

delayRatio: A Gravitational Wave Event Physical Likelihood Estimator Based on Detection Delays and SNR Ratios



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Abstract

The delayRatio is an efficient physical likelihood estimator which is designed to be used as part of a candidate coincident burst/inspiral gravitational wave event follow-up investigation. Given the event parameter estimation of detection delays and the SNR ratio between a pair of detectors, delayRatio returns a Boolean indicating if the parameters fall within physical ranges determined by using polarization averaged antenna patterns and assuming gravitational waves propagate at the speed of light. Since polarization effects can cause physical signals to lay outside the polarization

averaged bounds, probabilities determined from Monte Carlo simulations of physically motivated signals (all sky and random polarization) are also calculated to estimate the likelihood of a polarized signal that falls outside of the polarization averaged bounds. (If the signal falls within the polarization averaged bounds, the candidate is physically possible and the probability measure is not needed.) This work has also been generalized for 3+ detectors. For these combinations, the primary likelihood condition is the physicality of detection delays while SNR ratios give secondary validation measures. Therefore, this work has easily been extended to function as a simple source location tool based solely on the detection delays and timing errors.

Polarization Averaged Methodology

Based on the measured detection delay between two detectors, a circle on the sky where the gravitational wave source may have originated can be determined. The detection delay, Δt , is given by:

$$\Delta t = \frac{(\vec{V}_{\text{det2}} - \vec{V}_{\text{det1}}) \cdot \hat{V}_{\text{GW}}}{c}$$

where \mathbf{V}_{det1} and \mathbf{V}_{det2} are the vectors pointing from the center of the Earth to one of the detectors in question (det1 or det2), \mathbf{V}_{GW} is the unit vector of the gravitational wave source (pointing from the center of the Earth to the source) and c is the speed of light.

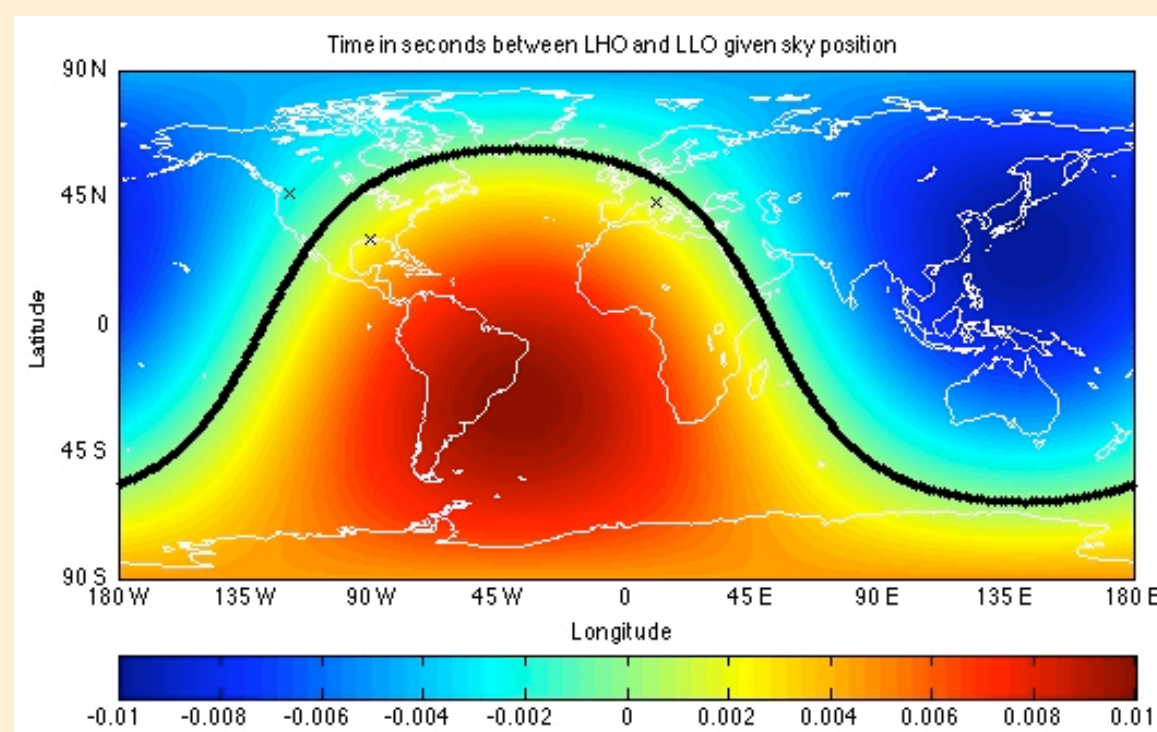
The circle on the sky where the source could have originated is determined by defining a coordinate system where this circle would lie on a constant declination, θ . This coordinate system, which will be referred to here as the baseline polar coordinates, is defined to be the basis where det1 and det2 lie on the vertical axis with the origin at the midpoint between the two (det1 is on +Z, det2 is on -Z). The declination corresponding to a detection delay, Δt , is given by:

$$\theta = \sin^{-1}\left(\Delta t \frac{c}{D}\right)$$

where D is the distance between the detectors. Once the declination is determined, the circle is sampled to a given resolution and these coordinates are then rotated back into the Earth coordinate system (θ_{\oplus}) by rotating around the baseline polar Z axis an angle α counterclockwise and then rotating the baseline polar Y axis an angle β counterclockwise:

$$\theta_{\oplus} = \begin{pmatrix} \cos(\alpha)\cos(\beta) & \sin(\alpha) & -\cos(\alpha)\sin(\beta) \\ -\cos(\beta)\sin(\alpha) & \cos(\alpha) & \sin(\alpha)\sin(\beta) \\ \sin(\beta) & 0 & \cos(\beta) \end{pmatrix} \theta$$

where α is the angle between the baseline polar X axis and the Earth X axis (the intersection of the Prime Meridian and the Equator) and β is the angle between the baseline polar Z axis and the Earth Z axis ($Z = X \times Y$).



The black line on this plot of the detection delays between LHO and LLO is the sky circle that would produce a zero second detection delay.

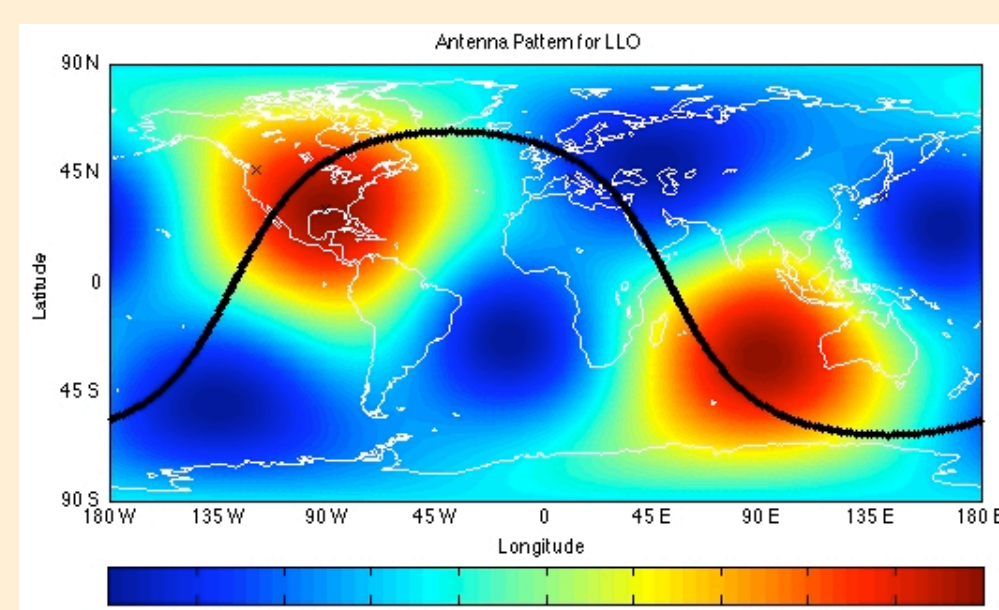
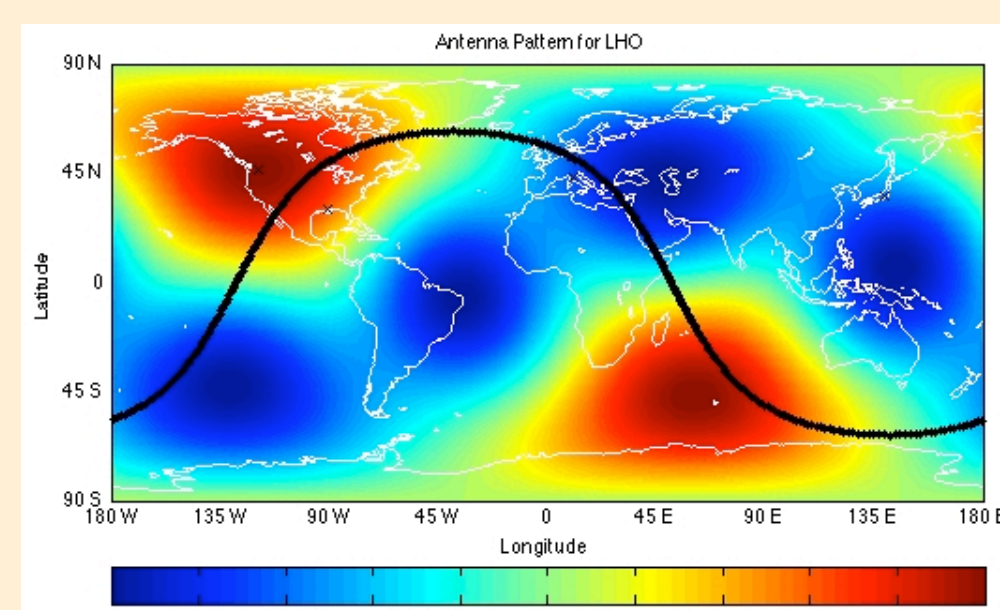
This circle is also superimposed on the polarization averaged antenna patterns below.

Project the determined points of the sky location circle onto the antenna pattern of each detector and determine the polarization averaged coefficient (ρ) for each point:

$$\rho = F_+^2 + F_{\times}^2$$

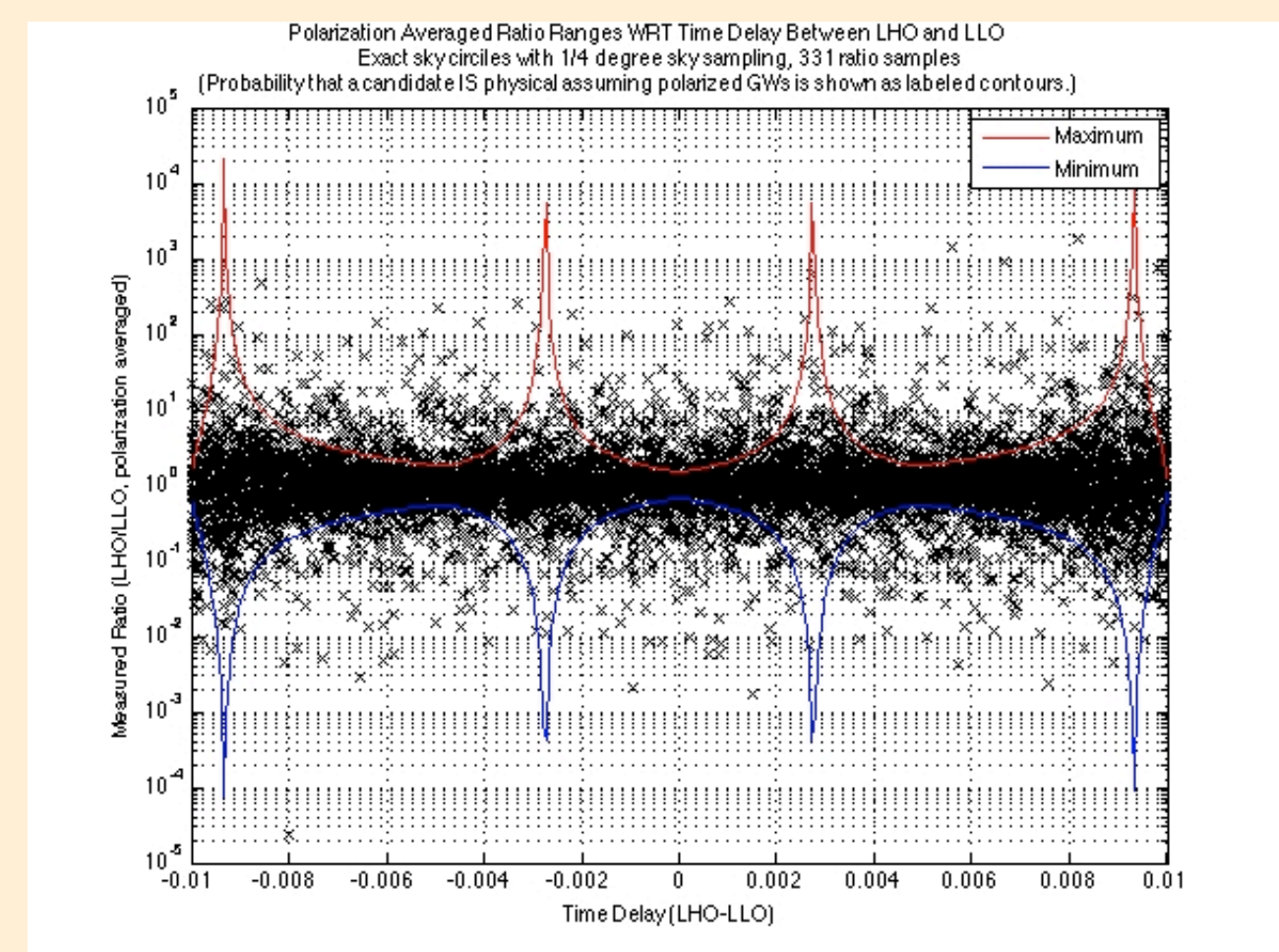
where F_+ and F_{\times} are the + and \times polarization beam patterns.

Point-by-point, compute the ratio of the coefficient from the first detector to the coefficient of the second detector (ρ_1/ρ_2). The minimum ratio and the maximum ratio establish the range of SNRs that are *always* physically possible for this given detection delay.

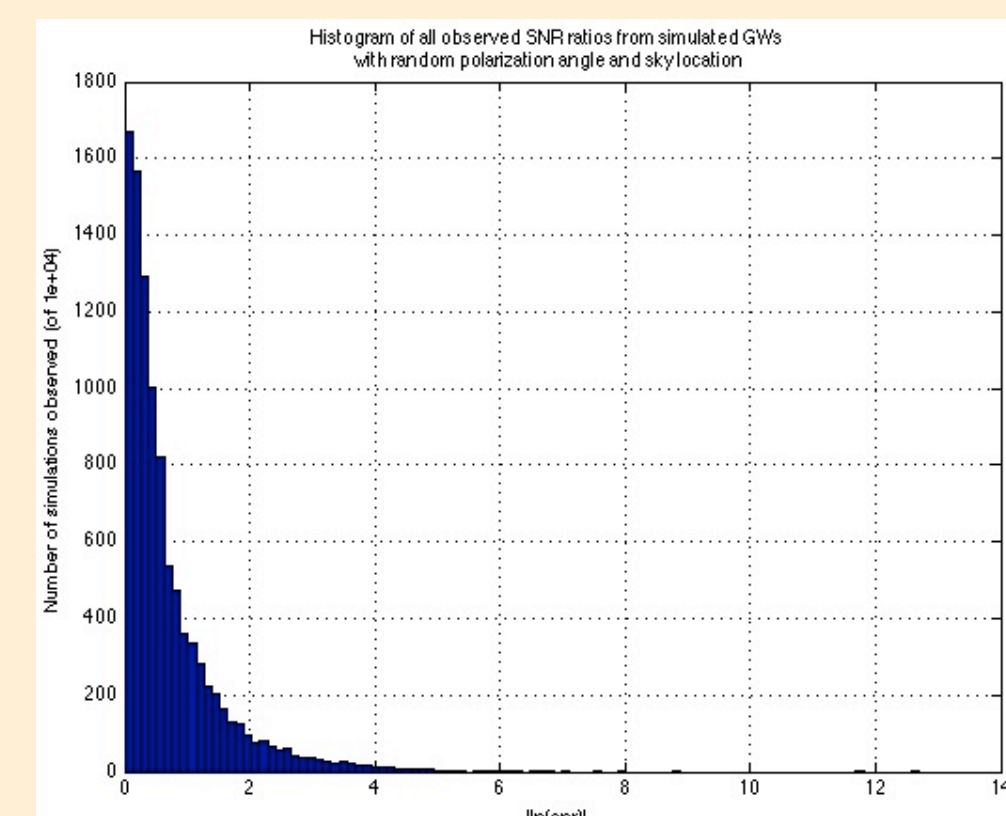


Effects of Polarization

The effects of polarization can cause a physical gravitational wave signal to have a SNR that lies outside of the bounds determined using polarization averaged antenna patterns. To determine the magnitude of this effect, 10,000 physical gravitational wave signals with random sky location and polarization angle were simulated using the GravEn simulation package. The resulting SNR ratios were then plotted against their detection delay and compared to the measured polarization averaged bounds:



As can be seen here, the distribution is even about the unity SNR ratio (ρ_1/ρ_2) and independent of the detection delay. The distribution of $|\ln(\rho_1/\rho_2)|$ is exponential:

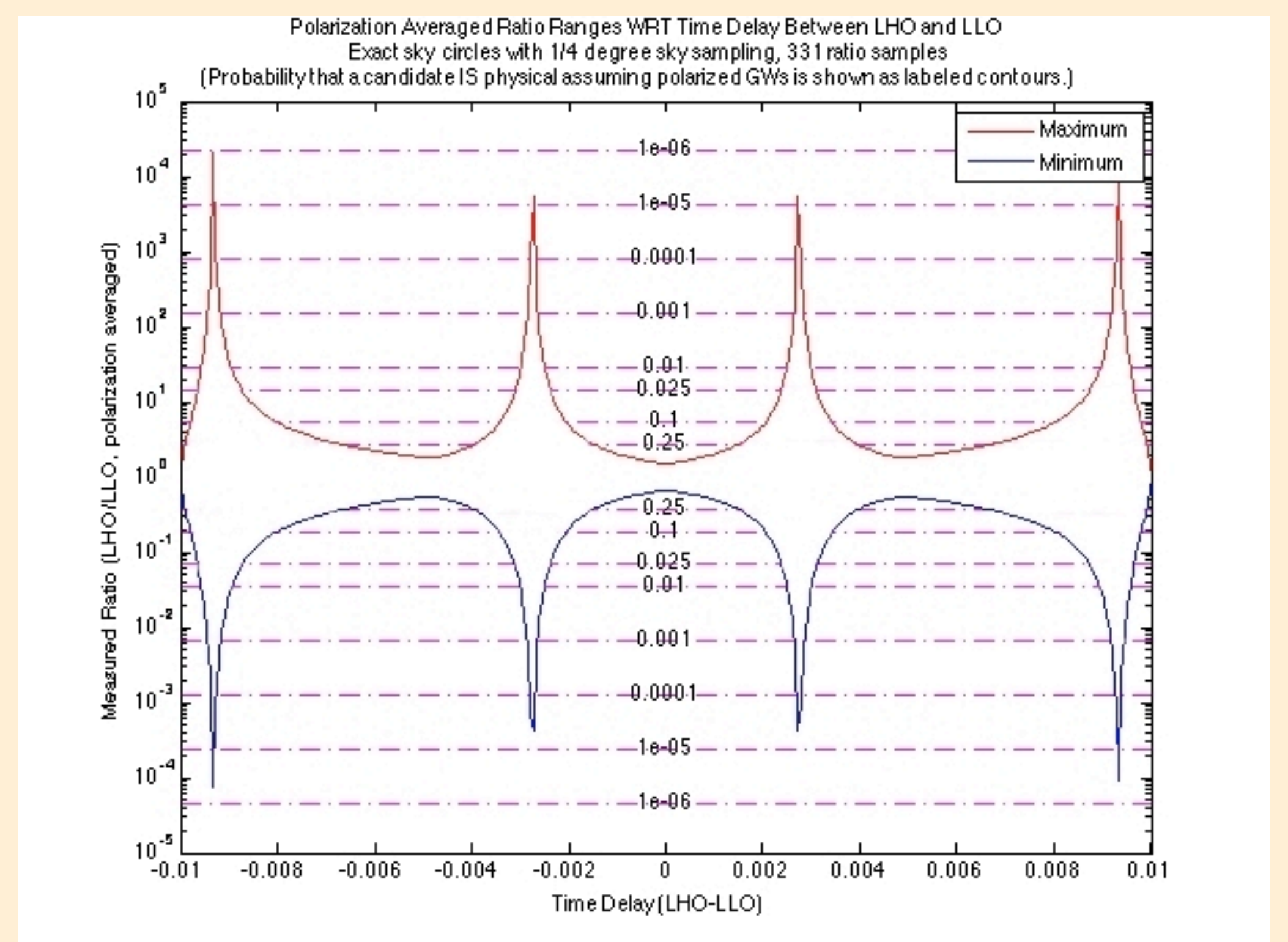


Therefore, the probability of measuring a physical gravitational wave greater than a given SNR ratio is:

$$P\left(> \left| \ln\left(\frac{\rho_1}{\rho_2}\right) \right|\right) = 1 - \int_0^{\left| \ln\left(\frac{\rho_1}{\rho_2}\right) \right|} \frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right) dx = \exp\left(-\frac{\left| \ln\left(\frac{\rho_1}{\rho_2}\right) \right|}{\mu}\right)$$

where μ is the scale factor maximum likelihood estimate.

The contour lines of this probability are superimposed on the polarization averaged bounds to form a combined figure of merit:



delayRatio (continued)

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Application to Detector Pairs

The combined figure of merit for a given pair of detectors was designed to be used as part of a candidate gravitational wave follow-up analysis. The results are in two parts: a Boolean is returned as true if the SNR ratio for the detection delay falls within the polarization averaged bounds and false if it does not. However, a false Boolean only means that the parameters are not *always* physical. A probability is also returned measuring the probability that the parameters are physical given the effects of polarization. This is not a “yes/no” follow-up module; it is a “yes/maybe” test with a measure of how *maybe*.

This module has already been incorporated into the LIGO/VIRGO CBC (Compact Binary Coalescence) follow-up pipeline and is also suitable for a Burst follow-up analysis.

Special thanks to **Cristina Torres** and **Romain Gouaty** for their work incorporating this module into the CBC follow-up pipeline.

Application to 3+ Detectors

This physical likelihood estimation can be extended to networks of 3+ detectors. In these cases, the likelihood is dominated by determining if the detection delays between the detectors are physical; consideration of the physicality of the measured SNR ratios are secondary.

However, delayRatio is easily adapted to take a set of detection delays and locating the intersections of their corresponding location sky circles:

