Detection and Parameter Estimation With Amplitude Corrected Waveforms.

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What are Compact Binary Coalescences?

Brief Recipe…

- Take two compact stellar objects –
  Black Holes and/or Neutron Stars.

- Place them in a binary orbit with each other.

- As the system evolves it will emit gravitational radiation in accordance with General Relativity.

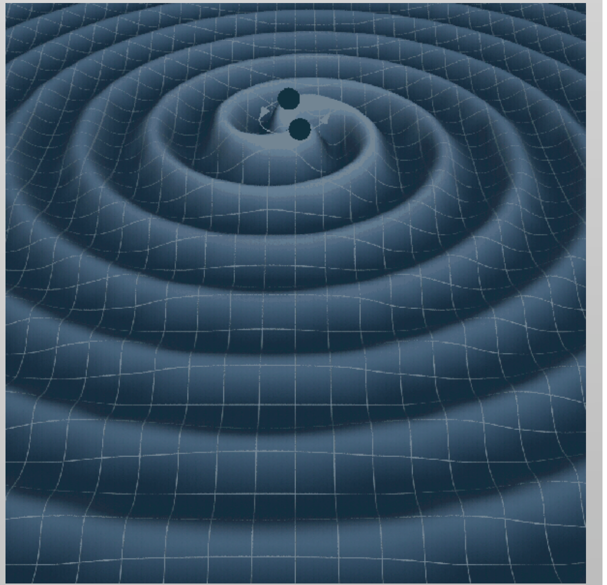


Image credits: K. Thorne (Caltech), T. Camahan (NASA/GSFC) Taken from "Searching for Gravitational Waves with LIGO" by Laura Cadonati

- The gravitational radiation will take energy away from the system, reducing the orbital separation and period until eventually the objects merge.

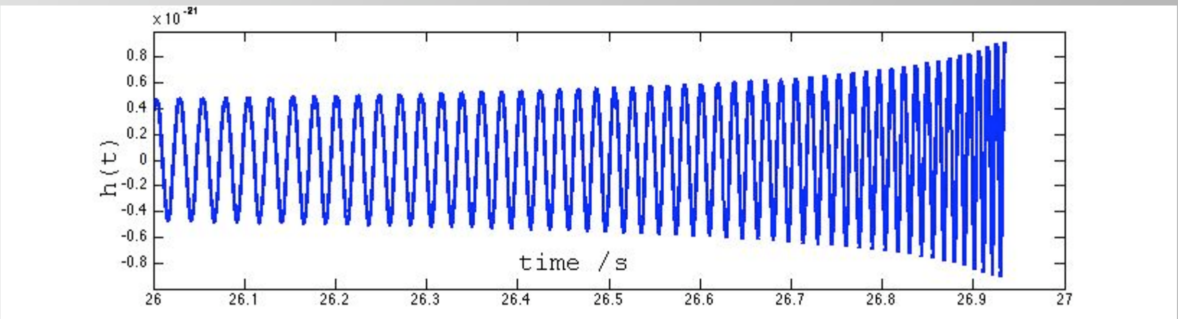
Inspiral stage of waveform is modeled accurately by PN theory.

- Eintstein's equations are solved perturbatively.

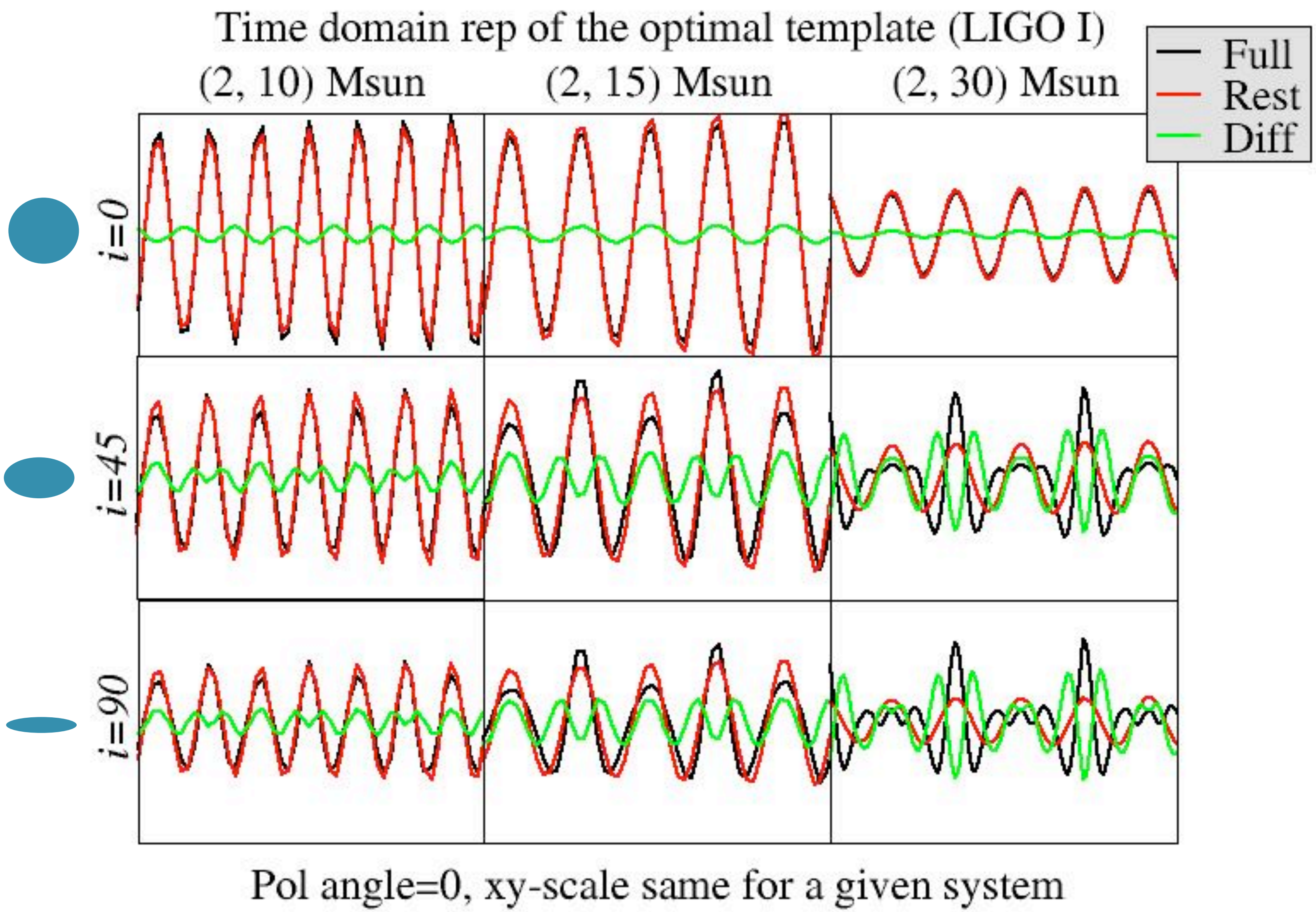
- The *phase* and *amplitude* of the waveforms are obtained as series expansions in *(v/c).*

- *Restricted* waveform retains only the leading order term in the amplitude.

- *Full* waveform includes the higher order amplitude terms, which contain other harmonics of the orbital frequency and amplitude corrections.



Full vs Restricted waveforms – *inclination angle*



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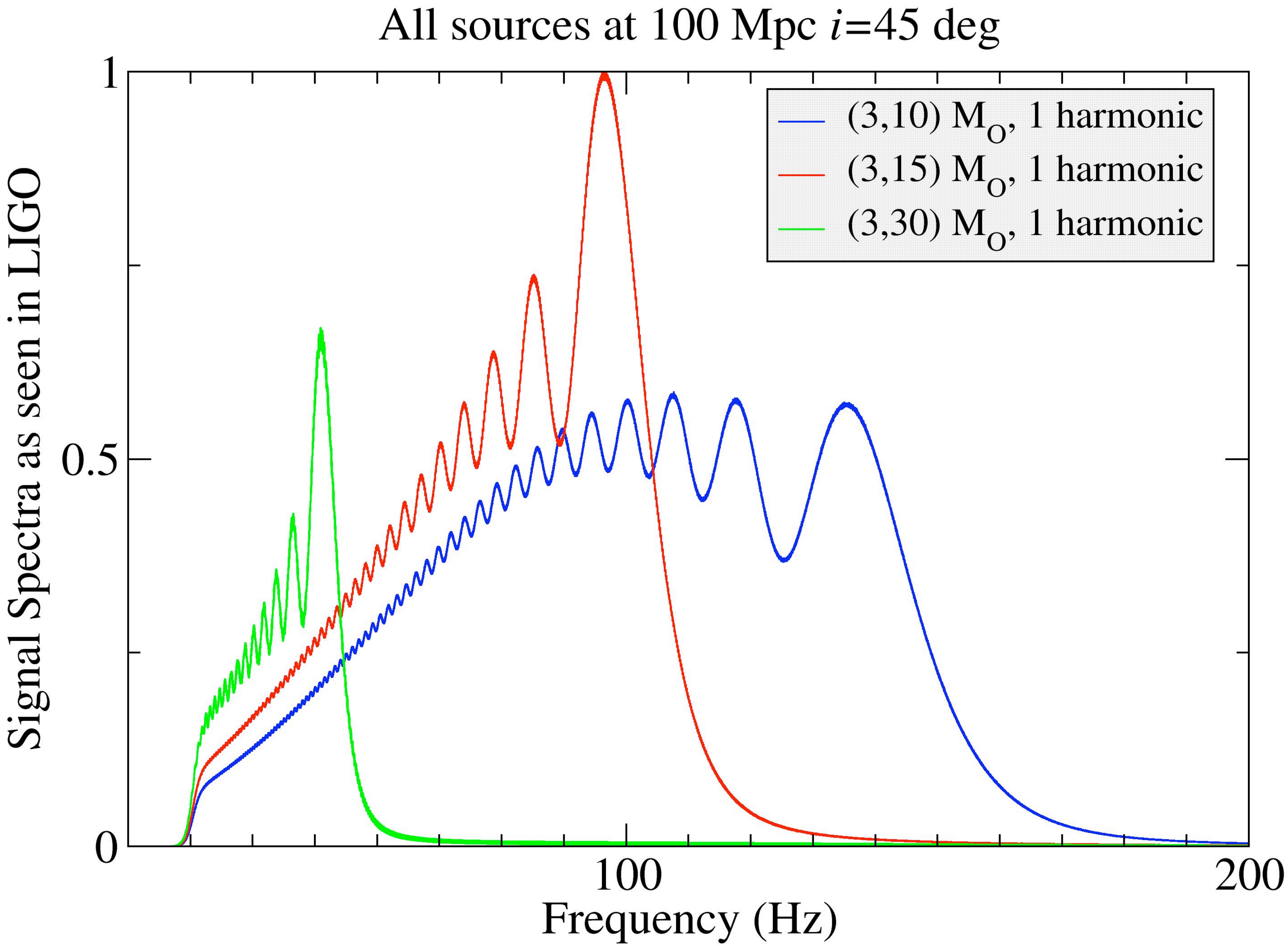
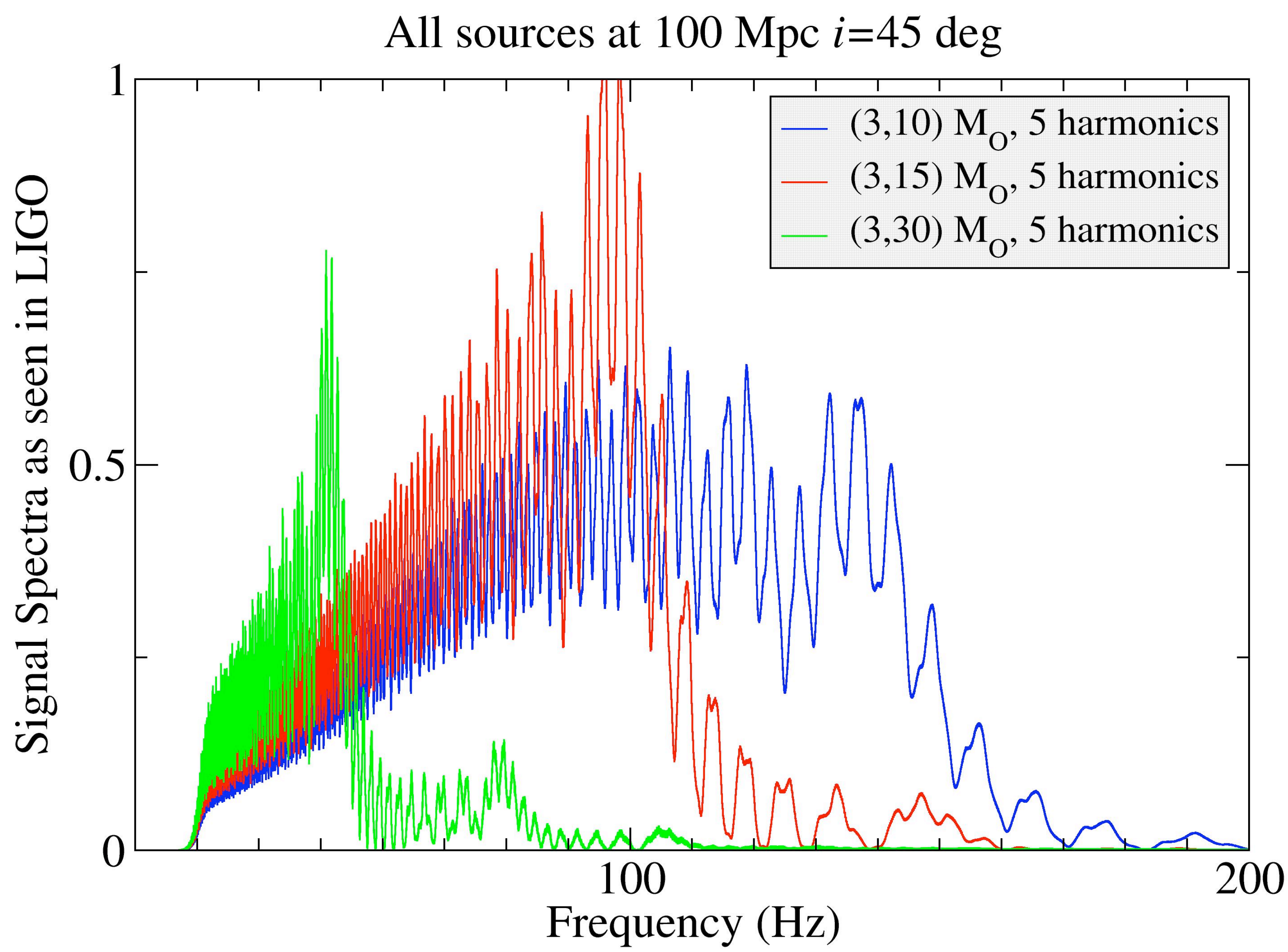

All sources at 100 Mpc *i*=45 deg
(3,10) M_O, 1 harmonic
(3,15) M_O, 1 harmonic
(3,30) M_O, 1 harmonic
Signal Spectra as seen in LIGO
Frequency (Hz)
1
0.5
0
100
200



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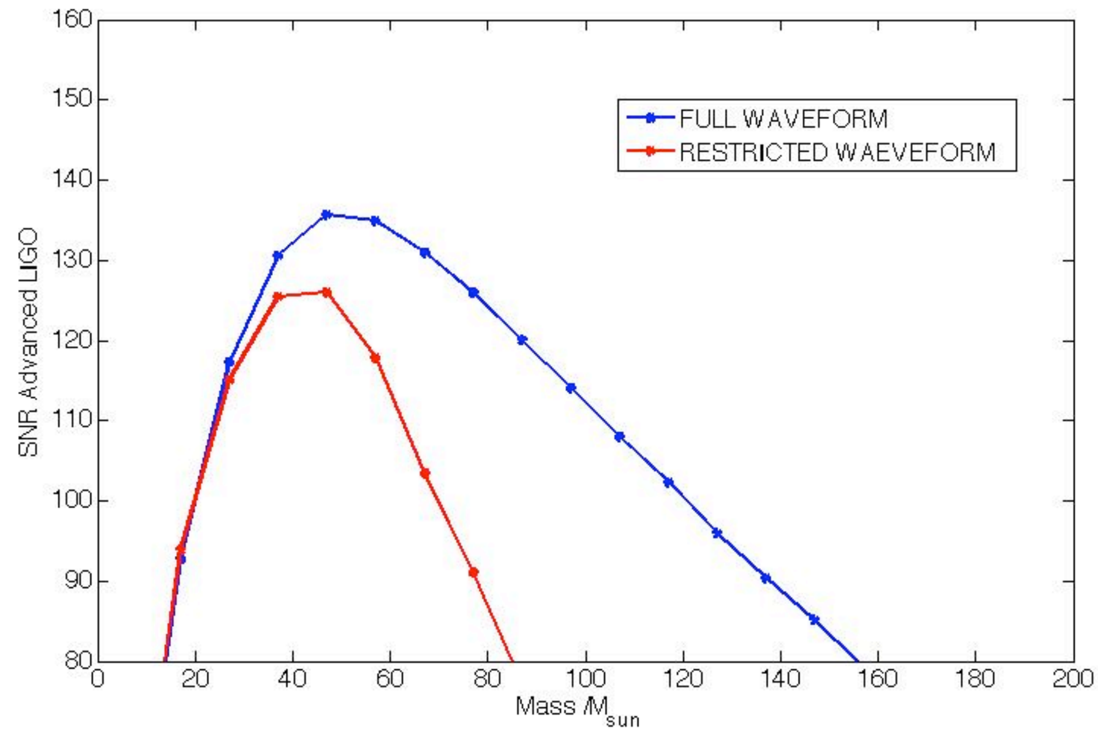
What can we do with the *Full* waveform?

Extend the mass reach…

- Detectors are limited by a lower cut off frequency - LIGO ~ 40Hz.

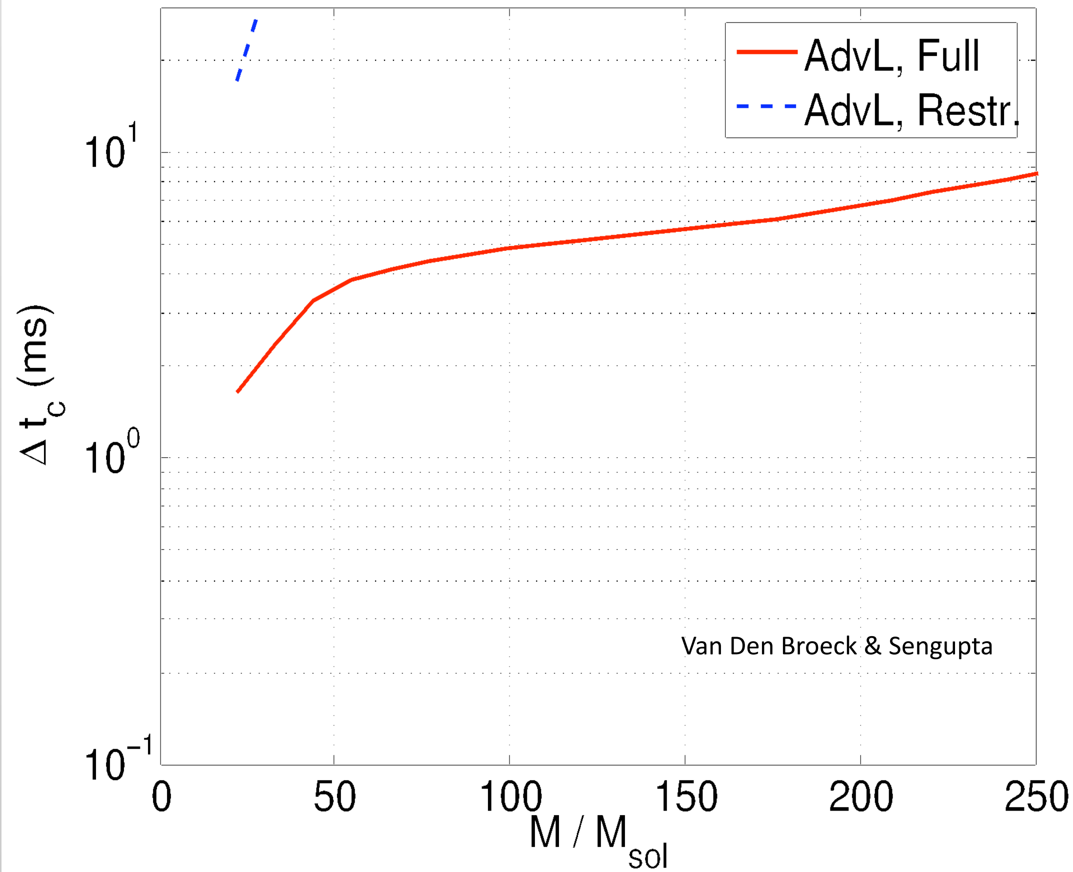
- Heavier systems reach the merger stage emitting GW at lower frequencies than lower mass systems.



- The higher harmonics in the full waveform can be above the detectors' lower cut off frequency, whilst the dominant harmonic of the restricted waveform is below.

Parameter Estimation – Accuracy in arrival times…

- The full waveform contains more information than the restricted waveform.
- In the case of the restricted waveform intrinsic information about a source can only be obtained from the phasing.
- Taking into account the amplitude corrections present in the full waveform can dramatically reduce the uncertainties in parameter estimation.



Detection with the restricted waveform:

- 2 orthogonal polarisations: h_+ and $h_x = ih_+$
- Write template as:

$$h(t) = A_0(t)\cos[\,2\phi(t) + \varphi_2\,]$$

- Easy to analytically maximise SNR for both polarizations over the angle φ_2.
- Easy to normalise such that $\langle h,h \rangle = 1$.

Detection with the full waveform at one order beyond leading (0.5PN):

- 3 harmonics, contribute to h_+ and h_x at different phases.
- Write template as:

$$h(t) = A_1(t)\cos[\,\phi(t) + \varphi_1\,] + A_2(t)\cos[\,2\phi(t) + \varphi_2\,] + A_3(t)\cos[\,3\phi(t) + \varphi_3\,]$$

- Difficult to analytically maximise SNR over the *different* angles $\varphi_{1,2,3}$.
- What about normalisation?

Start from the beginning…

- We have 3 harmonics at two polarizations, so *there are 6* filters.

$$h_{+1} \quad h_{x1} \quad h_{+2} \quad h_{x2} \quad h_{+3} \quad h_{x3} \qquad \text{or} \qquad h_j \quad j = 1..6$$

- The filters need to be orthogonal and we require normalisation, such that:

$$\langle h,h\rangle = \langle h_j, h_k\rangle = 1$$

- Apply *Gram-Schmidt* orthonormalisation on each filter so that they are orthogonal.

- We can then write the template as:

$$h = \Sigma_{j=1..6}\, \alpha_i\, h_j$$

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Detection Templates – *full waveform*

Maximisation of SNR…

- We require $\langle h,h \rangle = \Sigma_{j=1..6}\ \alpha_i^2 \langle h_i, h_i \rangle = 1$
- Since we have orthonormalised h_i, the maximisation constraint is simply:

$$\Sigma_{j=1..6}\ \alpha_i^2 = 1$$

- $SNR = \langle\ data,\ template\ \rangle = \langle\ S, h\ \rangle = \Sigma_{j=1..6}\ \alpha_i \langle\ S, h_i\ \rangle$
- It is fairly simple to maximise the SNR with the above constraint, using a Lagrange multilplier.
- We find that the maximised SNR is simply:

$$\mathrm{SNR}_{\mathrm{max}} = \left[\ \Sigma_{j=1..6} \langle\ S, h_i\ \rangle^{2}\ \right]^{1/2}$$

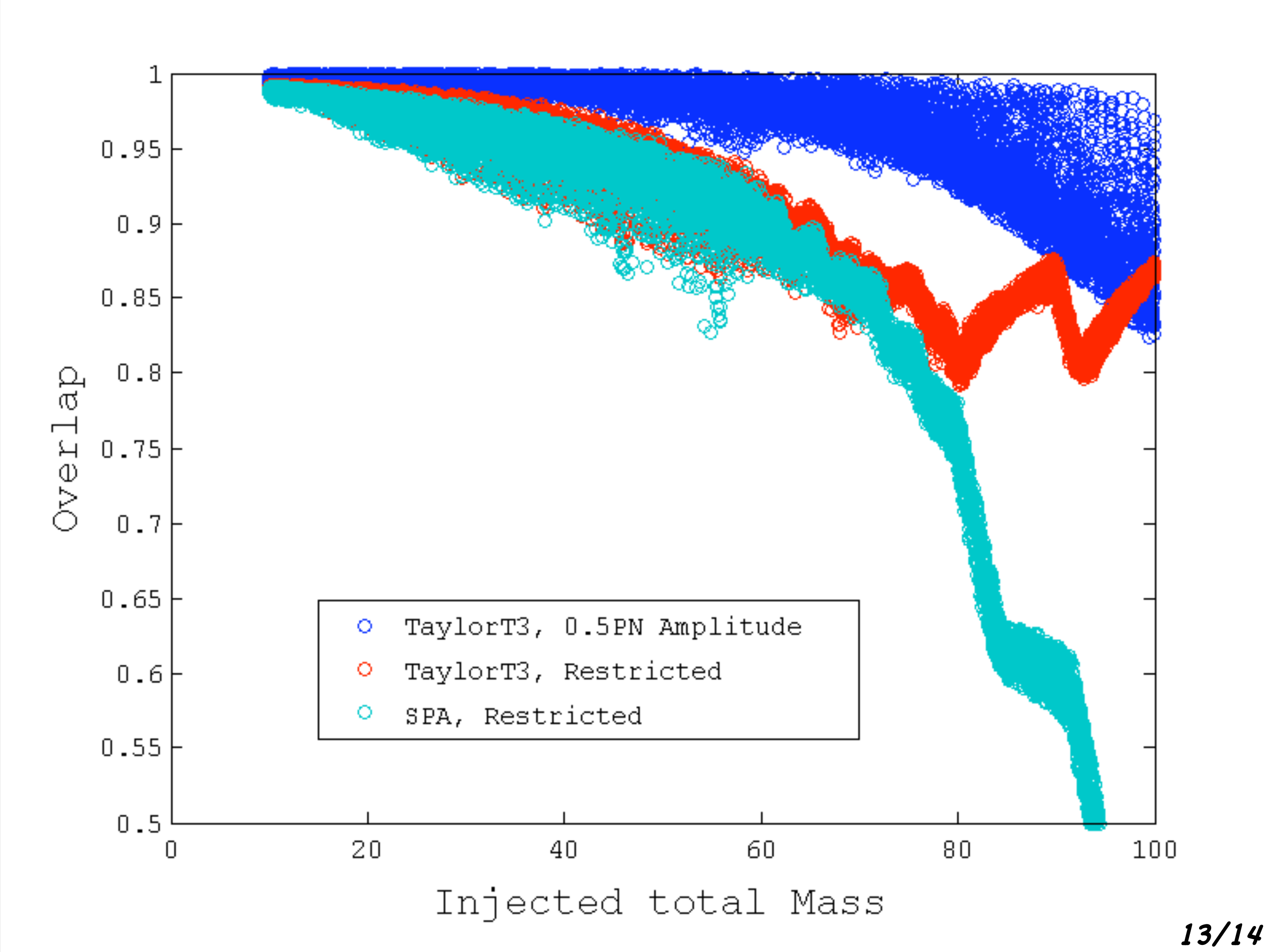
A Simulation Using the 0.5PN Amplitude Corrected Templates

Compute the template-signal overlap…

- 10,000 simulated signals at 2PN in phase *and* amplitude using TaylorT3 phasing model.
- Signal total mass between 10 and 100 M_{sun}.
- Use one SPA template bank with the same mass range.
- Compute best overlap for:
  - *Restricted* TaylorT3 templates at 2PN in phase, 0PN in amplitude.
  - TaylorT3 templates at 2PN in phase, 0.5PN in amplitude.
  - *Restricted* SPA templates at 2PN in phase, 0PN in amplitude.

A Simulation Using the 0.5PN Amplitude Corrected Templates

Compute the template-signal overlap…



- Investigate the use of the amplitude corrected filter in a real search for gravitational waves. What do we gain? Is it worthwhile?
- Search for simulated signals using template banks with these filters to study the parameter estimation benefits using a Monte Carlo style method.
- Can we reduce false alarm rate (by using parameter-based vetoes) in gravitational wave searches with amplitude corrected templates?
- Can we filter with *even* higher order amplitude corrected templates?
- The use of higher harmonics is expected to become more significant for advanced detectors.