Cross-Correlation Methods in Continuous Gravitational-Wave Searches

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Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

Outline

Gravitational-Wave Correlations

- Noisy data from GW Detector:

$$x(t) = n(t) + h(t) = n(t) + \overset{\leftrightarrow}{h}(t) : \overset{\leftrightarrow}{d}$$

- Correlate data btwn detectors (Fourier domain)

$$\langle \tilde{x}_1^*(f) \tilde{x}_2(f') \rangle = \langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle = \overset{\leftrightarrow}{d}_1 : \langle \overset{\leftrightarrow}{\tilde{h}}_1^*(f) \otimes \overset{\leftrightarrow}{\tilde{h}}_2(f') \rangle : \overset{\leftrightarrow}{d}_2$$

- For stochastic backgrounds

$$\langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle = \delta(f - f') \gamma_{12}(f) \frac{S_{\text{gw}}(f)}{2}$$

$S_{\text{gw}}(f)$ encodes spectrum; $\gamma_{12}(f)$ encodes geometry

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Applications and Outlook

Application to Stochastic Background

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Detection Statistic

- Optimally filtered cross-correlation statistic

$$Y = \int df\, \tilde{x}_1^*(f)\, Q(f)\, \tilde{x}_2(f)$$

- Filter encodes expected spectrum & spatial distribution (isotropic, pointlike, spherical harmonics ...)

$$Q(f) \propto \frac{\gamma_{12}^*(f) S_{gw}^{exp}(f)}{S_{n1}(f) S_{n2}(f)}$$

- "Radiometer" search for ptlike srcs incl targeting Sco X-1: known sky location, unknown frequency
  Ballmer, *CQG* **23**, S179 (2006); LSC, *PRD* **76**, 082003 (2007)

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Signal Model
Cross-Correlation Search
Relationships Among CW Searches

Gravitational Waves from Quasiperiodic Sources

- Sco X-1 is Low-Mass X-Ray Binary:
  accreting neutron star in orbit w/companion
- Rotating NS w/deformation emits nearly sinusoidal signal

$$\overset{\leftrightarrow}{h}(t) = h_0 \left[\frac{1 + \cos^2 \iota}{2} \cos \Phi(\tau(t)) \overset{\leftrightarrow}{e}_+ + \cos \iota \sin \Phi(\tau(t)) \overset{\leftrightarrow}{e}_\times \right]$$

  - $\Phi(\tau)$: phase evolution in rest frame;
  - $\tau(t)$: Doppler mod from detector motion (& binary orbit)
- Features of signal model missing from stoch search:
  - Doppler shift @ each detector:
    correlations peaked @ different freqs
  - Long-term coherence:
    can also correlate data @ different times

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Cross-Correlation of Continuous GW Signals

- Cross-correlation of signal w/intrinsic frequency f_0:

$$\tilde{h}_1^*(f_1)\tilde{h}_2(f_2) = h_0^2 \tilde{\mathcal{G}}_{12}\delta_{\Delta T}(f_0 - f_1 - \delta f_1)\delta_{\Delta T}(f_0 - f_2 - \delta f_2)$$

  - $\tilde{h}_I(f)$ is Short Fourier Transform, duration ΔT
  - $\delta_{\Delta T}(f - f') = \int_{-\Delta T/2}^{\Delta T/2} dt\, e^{i2\pi(f-f')t}$
  - $\tilde{h}_1$ & $\tilde{h}_2$ can be same or different times or detectors
  - δf_I is relevant Doppler shift

- For given set of params, can add products of all SFT pairs

$$Y = \sum_{IJ} Q_{IJ}\tilde{x}_I^*(f_0 - \delta f_I)\tilde{x}_J(f_0 - \delta f_J) \qquad Q_{IJ} \propto \frac{\tilde{\mathcal{G}}_{IJ}^*}{S_{n,I}(f_0)S_{n,J}(f_0)}$$

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Computational Costs and Frequency Resolution

- Correlating arbitrary times fine if know freq, sky pos, etc
- Fine param space resolution problematic if have to search
  e.g., tmplts close in freq lose phase coherence over time

$$N_{\text{tmplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta \dot{f}} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches are semi-coherent: deliberately limit
  coherent integration time & param space resolution
  to keep number of templates manageable

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Synchronous Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	N	Y	N	N	N	N	N	N
$x_2(t_0)$	Y	N	N	N	N	N	N	N
$x_1(t_1)$	N	N	N	Y	N	N	N	N
$x_2(t_1)$	N	N	Y	N	N	N	N	N
$x_1(t_2)$	N	N	N	N	N	Y	N	N
$x_2(t_2)$	N	N	N	N	Y	N	N	N
$x_1(t_3)$	N	N	N	N	N	N	N	Y
$x_2(t_3)$	N	N	N	N	N	N	Y	N

"Stochastic-style": correlate data @ same time, diff detectors

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Fully Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y

Combine all SFT pairs; as with standard $\mathcal{F}$-statistic,
quadratic combination of all SFTs

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Excess Power Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	N	N	N	N	N	N	N
$x_2(t_0)$	N	Y	N	N	N	N	N	N
$x_1(t_1)$	N	N	Y	N	N	N	N	N
$x_2(t_1)$	N	N	N	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	N	N	N
$x_2(t_2)$	N	N	N	N	N	Y	N	N
$x_1(t_3)$	N	N	N	N	N	N	Y	N
$x_2(t_3)$	N	N	N	N	N	N	N	Y

Only consider "diagonal" auto-correlations

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Semi Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

Coherently combine within epochs

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Lag-Limited Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_1(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

"Sliding" semi-coherent search

Applying the Cross-Correlation Method

- Can apply this to blind searches,
  or to searches where some but not all params known:
  - Directed search: known interesting location
    (e.g., galactic center, SN remnant)
  - LMXBs: know sky pos, no spindown;
    unknown freq & orbital params
- See Amaldi talk next week by
  JTW, Chung, Krishnan, Melatos, Owen, Peralta