is designed for the Y-arm while SRC is designed for the X-arm. For the folded cavity, PRC is designed for the X-arm while SRC is designed for the Y-arm. The difference in the resulting ROC for the cavity mirrors is very small and well within the proposed tolerance.

One important thing to note that the optical thicknesses play a different role for the cavity length (or phase) locking and the mode matching. For the optical phase or cavity length calculations, when the light beam passes through a substrate, the optical phase accumulated as n*d where n is the refractive index while d is the thickness of the material. For the case of mode matching, a substrate of thickness d is modeled as n/d. So the effective thickness is reduced. This has an important significance when considering the BS thickness and the Schnupp asymmetry. When considering the PRC X-arm, this arm travels through the BS. So the 'optical thickness' is larger than the actual thickness of the BS by a factor of n, i.e., the refractive index. In assigning the Schnupp asymmetry, currently X-arm has a longer arm length than the Y-arm when considering the optical phases and the cavity lengths. However, when considering the beam propagation, because of the n/d effect in the BS thickness, the beam propagating through the BS sees n/d as the thickness. This difference more or less makes the mode matching same into the X-arm and the Y-arm. The Schnupp asymmetry is reversed in the case of folded IFO which preserves the advantage because now the Y-arm beam passes through the BS.

2.2 Low Power Operation

Parameters given in Section 2 are basically for the cold IFO state. TCS is supposed to preserve the IFO mode as the IFO is locked at higher power. This is done by using ring heaters on the test masses and by using CO_2 beam on the compensation plate.

However, we can envision that we may want to operate the IFO at low power to take advantage of the various IFO configurations available. As mentioned in Ref. 3, there are significant astronomical interest in operating at or near 25 W. The thermal lens in the ITMs is the main factor that would change the mode matching. It would be desirable to design the RC to match some intermediate power level such that we can operate the IFO without engaging TCS at low powers. Since we want good mode matching at the beginning also for locking purposes, it seems prudent to design the RC for some intermediate power level between the cold and the 25 W operation. We have chosen to present the design of the RCs for 12.5 W assuming that we do not engage the TCS. Therefore, we consider that there will be a thermal inside the ITM. The change of mode due to ITM HR ROC change is very insignificant as compared to the ITM substrate thermal lens and therefore can be neglected. From various simulations and thermal modeling, thermal lens of about 5 km at 125 W for 5.3 cm beam size is expected. This translates into a 50 km thermal lens at 12.5 W. Therefore, we can design the RC such that we include a 50 km thermal lens in the ITM substrate. The mode matching between the arm cavity mode and the recycling cavity, and mode matching product of AC mode, RC mode, and IMC mode is presented in Fig. 3.

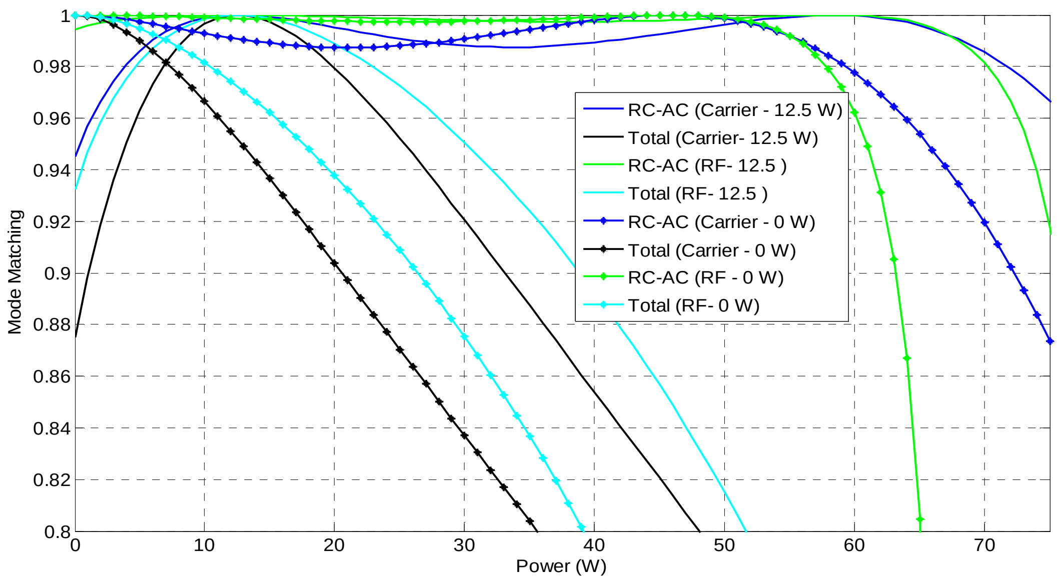


Fig. 3: Coupling between various modes for both carrier and the RF sidebands. Here RC-AC represents the coupling between the arm cavity mode and the recycling cavity mode. Total represents the product of coupling between AC-RC and IMC-RC. The dotted curves represent the mode matching for the IFO at 0 W. The solid lines without markers are for 12.5 W.

Table 3: ROC Values for the RC Mirrors Optimized for 12.5 W Operations

Optics	ROC (m)		Beam Size (mm)		Sag (µm)		ROC Tolerance in % and mm			Tol. Sag (nm)	
	Straight	Folded	Straight	Folded	Straight	Folded	Both (%)	Straight (mm)	Folded (mm)	Straight	Folded
PRM	9.44	9.49	1.9	2.1	0.20	0.21	1	94.4	94.9	2.0	2.1
PR2	-2.25	-1.89	3.3	3.0	-2.33	-2.33	1	-22.5	-18.9	-23.1	-23.1
PR3	34.00	31.80	54.0	54.5	42.87	46.63	0.5	170.0	159.0	213.3	232.0
SRM	-12.10	-166.70	2.1	3.1	-0.18	-0.03	1	-121.0	-1667.0	-1.8	-0.3
SR2	-3.74	-2.45	5.5	3.8	-3.94	-2.95	1	-37.4	-24.5	-39.0	-29.2
SR3	34.00	34.00	54.0	54.5	42.86	43.68	0.5	170.0	170.0	213.2	217.3

As is evident from Fig, 3, there is considerable advantage in designing the system for low power operation. We can gain almost 8% mode matching improvement at 25 W if we design the system for 12.5 W. Note that 12.5 W is chosen such that the mode matching is good for both zero power and 25 W. the data in Fig. 3 has been generated using modal model of the IFO developed at UF. This should be checked against FFT model.

Table 3 presents the ROC values of the RC mirrors for the low power design. These values should be compared with Table 2. The large P(S)R_3 mirror ROC is same as in the old case. The P(S)R_2 and P(S)RM gets changed a little bit. In most of the cases, the new ROCs compared to the zero power design are exactly at the limit of the tolerance of the cold case. So in principle the change in ROC of these two mirrors can be adjusted by changing the optical length between the cavity mirrors. This however, will eat-up the range available for mode matching adjustments. So leaving the motion range available for adjusting the mirrors for low power operation, does not leave a whole lot for the large mirror ROC tolerance or other tolerances.

2.3 Adaptive Mode Matching from IOO to IFO at Low Power

IOO plans to install one (possibly two) adaptive lenses in the IOO chain so that the mode matching at all powers can be adjusted for the thermal effects. A new material (SF57 from SCHOTT) is undergoing tests for adaptive mode matching. So far all the results indicate that this would be a very good candidate for the adaptive lenses. If we use two, we can easily mode match perfectly at all power levels. Figure 4 shows the mode matching for various levels of adaptive elements in operation. The black curve shows how the mode-matching would be affected from IOO to the IFO if we have no adaptive lens in the IOO chain. Red curve shows the case for one adaptive element while the blue curve indicates the mode matching if we have two adaptive elements in the IOO chain. The location of the two adaptive elements chosen for this simulation is one after the $PMMT_2$ and the second just before SM_2. Figure 5 shows the ROC change required from the adaptive elements at these locations.

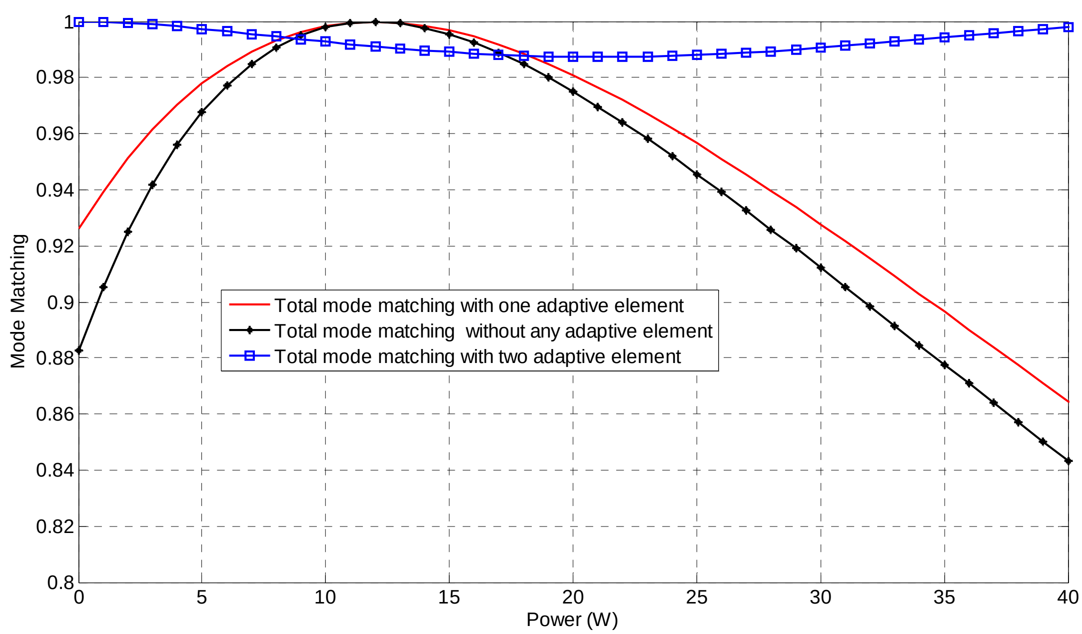


Fig. 4: Total mode matching from IOO to IGO including both IOO-RC and RC-AC mode matching for various adaptive compensation schemes. Mode matching is for the carrier.

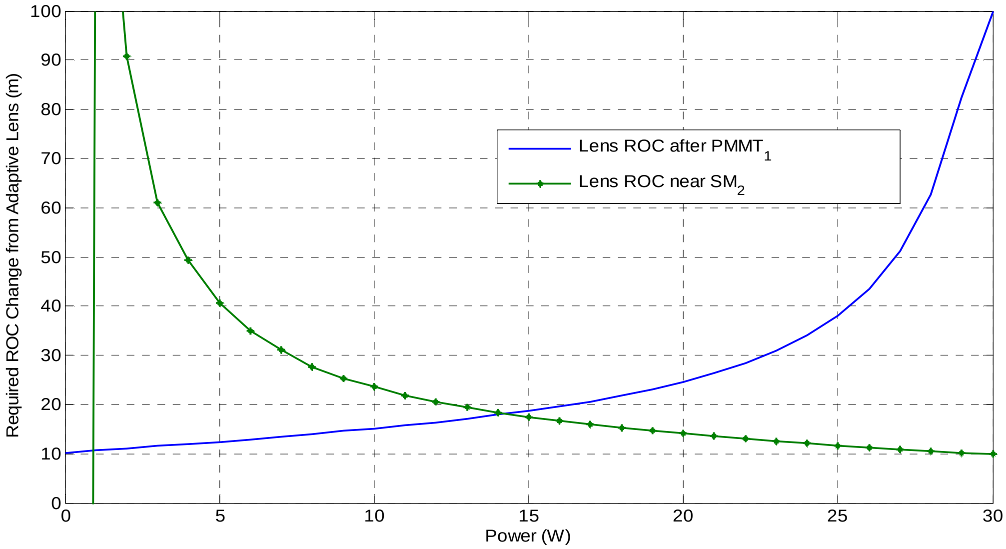


Fig. 5: ROC change required at from two adaptive lenses located after $PMMT_2$ and just before SM_2 respectively to get the mode matching as indicated by the blue curve in Fig. 4. The values shows are the ROC values at about 2 mm beam size.

3 Derived Cavity Parameters

To derive cavity parameters we have to use some mirror transmittances and distances. These are given in Table 4 and are taken from Ref. [1-3].

Table 3: Derived Cavity Parameters

Quantity	***Unit***	***Straight IFO (Folded)***
ITM Transmittance	%	1.4
PRM Transmittance	%	3.0
SRM Transmittance	%	20.0
ETM Transmittance	ppm	10
PRC Length	M	57.676 (60.398)
SRC Length	m	56.028 (62.124)
Arm cavity length	m	3994.75 (3996)
Lower Mod. Frequency	MHz	9.096270 (8.686280)
Distance b/w P(S)R_3 and BS	MHz	45.481350 (43.431400)
Arm cavity Finesse		443
Arm cavity FSR	KHz	37.52
Arm cavity TMS	KHz	32.69
Arm cavity Linewidth	Hz	42.37
Arm cavity G-factor		0.845

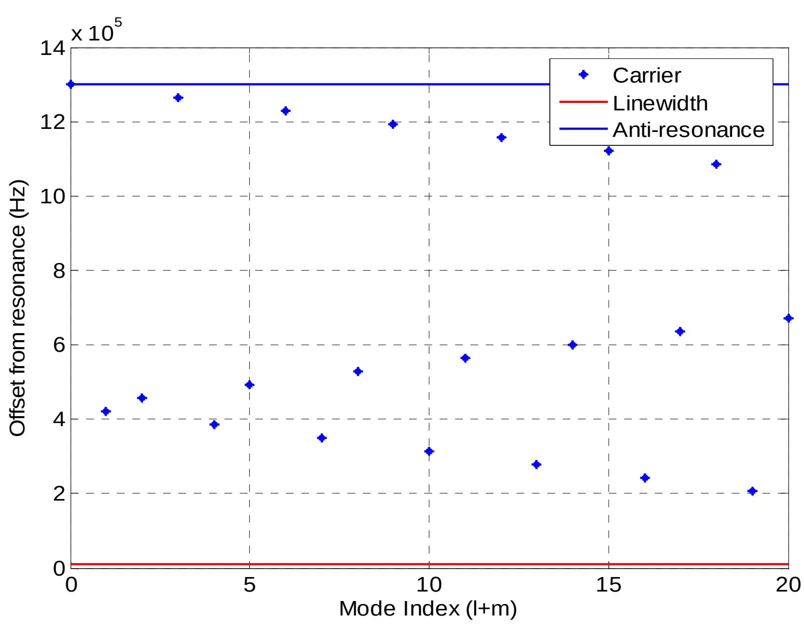
		PRC	*SRC*
Carrier Recycling cavity Finesse		114	25.36
Recycling cavity FSR	MHz	2.6	2.67
Recycling cavity TMS	MHz	1.722	0.434
Carrier Recycling cavity Linewidth	KHz	11.35	52.7
RF Recycling cavity Finesse		141	26.46
RF Recycling cavity Linewidth	KHz	9.22	50.57
G-factor		0.2376	0.7617
One way Gouy Phase	Radian	2.08	0.51

Note that there is a little difference between the Finesse for the carrier and the RF sidebands in the recycling cavities. This happens because the carrier is resonant in the arms while the sidebands are not. Therefore we use effective reflectance for the over-coupled arm cavity. The amplitude reflectivity of ITM based upon 1.4% power transmittance is 0.9930 while the amplitude reflectivity of the combined ITM-ETM combination when the carrier is resonant inside the arm cavity is -0.9879. Thus the slightly higher losses for the carrier accounts for slightly reduced Finesse of the PRC and SRC for the carrier as compared to the sideband Finesse in these cavities. Please note that the derived values, especially the Finesses and the cavity linewidth may change a little because of the round-trip loss in the cavities. The true values should be determined using FFT analysis.

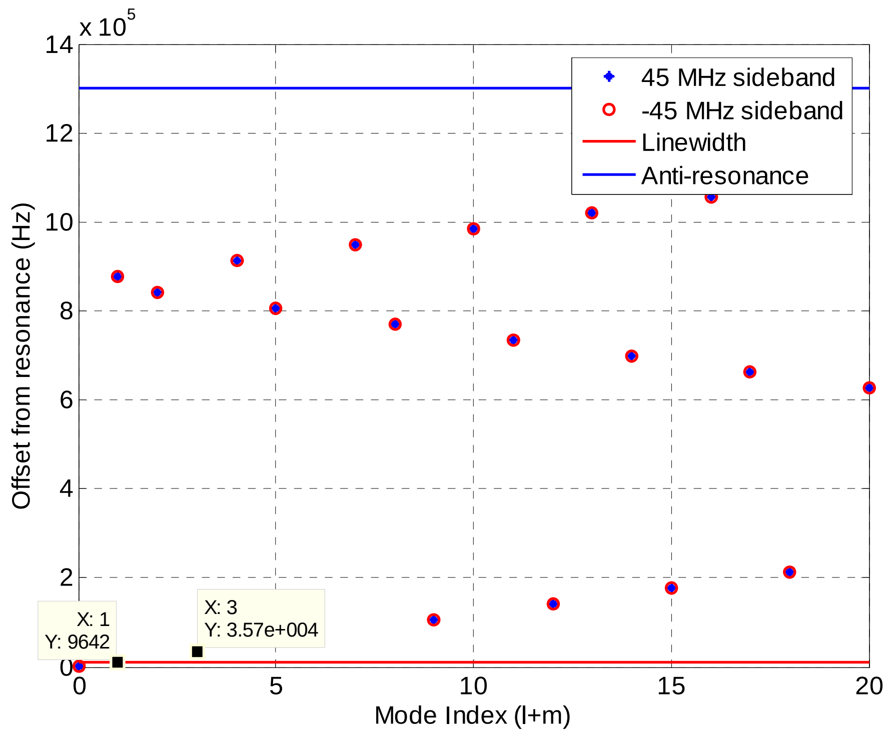
4 Higher Order Mode Resonance

Next we have used straight IFO to check the higher order mode resonance in the recycling cavities. The results are shown in the figures below.

4.1 Carrier in the PRC

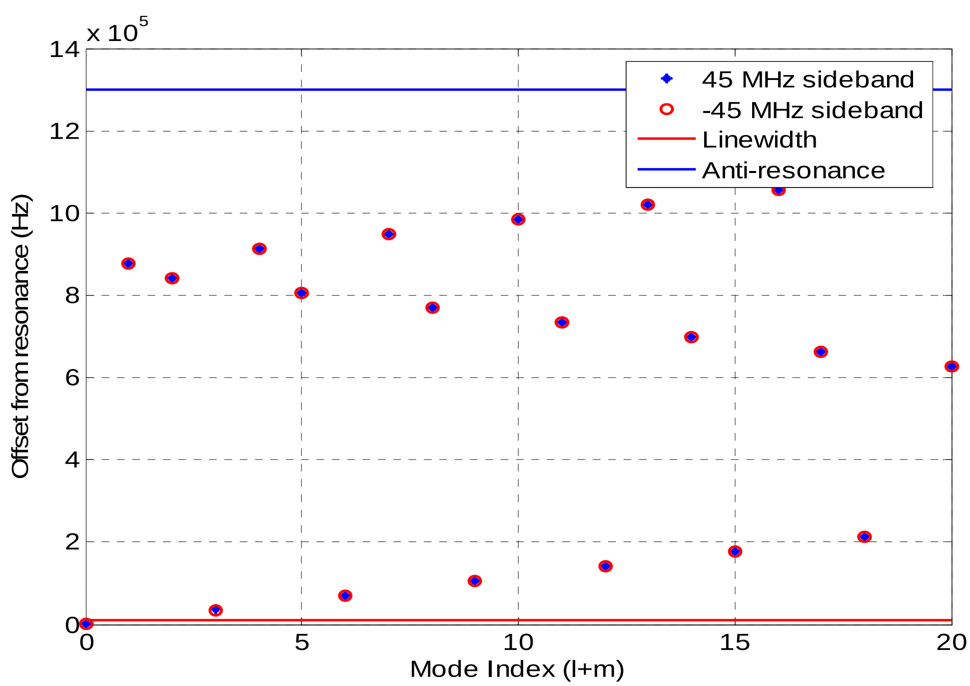


4.2 Low Frequency Sideband in the PRC

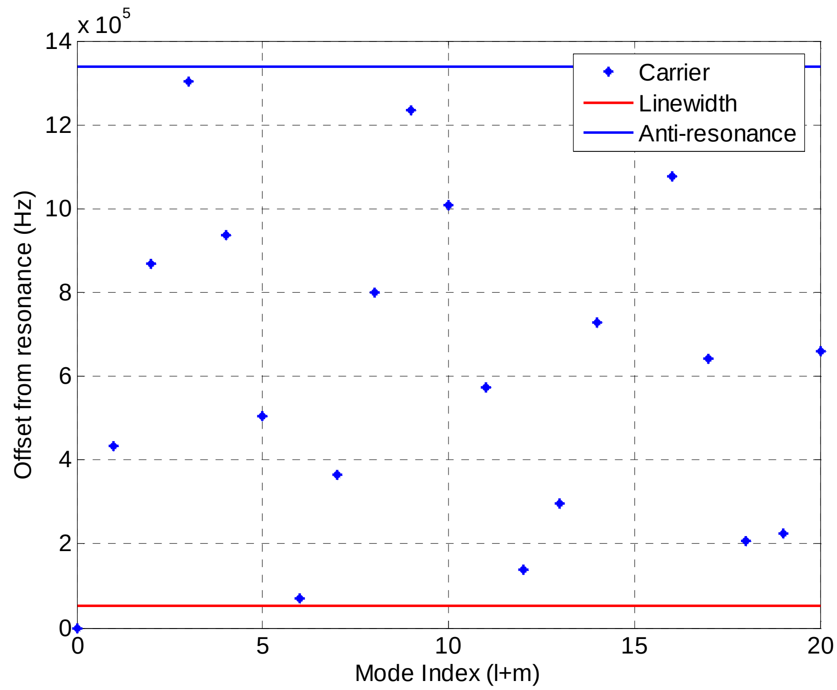


Here the 3rd order mode is about 3.7xlinewidth away from the resonance.

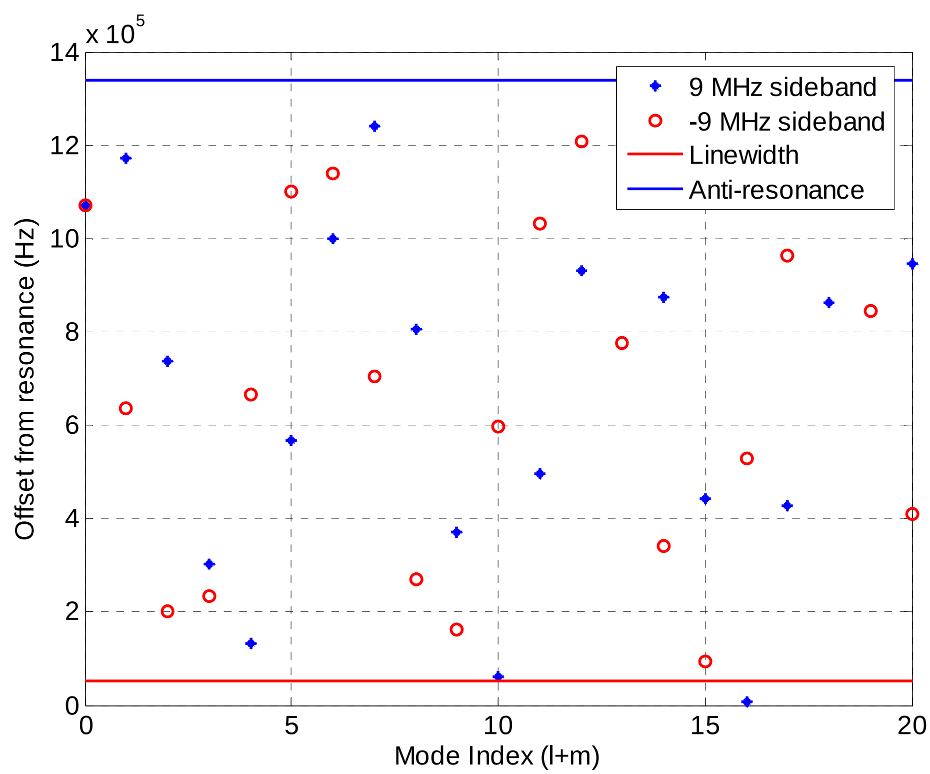
4.3 High Frequency Sideband in the PRC



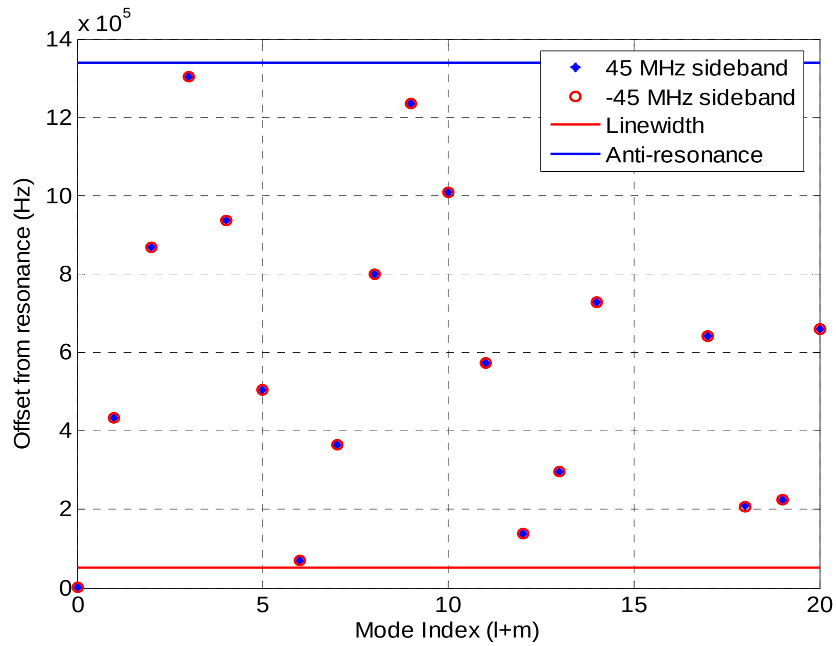
4.4 Carrier in the SRC



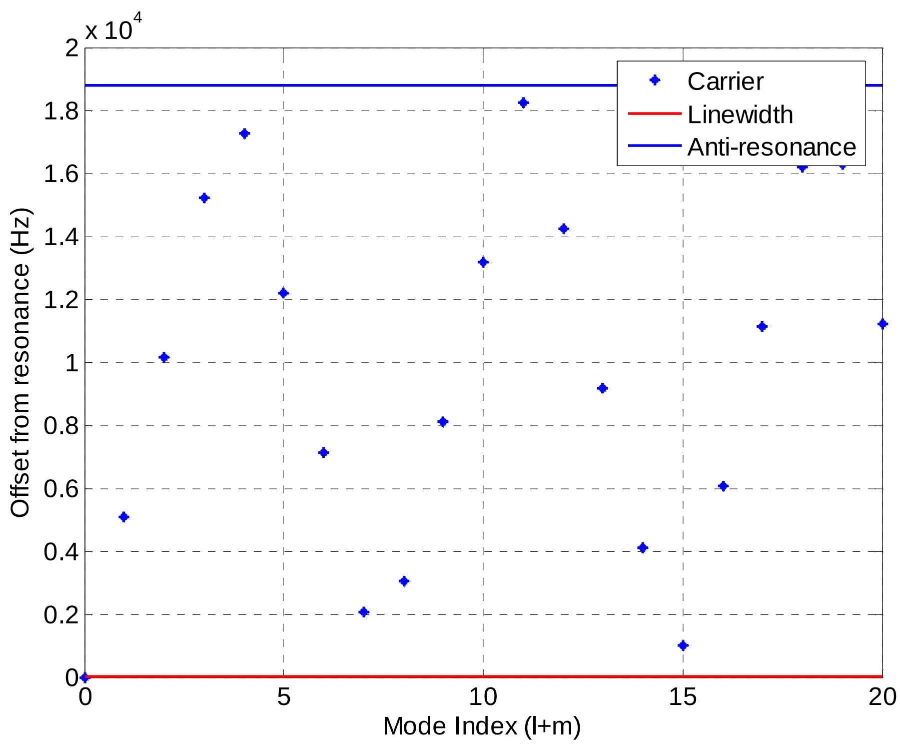
4.5 Low Frequency Sideband in the SRC



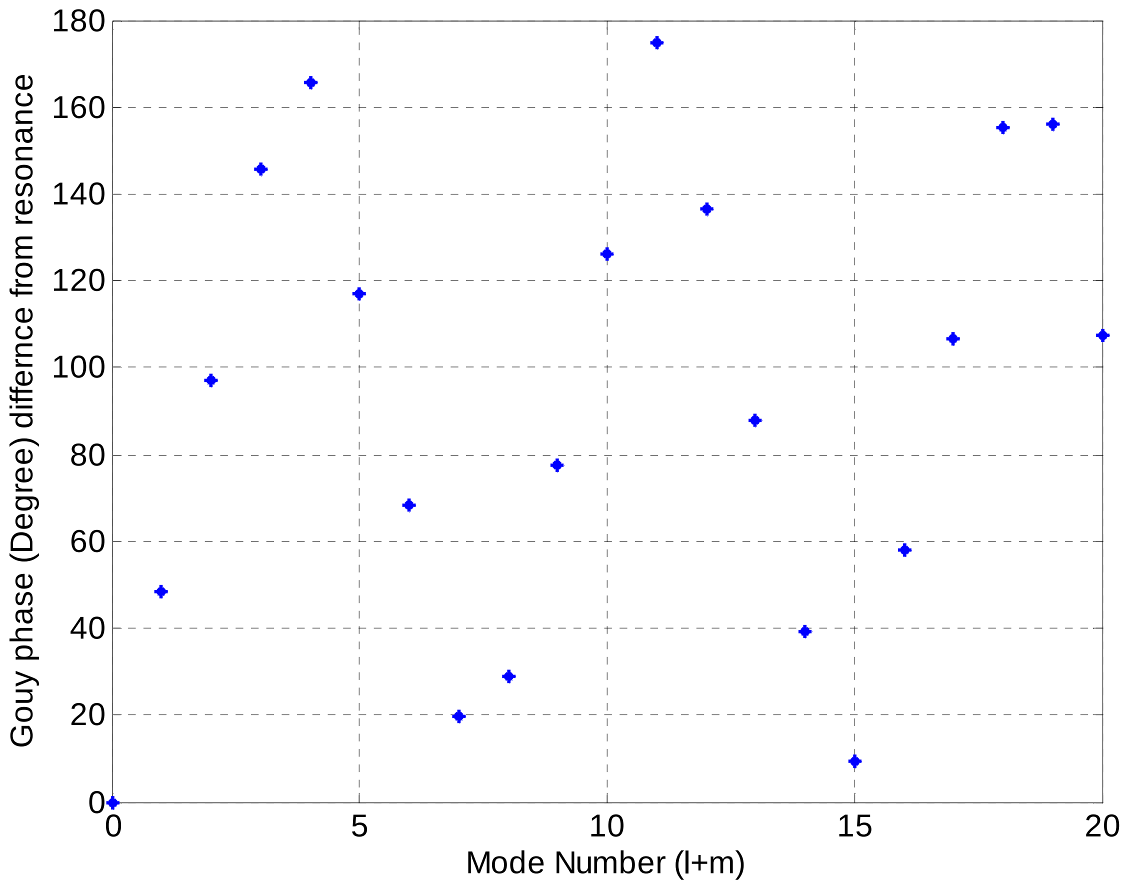
4.6 High Frequency Sideband in the SRC



4.7 Carrier in the Arm Cavity



Since there has been considerable interest in the 8th order mode resonance, the modal calculations for the Gouy phase of the AC HOMs is also given. This graph should be checked against probable parametric instabilities in the arm cavity.



5 Summary

We have presented the optical parameters for the various cavities in Advanced LIGO. We have checked the possibility of designing the system for reduced power operation such that we do not have to engage TCS for correction. We have evaluated the higher order mode resonance for the carrier and the sidebands in the PRC and SRC. None of the higher order mode with a mode index of less than 15 is within the cavity linewidths anywhere. The l+m=6 mode in the signal recycling cavity for the carrier and the 45 MHz sideband is almost at the edge of the cavity linewidth. The choice of PRC and SRC Gouy phase seems appropriate. The higher mode resonances in the arm cavity are also checked using standard ABCD model. The exact values of the resonances should be checked using FFT. Similarly, the HOMs in the AC should also be checked for parametric instabilities.