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LIGO I mirror scattering loss by microroughness

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1. Introduction

Various measurements indicate that the total loss in the initial LIGO arm is around 150ppm per arm or 75ppm per mirror [1,2] with an uncertainty of around 15ppm. Out of these 75ppm loss per mirror, 20-30ppm can be explained by the scattering loss due to mirror surface errors with spatial wavelength > a few millimeters [2]. Other losses, including transmission of ETM (7ppm/2), the absorption loss (4ppm) and diffractive loss (1-2ppm), account for around 9ppm. The loss due to microroughness was originally estimated to be 4.6ppm, based on microroughness data from the substrate polisher (CSIRO). The source of the remaining loss of 30-40ppm is unknown.

This unexpected loss was not a serious problem for initial LIGO, because the larger scale substrate errors were quite small, so that the target recycling gain of 30 could be met. For Advanced LIGO, a goal of reducing the arm loss by a factor of 2 was set, for a loss budget of 35ppm per arm mirror. By combining the ETM transmission (7ppm), absorption (0.5ppm), mirror surface figure loss (20ppm) and the diffractive loss (0.2ppm), there is only 7ppm left for other losses. If the 30-40 ppm of unknown, extra loss seen in initial LIGO still remains in the Advanced LIGO mirrors, the maximum potential stored power in the arms would be reduced by a factor of 2.

Various efforts are going on in order to uncover the source of the unknown losses of the initial LIGO mirror [1]. This note is concentrated on re-evaluation of the microroughness of the initial LIGO mirror surface. Originally, the loss contribution was estimated to be 4.6ppm from the microroughness of the polished mirror surface reported by CSIRO. This note shows that this loss may be underestimated by factor of 4, and points out that the specification of the microroughness requirement for the advanced LIGO mirror needs to be revised so that the surface roughness can be specified to match the loss requirement.

In section 2, basic formulations are reviewed to derive the relationship (Eq.(8)) between the one dimensional power spectral density (PSD) of the surface, which is directly related to the TOPO measurements along lines, and the scattered loss characterized by the scattering angle or minimum spatial frequency. The analysis of the LIGO I mirror PSD by CSIRO [4] and their measurement of RMS [5] are summarized in section 3. In the following section, the bias of their measurements of the RMS using TOPO data is explained and estimations of the more accurate RMS of the roughness are presented. A summary and implication for advanced LIGO are given in the summary section.

2. Basic formulations

This section summarizes relationships between three quantities: one dimensional ($S_1(f)$ or PSD(f)) and two dimensional ($S_2(fx,fy)$) power spectral densities and BRDF [3].

The one dimensional PSD, $S_1(f)$, is defined to be single sided, i.e., $f \geq 0$. S_1 and S_2 are related by the following formula.

$$S_1(f_x) = 2\int_{-\infty}^{\infty} S_2(f_x, f_y) df_y \tag{1}$$

The factor 2 comes because S_1 is single sided ($f_x > 0$) while S_2 is fully two dimensional ($-\infty < f_x, f_y < \infty$). When the surface topography is isotropic, i.e., S_2 depends only on f, the above equation becomes, substituting $S_2(f_x, f_y) = S_2(f)$:

$$S_1(f_x) = 4\int_{f_x}^{\infty} S_2(f)\frac{fdf}{\sqrt{f^2 - f_x{}^2}} \tag{2}$$

When the functional form of $S_1(f)$ can be parameterized as

$$S_1(f) = A(1+(Bf)^2)^{-C/2} \tag{3}$$

the two dimensional spectrum can be related to S_1 in the following way:

$$\begin{aligned} S_2(f) &= D\frac{B}{\sqrt{1+(Bf)^2}}S_1(f) \underset{f \gg 1/B}{\approx} D \cdot S_1(f)/f \\ D &= \Gamma(\frac{C+1}{2})/(2\sqrt{\pi}\Gamma(\frac{C}{2})) \end{aligned} \tag{4}$$

The scattered energy propagating in a certain direction is expressed using BRDF and by the two dimensional PSD in the following way:

$$BRDF(\theta)\theta d\theta d\varphi = \left(\frac{4\pi}{\lambda}\right)^2 S_2(f)fdfd\varphi \tag{5}$$

By substituting the relation between the scattering angle and the corresponding frequency, θ = f λ, BRDF can be related to S_2 and S_1 as follows.

$$\begin{aligned} BRDF(\theta) &= (\frac{4\pi}{\lambda^2})^2 S_2(f) \\ &= (\frac{4\pi}{\lambda^2})^2 D\frac{S_1(f)}{f} \end{aligned} \tag{6}$$

The loss due to the scattering into a larger angle or a larger frequency region is given by

$$\begin{aligned} loss &= \int_{\theta} BRDF(\theta)\theta d\theta d\varphi \\ &= 2\pi(\frac{4\pi}{\lambda^2})^2 D\int_{f_0}\frac{S_1(f)}{f}\lambda^2 fdf \\ &= 2\pi D \times (\frac{4\pi}{\lambda})^2 \int_{f_0} S_1(f)df \end{aligned} \tag{7}$$

This formula shows that the loss is related to the one dimensional rms in the following way by a factor 2πD which depends on the slope of the spectrum.

$$\begin{aligned} loss &= 2\pi D(\frac{4\pi\sigma_1}{\lambda})^2 \\ \sigma_1{}^2 &= \int_{f_0} S_1(f)df \end{aligned} \tag{8}$$

For the slope discussed in the following sections, i.e., C=1.45, 2πD = 1.2. Note that the value of this factor usually assumed in calculating loss from a microroughness level, 2πD = 1, is correct only for a PSD slope of f^{-1}.

3. CSIRO data and their analysis

In [5], details of "measuring" the rms value using the two sets of the TOPO (a three-dimensional noncontact optical profiler) measurements are explained. One set of TOPO data with 2.5x magnification is measured at 1024 points along a line of length 5.28mm, and one set with 40x magnification is measured at 1024 points along a line of length 0.323mm [6]. They measured these data sets at several locations on the mirror. For each data set, they subtracted tilt and curvature and calculated the rms as the square root of the sum of the squared rms of each magnification data. This process of subtracting tilt and curvature is needed to remove measurement artifacts [8].

They reported the measured rms of initial LIGO mirrors to be around 0.17nm, for spatial frequencies greater than 3.8cm^{-1} (e.g., [7]), which satisfied the requirement imposed by LIGO lab that the rms be < 0.2 nm in a frequency range of 4.3 – 7500 cm^{-1}.

Using this rms the loss due to microroughness was estimated to be 4.6ppm, using Eq.(8) without the proper conversion factor, i.e., by using 2πD = 1, instead of 1.2. This underestimated the loss by 20%.

The CSIRO group published a paper analyzing the uncoated surface of the initial LIGO mirror [4] polished by CSIRO. The PSD of ETM and ITM mirror published in their paper are show in Figure 1. Each PSD is consisted of three sets of measurements, LADI (large-aperture digital interferometer) and TOPO with two magnifications.

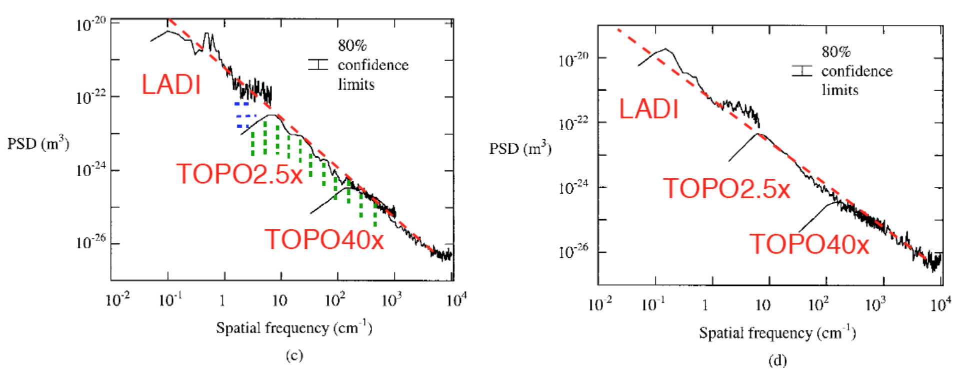

Figure 1 PSD functions for (c) the ETM and (d) the ITM. These are Fig.10 in reference [4].

They found that all kinds of mirrors (RM, ETM, ITM and FM) can be fit by a simple functional form

$$PSD(f[m^{-1}]) = A \cdot f^{-n}[m^3] \tag{9}$$

Red lines in Figure 1 are a fit with A = 7e^{-19} and n = 1.45.

4. Re-evaluation of the rms measurement

In Figure 1, one can see that low frequencies of the PSDs obtained by using two TOPO measurements show discontinuities with the higher frequencies of PSDs calculated using other data sets. The rms measurement corresponds to the area with vertical dotted lines [4].

It needs to be emphasized that (1) the removal of tilt and curvature is the process of removing unreal data and is a necessary process, but (2) this process introduces a bias to suppress the numerical value of rms because of finite data length.

The following calculations were done to estimate the bias introduced by the process of removing the tilt and curvature.

The first calculation estimates the bias of the mathematical process itself. A set of random numbers, total of 1024, were generated which follow the spectral shape of f^{-2}. Then the rms values were calculated in two ways, (1) after subtracting only mean values, and (2) after subtracting mean, tilt and curvature. The ratio of the rms for (1) is 1.9 times larger than the rms for (2). The calculation of these rms's does not use any spectrum calculation, and should be close to the process CSIRO used to measure the rms of LIGO mirrors. The loss is proportional to the square of the rms values, and this result suggests that the tilt and curvature subtraction introduces a bias to underestimate the loss by a factor of 4.

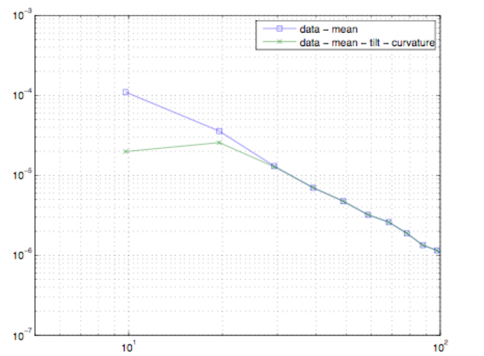

Figure 2 Effect of subtraction of tilt and curvature on power spectrum density.

Figure 2 shows the power spectral density of data mentioned above, with and without tilt and curvature subtraction. As can be seen from this figure, the subtraction reduces the values of psd at the lowest frequency point. This is the same trend observed at the lower frequencies of the two TOPO data sets in Figure 1. The low end of the spectral density suffers from low statistics of the sampled data. This can be improved by using different sampling methods as is discussed in [4]. But the mean values in the lower frequency region do not change much, as shown in Fig. 3 of the same reference, and this region contributes strongly to the rms.

The second method is to use the smooth fit of three data sets, i.e., to calculate the area marked by horizontal blue lines and vertical green lines by using the red lines in Figure 1. By using the loss formula, Eq.(8), and the fitted spectrum, Eq.(9), the loss comes out to be 20ppm for $f > 3.8\text{cm}^{-1}$, 4 times larger than the original estimation. This is consistent with the estimation given by the first method.

5. Summary

As has been demonstrated in the previous section by using two independent methods, the data set used for the microroughness rms measurement suffers a bias introduced by the process of subtracting tilt and curvature. The size of the bias is large enough to suppress the measured value of rms to be half of the true rms value. This bias means the loss due to microroughness has been underestimated by a factor of 4.

In the original estimation of the loss per mirror, the effect of the microroughness was estimated to be 4.6ppm and contributed only a fraction of the unknown loss of 30-40ppm. Now the estimation is 20ppm, and the remaining unaccounted for loss is 10-20 ppm. Two sources of loss that need further investigation are: errors in the spatial frequency region around ~1mm^{-1} [2], and point scatter.

For the Advanced LIGO pathfinder substrates, the microroughness (actually 'high spatial frequency errors') is specified to be less than 0.16 nm, in the band 1-750 mm^{-1} [9]. The lesson learned here is that we need to pay attention to the methods used by the vendors to measure and calculate these errors.

6. References

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7. "2ITM01-C", LIGO-T990147-00-D (1999)
8. C.J.Walsh, private communication. "Tilt is purely a measurement artifact associated with set up of the substrate and should always be removed. Curvature is part measurement artifact associated with the set up of the substrate and may be part the contribution of a surface feature whose wavelength is considerably larger than the measurement aperture. In either case, removal of curvature is also justified. To repeat: these terms must be removed because they are not real." (excerpt from Chris Walsh e-mail on April 4, 2007)
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