Coating thermal noise of a finite-size cylindrical mirror

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Our paper of this topic coming soon in Physical Review D

… or gr-qc/ 0903.2902

0. Abstract

Mirror thermal noise related to coating

Finite-size mirror

Without half-infinite approximation

Applicable to thin mirror (Quantum experiment)

Contents

1. Introduction

Mirror thermal noise

Fundamental limit of interferometric gravitational wave detector

Calculation : Half-infinite approximation

Mirror > Laser beam

Quantum optics experiment with mechanical oscillators

ex.. 10 m prototype at Hannover, Germany

(Talks of Stefan Gossler, Fumiko Kawazoe, Kentaro Somiya)

Larger radiation pressure noise

↓

Smaller mirror mass

↓ **Mirror radius > Laser beam radius**

Thinner mirror

Half-infinite approximation is not appropriate.

Calculation with finite size mirror

(Accurate calculation for gravitational wave detection)

2. Review of coating thermal noise

Fluctuation Dissipation Theorem

Relation between thermal noise and loss

2-1. Kinds of dissipation

Structure damping (Brownian noise)
Thermoelastic damping (Thermoelastic noise and thermorefractive noise)

(1) Structure damping

Mechanism : unknown

Frequency independent dissipation of strain energy

(2)Thermoelastic damping

Mechanism : well known

(i) Inhomogeneous strain
(ii) Thermal gradient
(iii)Thermal relaxation

Frequency depend dissipation

2-2. Brownian noise

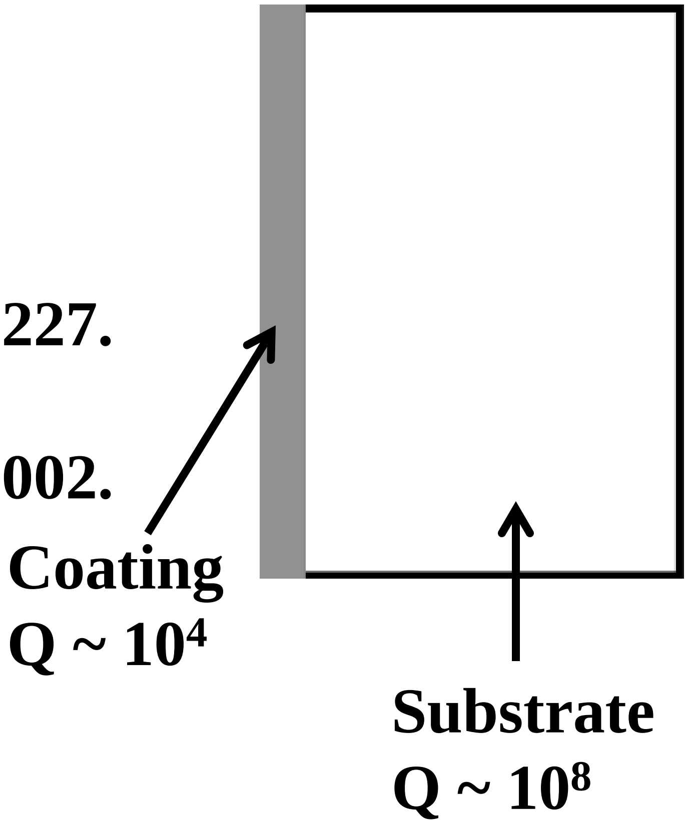
(1) Substrate Brownian noise

Finite size mirror

F. Bondu *et al*., Physics Letters A 246 (1998) 227.

Y. Liu *et al*., Physical Review D 62 (2000) 122002.

This is not a topic in this talk.



(2) Coating Brownian noise

K. Yamamoto *et al*., Physics Letters A 305 (2002) 18.

Finite size (not thin) mirror

Thin mirror is a topic in this talk.

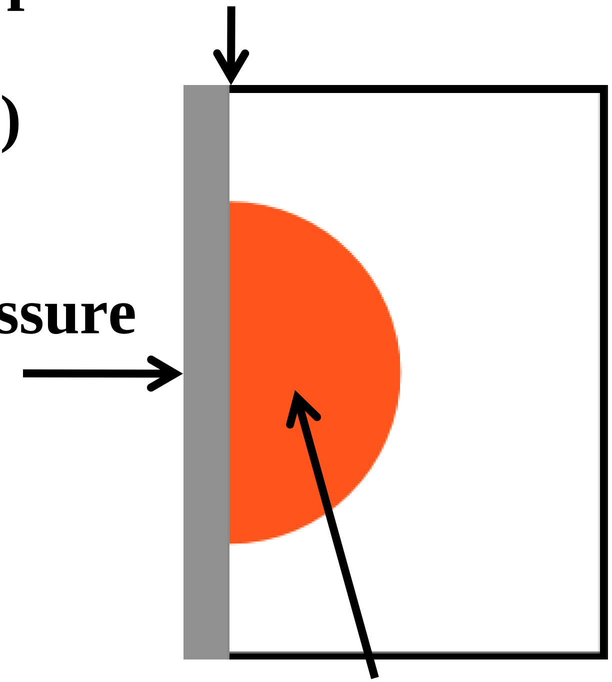
2-3. Thermoelastic noise

(1) Substrate thermoelastic noise (damping)

Temperature gradient in substrate

Finite size mirror

Y. Liu *et al.*, Physical Review D 62 (2000) 122002.



(2) Coating thermoelastic noise (damping)

Temperature difference between substrate and coating due to difference of material

Finite size mirror

V.B. Braginsky *et al.*, Physics Letters A 312 (2003) 244.

Summation of substrate and coating thermoelastic noise with correlation is a topic in this talk.

2-4. Correlation between substrate and coating thermoelastic noise

Another explanation for thermoelastic noise

Temperature fluctuation and thermal expansion (α)

↓

Fluctuation of surface of coating

If temperature near beam spot becomes higher,
both substrate and coating expand.

⟶ **Correlation**

2-4. Correlation between substrate and coating thermoelastic noise

Sign of correlation between substrate and coating thermoelastic noise

(1) If α of coating is smaller than α of substrate, coating expansion is not as well as substrate expansion.

→ Negative correlation

(2)If α of coating is larger than α of substrate, correlation is positive.

Substrate : Fused silica, smaller α than typical value

Correlation between substrate and coating thermoelastic noise is positive.

2-5. Thermorefractive noise

Coating thermoelastic noise

Temperature fluctuation and thermal expansion (α)

↓

Fluctuation of surface of coating

Thermorefractive noise

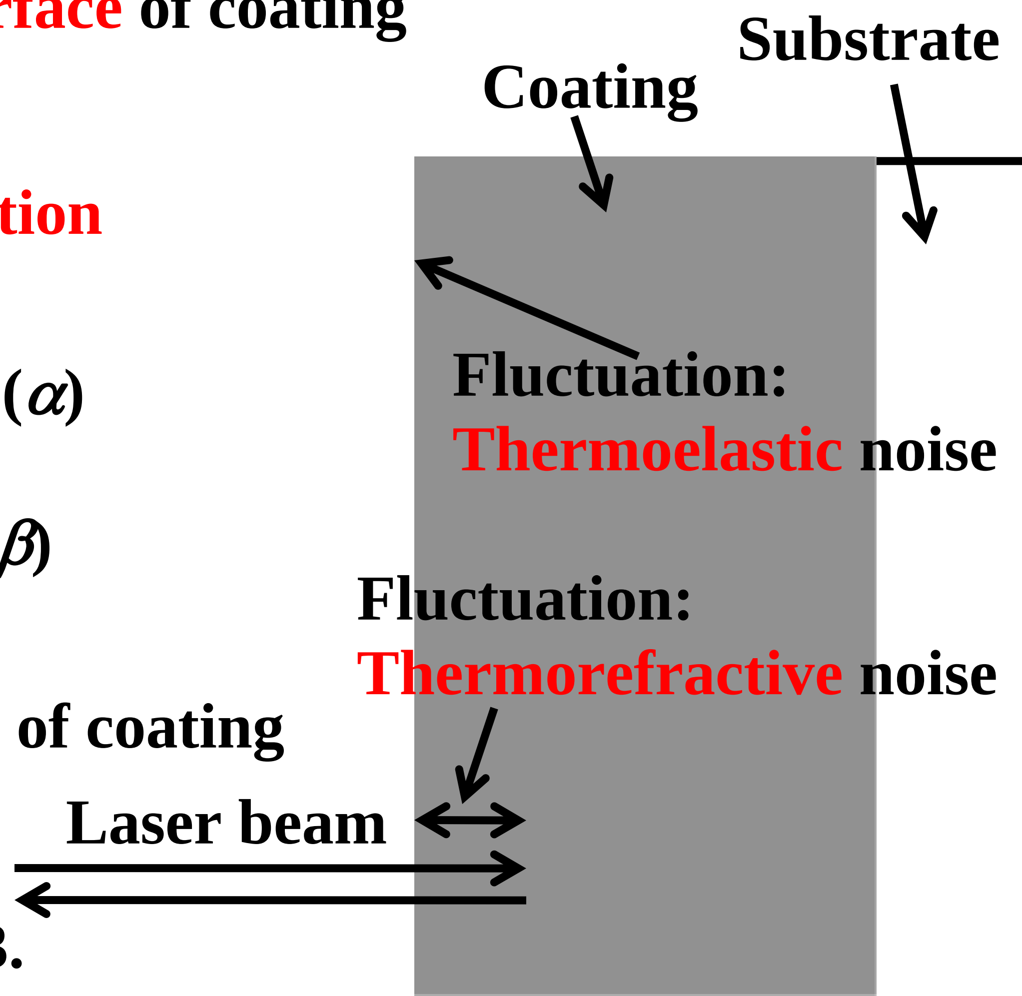
Temperature fluctuation

↓

Thermal expansion (α)
Thermal coefficient
of refractive index (β)

↓

Fluctuation of thickness of coating



V.B. Braginsky *et al.*,
Physics Letters A 271 (2000) 303.

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2-6. Thermo-optic noise

Thermo-optic noise

Summation of coating thermoelastic and thermorefractive noise

Correlation (same origin : temperature fluctuation)

Sign of correlation (in usual case)

Thermal expansion (α) : positive
Thermal coefficient of refractive index (β) : positive

2-6. Thermo-optic noise

If temperature becomes higher ...

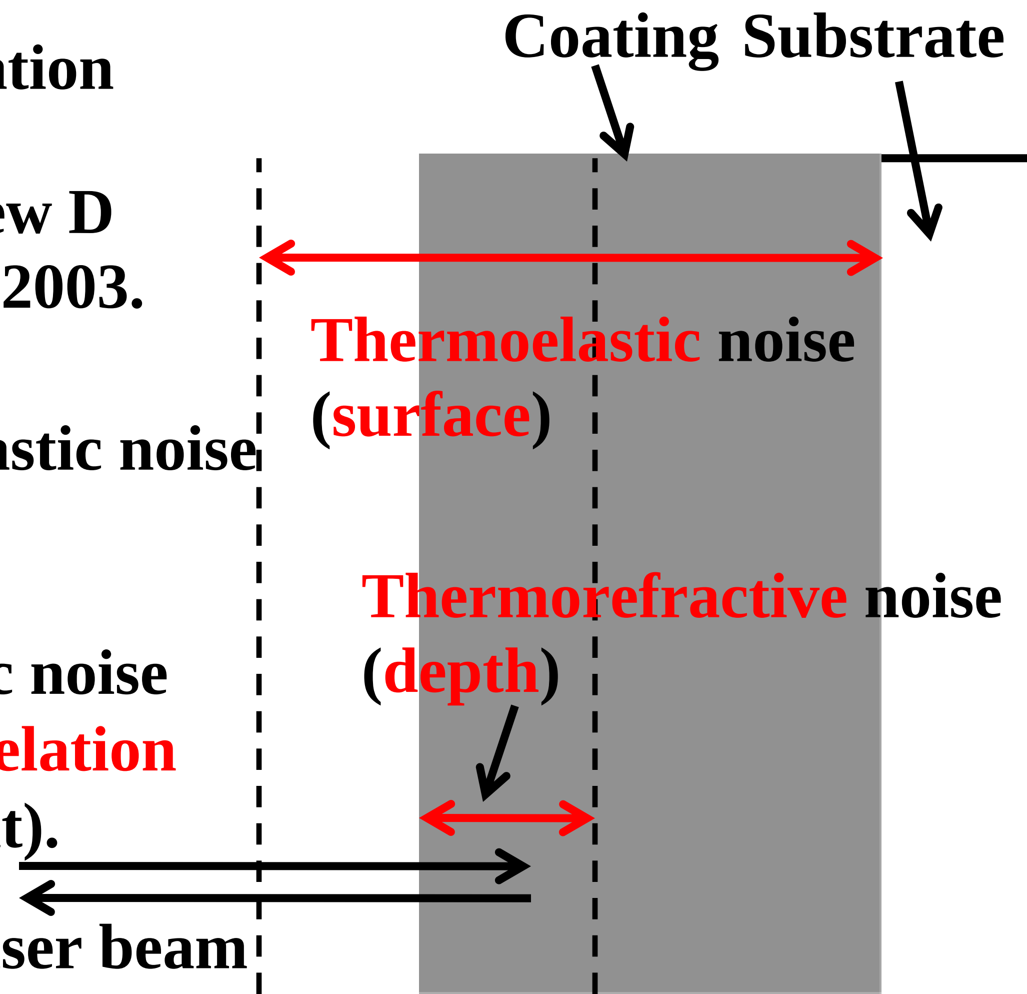
Thermoelastic noise : Shorter cavity length
Thermorefractive noise : Longer cavity length

Negative correlation

M. Evans *et al*., Physical Review D 78 (2008) 102003.

Half-infinite mirror
without substrate thermoelastic noise

Finite mirror
with substrate thermoelastic noise is a topic in this talk (Correlation must be taken into account).



3. Calculation method

Y. Levin *et al.*,
Physical Review D 271 (1998) 303.

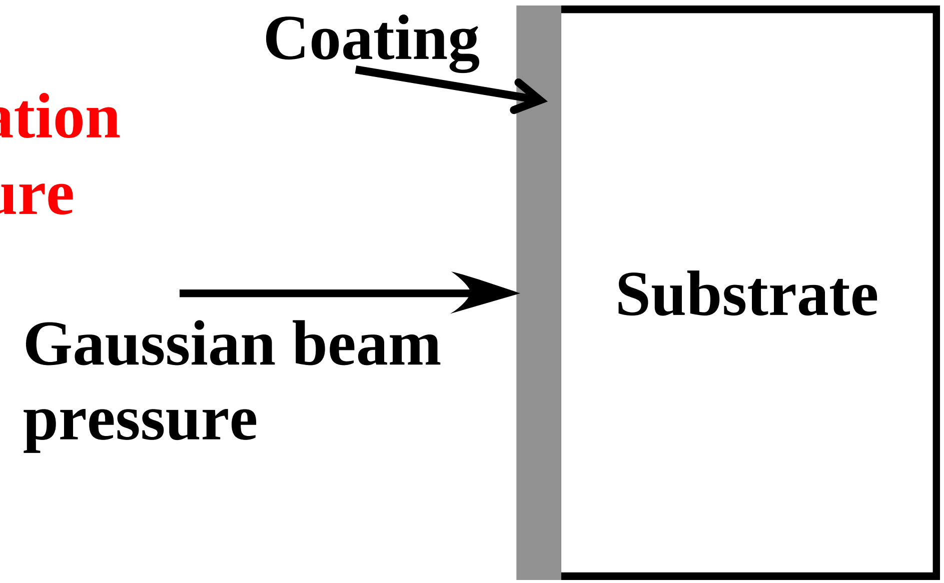
3-1. Levin method

Fluctuation Dissipation Theorem

Relation between (Brownian and thermoelastic) noise and loss under (beam profile) pressure on mirror surface

Levin method

Direct calculation of dissipation under (beam profile) pressure with equation of motion



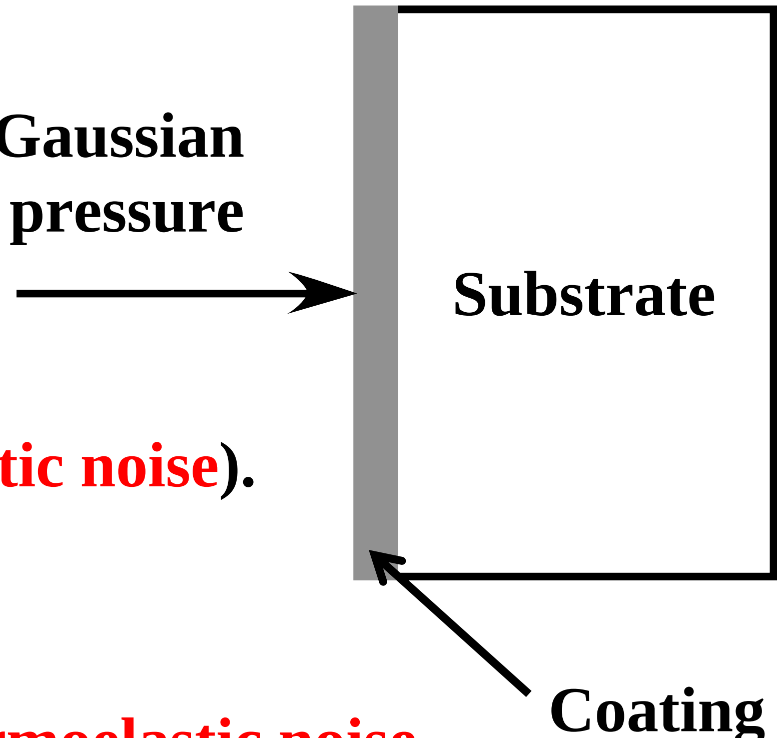
Half-infinite approximation : simple boundary condition

But, in this talk, mirror size is finite.

Complex boundary condition

3-2. Brownian noise and thermoelastic noise of thin mirror

Frequency of gravitational wave (100Hz) is smaller than resonant frequencies (10kHz) of mirror elastic modes.



Soft mirror has larger strain and dissipation (Brownian and thermoelastic noise).

Thinner mirror is softer and has larger Brownian and thermoelastic noise than half-infinite approximation.

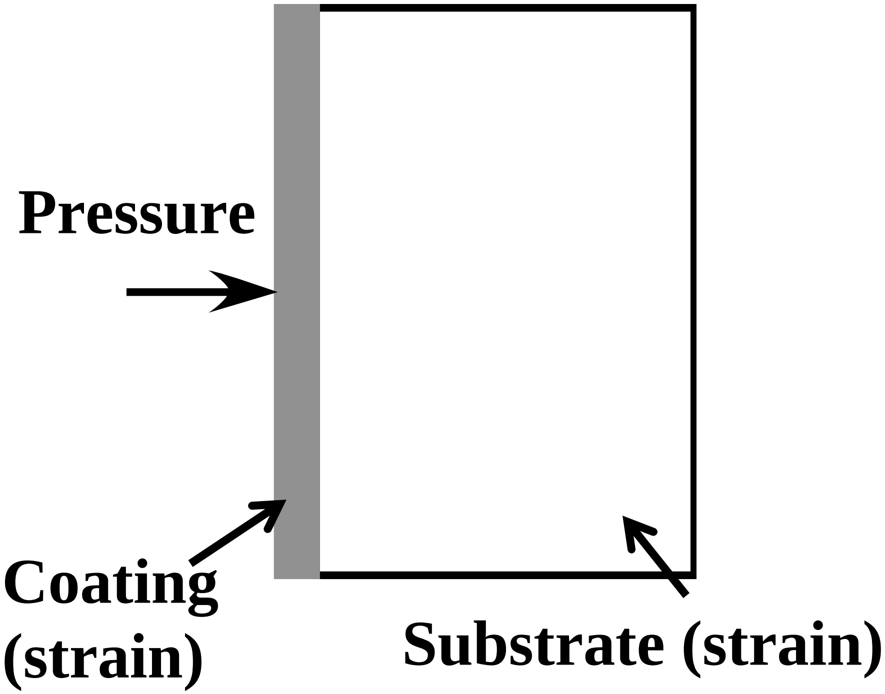
3-3. Thermorefractive noise of thin mirror

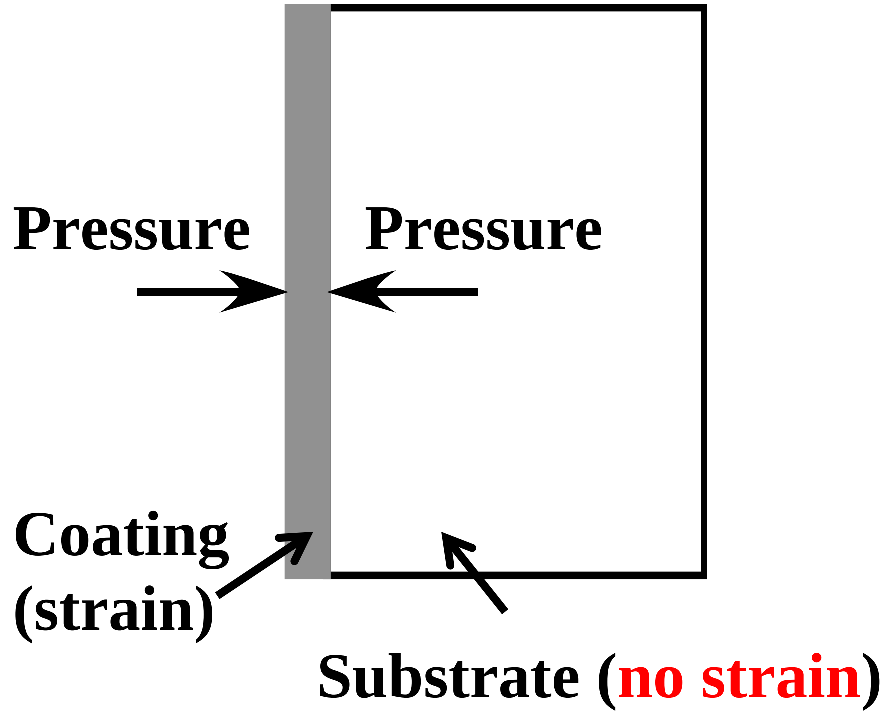
Thermoelastic noise :

fluctuation of surface

Thermorefractive noise :

fluctuation of coating thickness





Strain (of coating and substrate) and thermoelastic noise depend on the substrate size.

Thermorefractive noise is independent of the substrate size.

3-4. Cross check

Our calculation : finite mirror

If mirror is enough large,

our result must agree with half-infinite approximation.

If mirror is enough thin,

our result must agree with thin plate approximation.

Thin plate approximation : Thickness is smaller than radius.

Calculation method : Modal expansion, not Levin method

If our finite mirror result (Levin method) is consistent with thin plate approximation, it is a strong evidence of validity.

3-5. Thin plate approximation and modal expansion

Modal expansion : popular method long long time ago ...

P.R. Saulson, Physical Review D 42 (1990) 2437.

(1) Thermal motion of each resonant mode

(2) Summation

3-5. Thin plate approximation and modal expansion

Problems of modal expansion

(a) Many modes must be taken into account.

The order of 100 modes in the case of thick mirror

A. Gillespie *et al*., Physical Review D 52 (1995) 577.

F. Bondu *et al*., Physics Letters A 198 (1995) 74.

(b) Inhomogeneously distributed loss

Modal expansion is invalid

because of correlation between modes.

K. Yamamoto *et al*., Physics Letters A

280 (2001) 289; 305 (2002) 18; 321 (2004) 79.

K. Yamamoto *et al*., Physical Review D 75 (2007) 082002.

3-5. Thin plate approximation and modal expansion

How about modal expansion in thin plate approximation ?

Only fundamental mode dominates thermal noise.

(a) Only fundamental mode must be taken into account.

(b) Correlation between other modes are negligible.

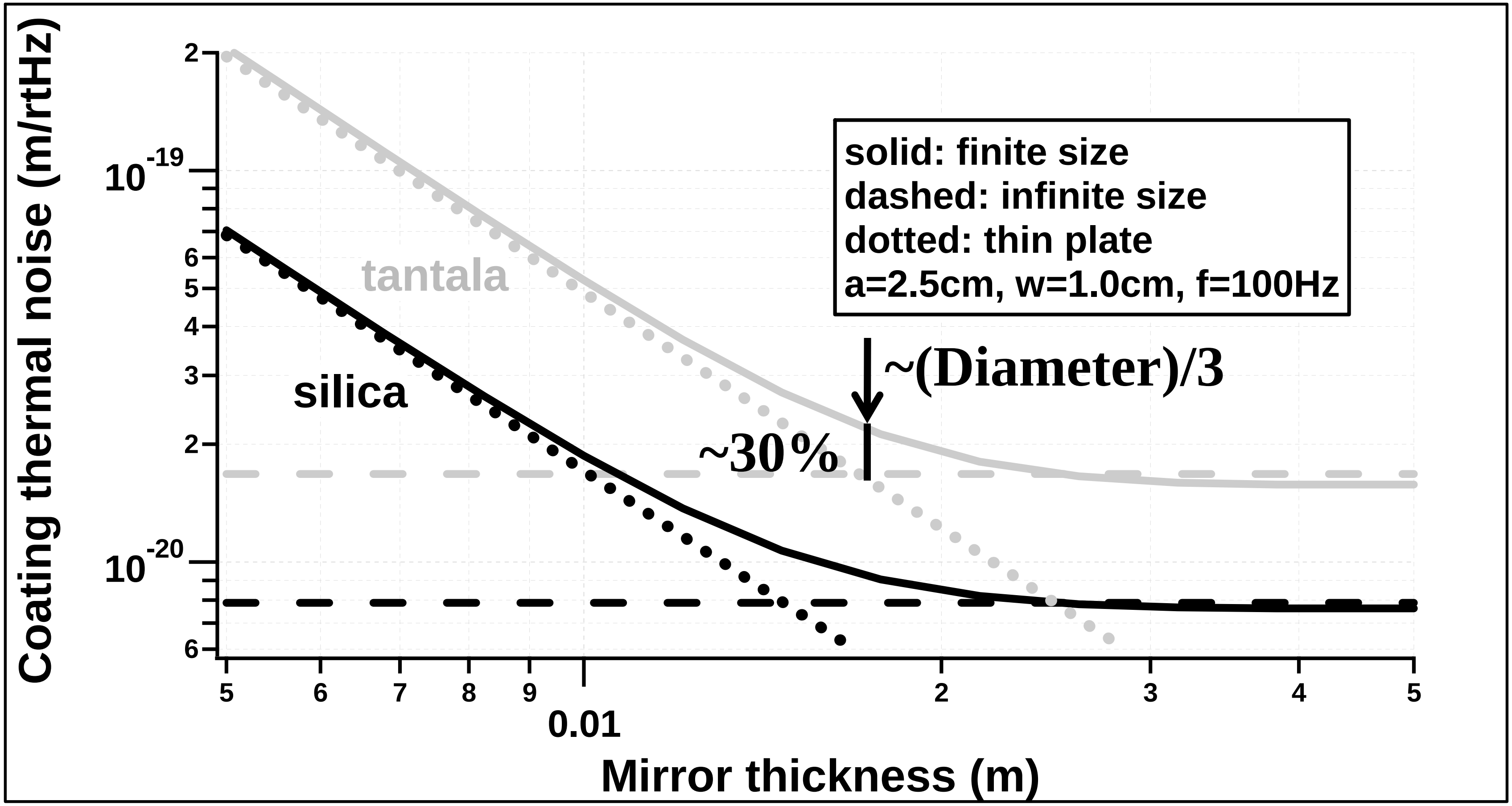
Calculation using modal expansion for thin plate is

(a) not hard and

(b) correct even if loss is inhomogeneous.

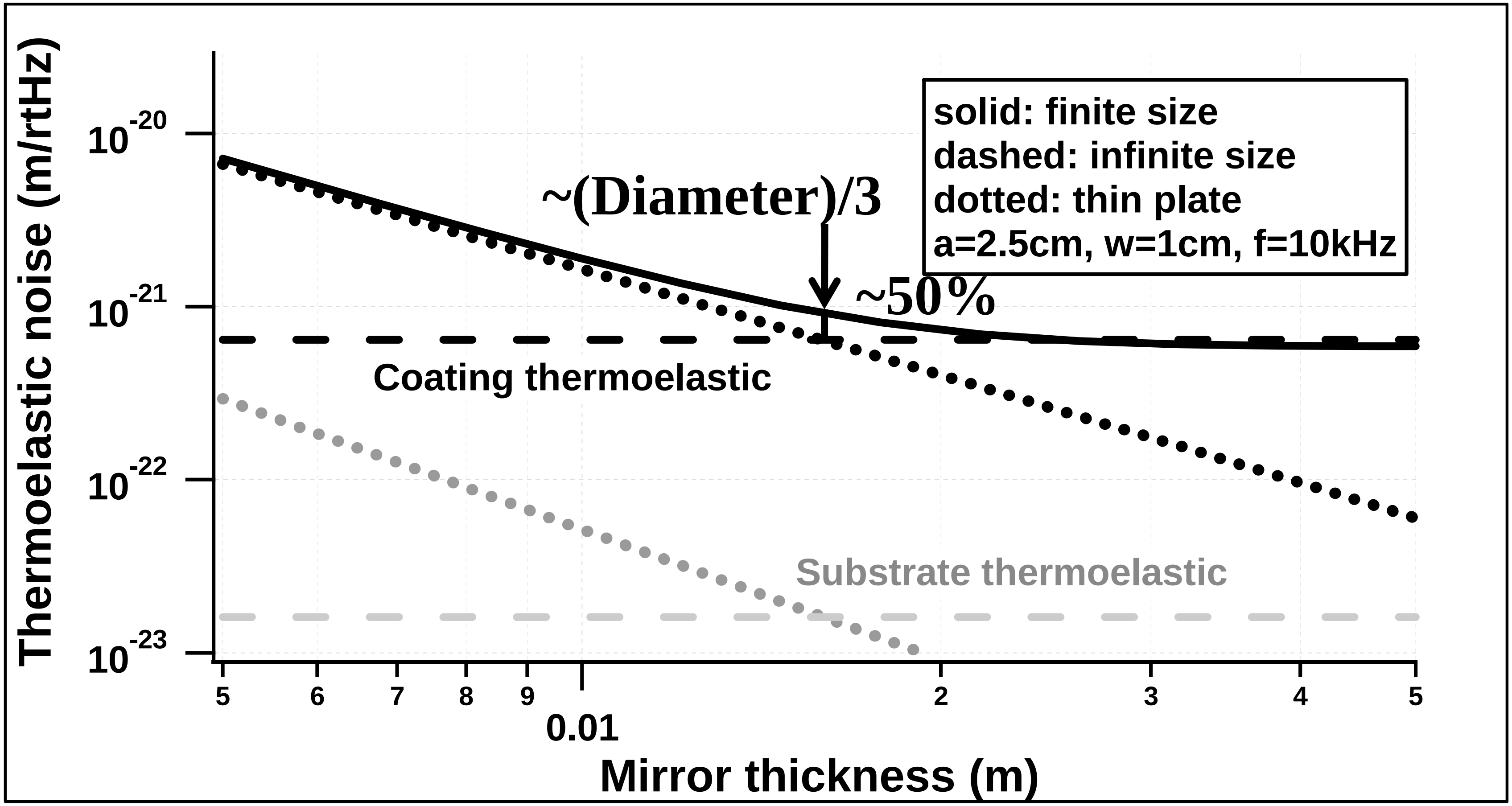
4. Results

4-1. Coating Brownian noise



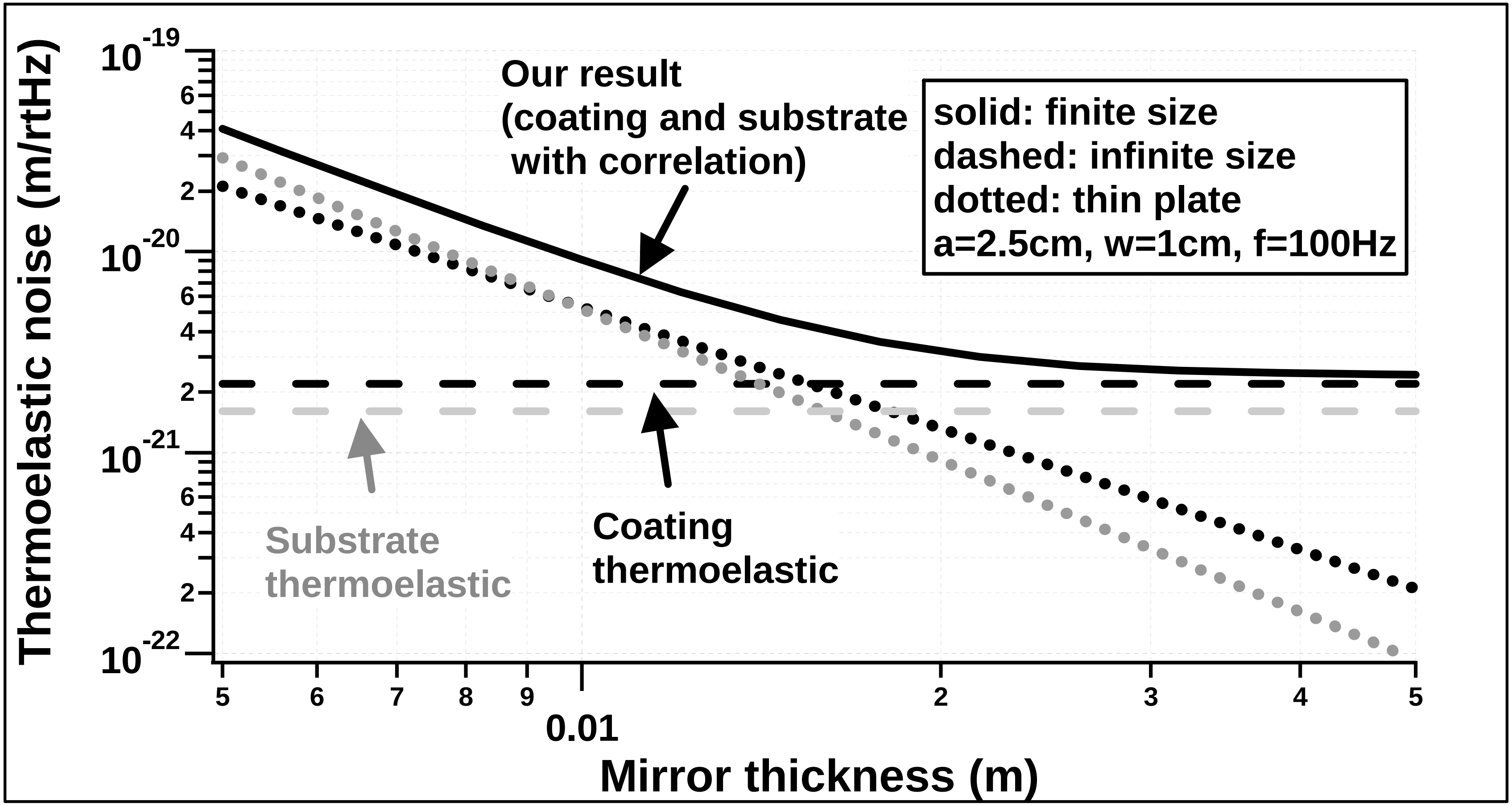
Our result **agrees with** **those of** **half-infinite**
and **thin plate** **approximations.**

4-2. Thermoelastic noise (substrate and coating) at 10 kHz



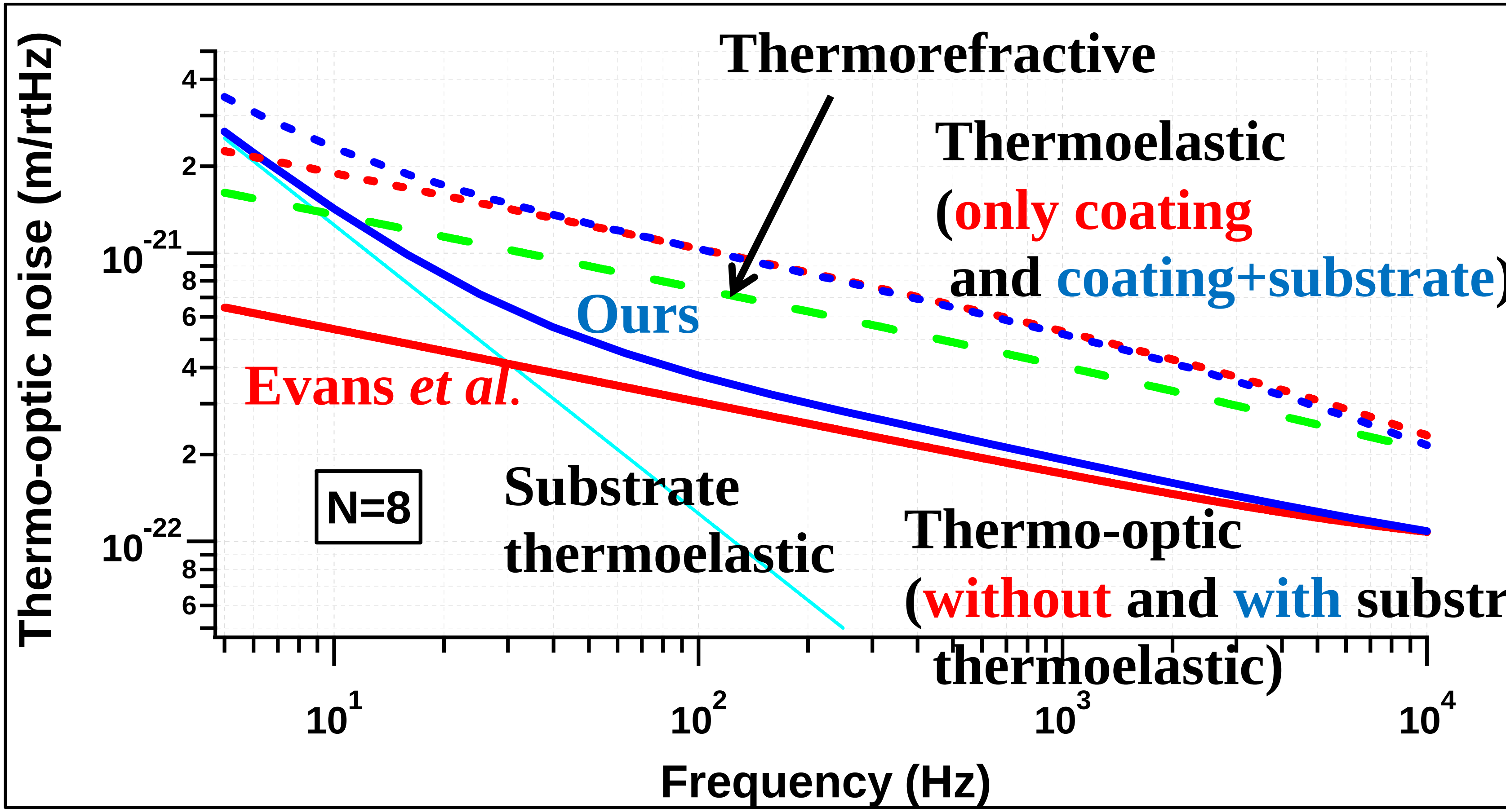
Our result agrees with those of half-infinite and thin plate approximations.

4-3. Thermoelastic noise (substrate and coating) at 100 Hz



Our result includes substrate thermoelastic noise and correlation (between substrate and coating thermoelastic noise).

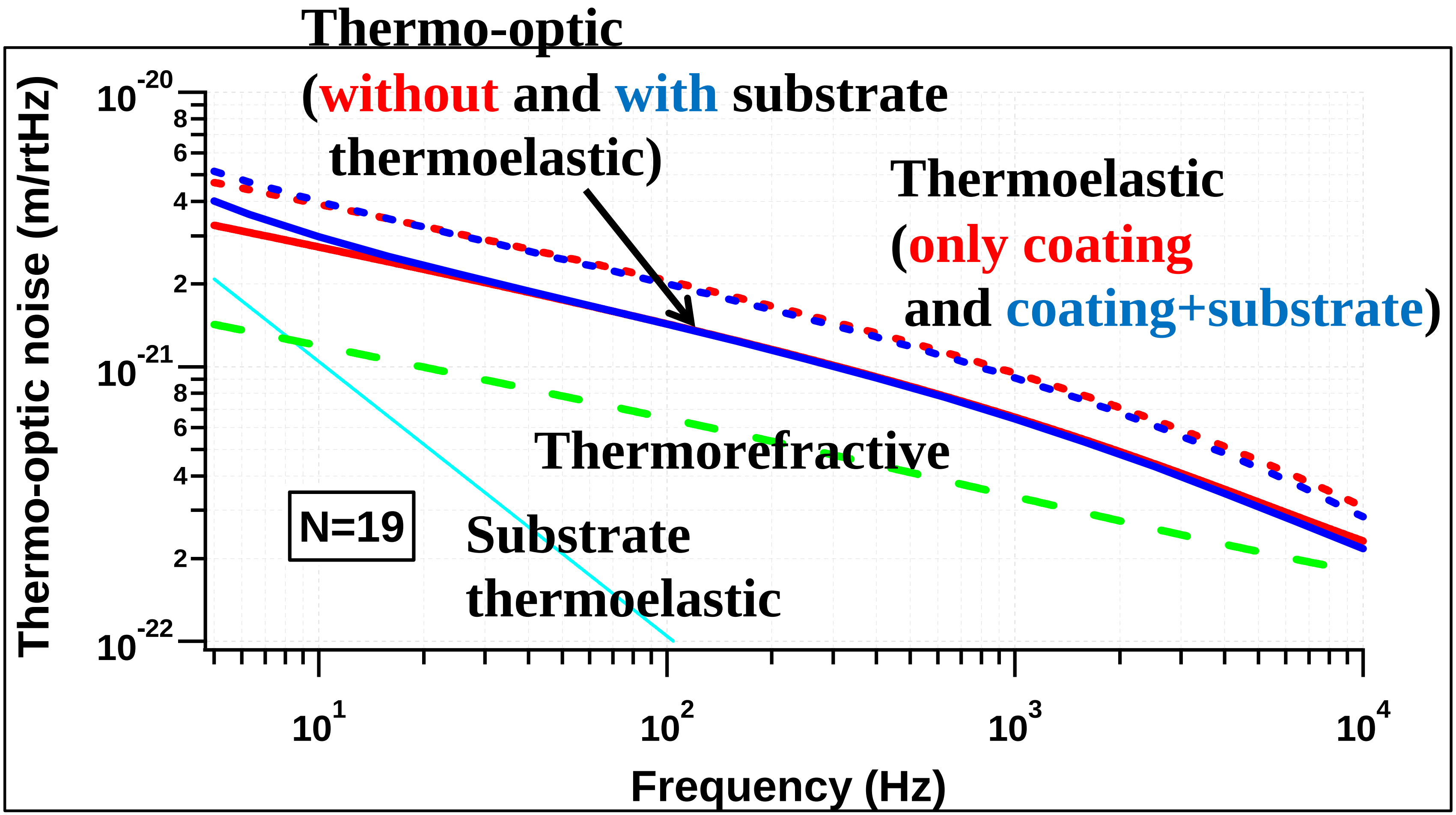
4-4. Thermo-optic noise with substrate thermoelastic noise (advanced LIGO front mirror)



Small thermo-optic noise (thermoelastic and thermorefractive noises cancel each other well).

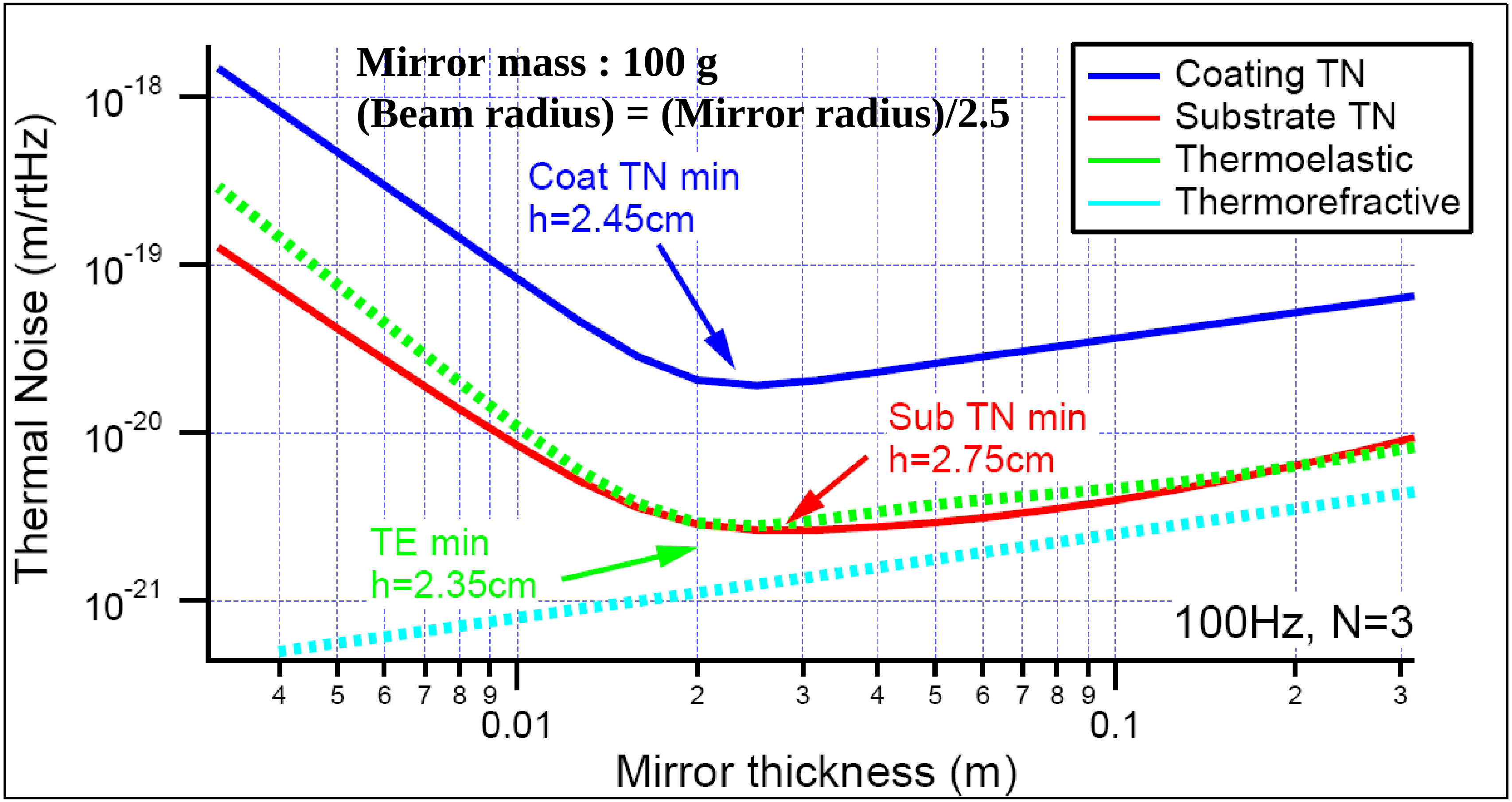
Our result (thermo-optic) includes substrate thermoelastic noise.

4-5. Thermo-optic noise with substrate thermoelastic noise (advanced LIGO end mirror)



Coating thermoelastic noise is larger than thermorefractive noise because of thick coating. Thermo-optic noise is larger.

4-6. Optimum mirror for quantum optics experiment (10m prototype, Hannover, Germany)



Thickness of optimum mirror is a half of the diameter.
(~4.8 cm)

5. Summary

(1) We calculated thermal noise of finite cylindrical mirror

and

summation of substrate and coating thermoelastic noise and thremorefractive noise with their correlations.

(2) Our results agree with half-infinite and thin plate approximation formulae.

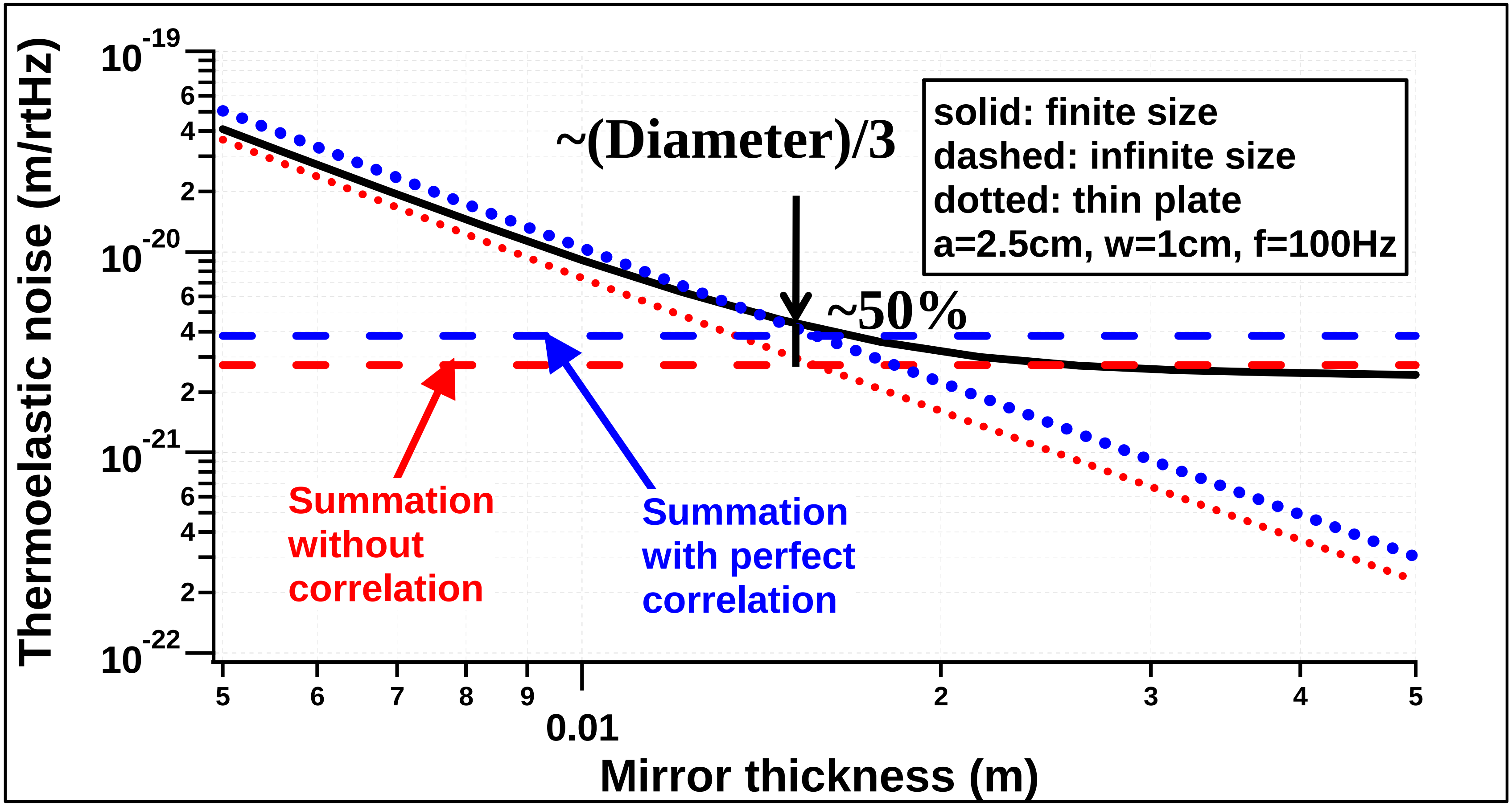
Half infinite approximation : (Thickness) > (Diameter)/3

Thin plate approximation : (Thickness) < (Diameter)/3

(3) Optimum specification of mirror for quantum optics experiment

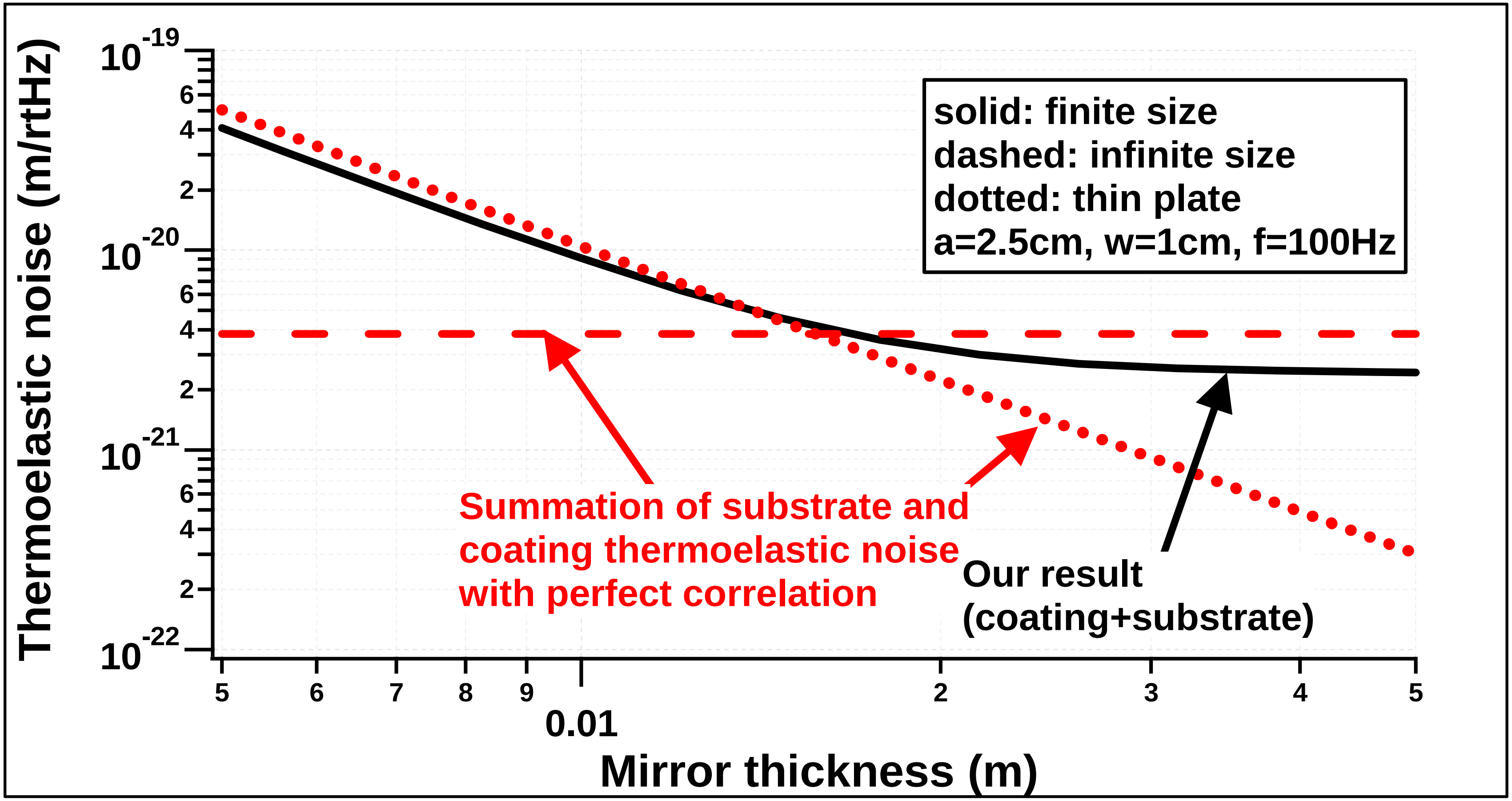
(Thickness) ~ (Diameter) /2

4-3. Thermoelastic noise (substrate and coating) at 100 Hz



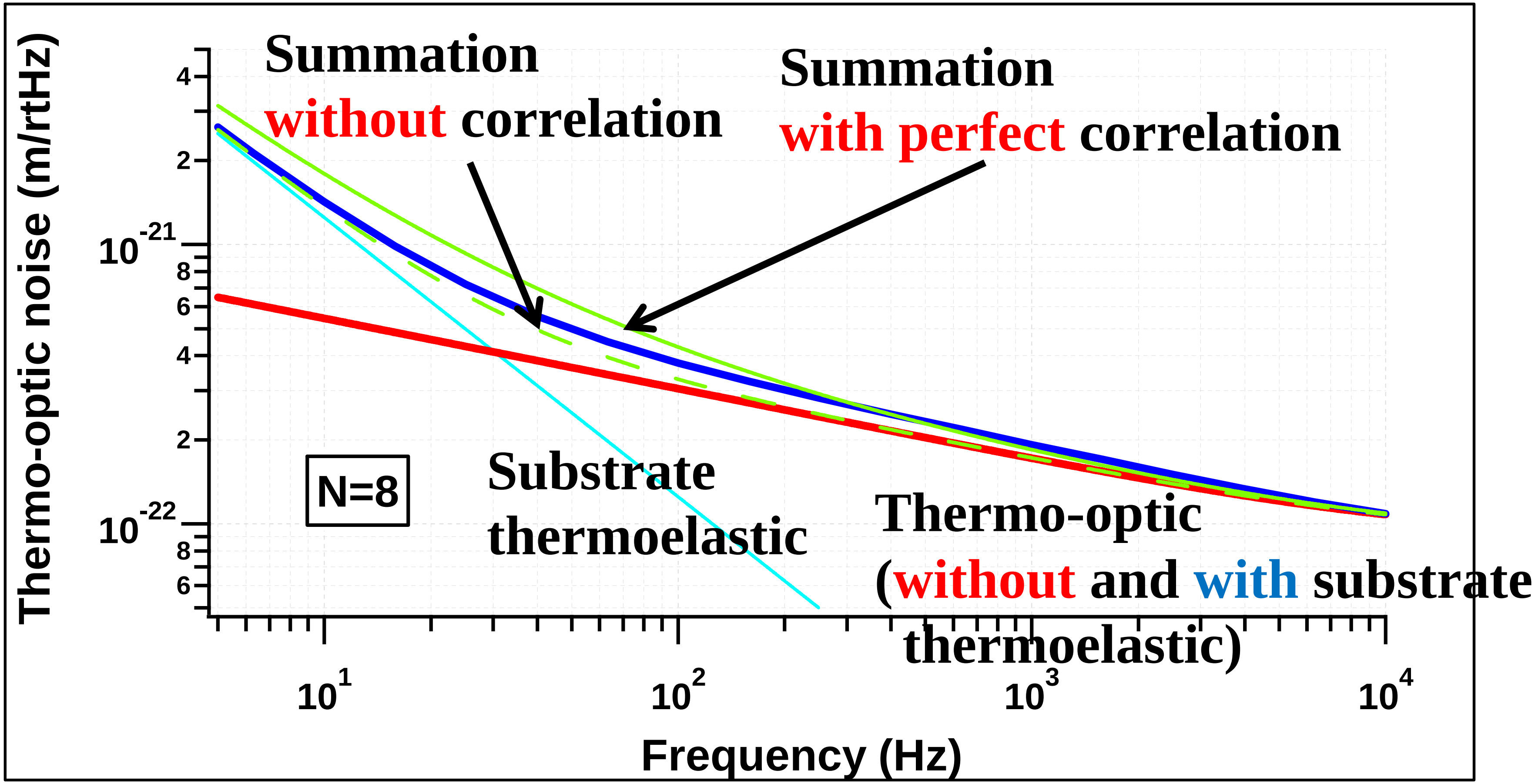
Correlation is not so strong.

4-3. Thermoelastic noise (substrate and coating) at 100 Hz



Our result is below summation with perfect correlation.

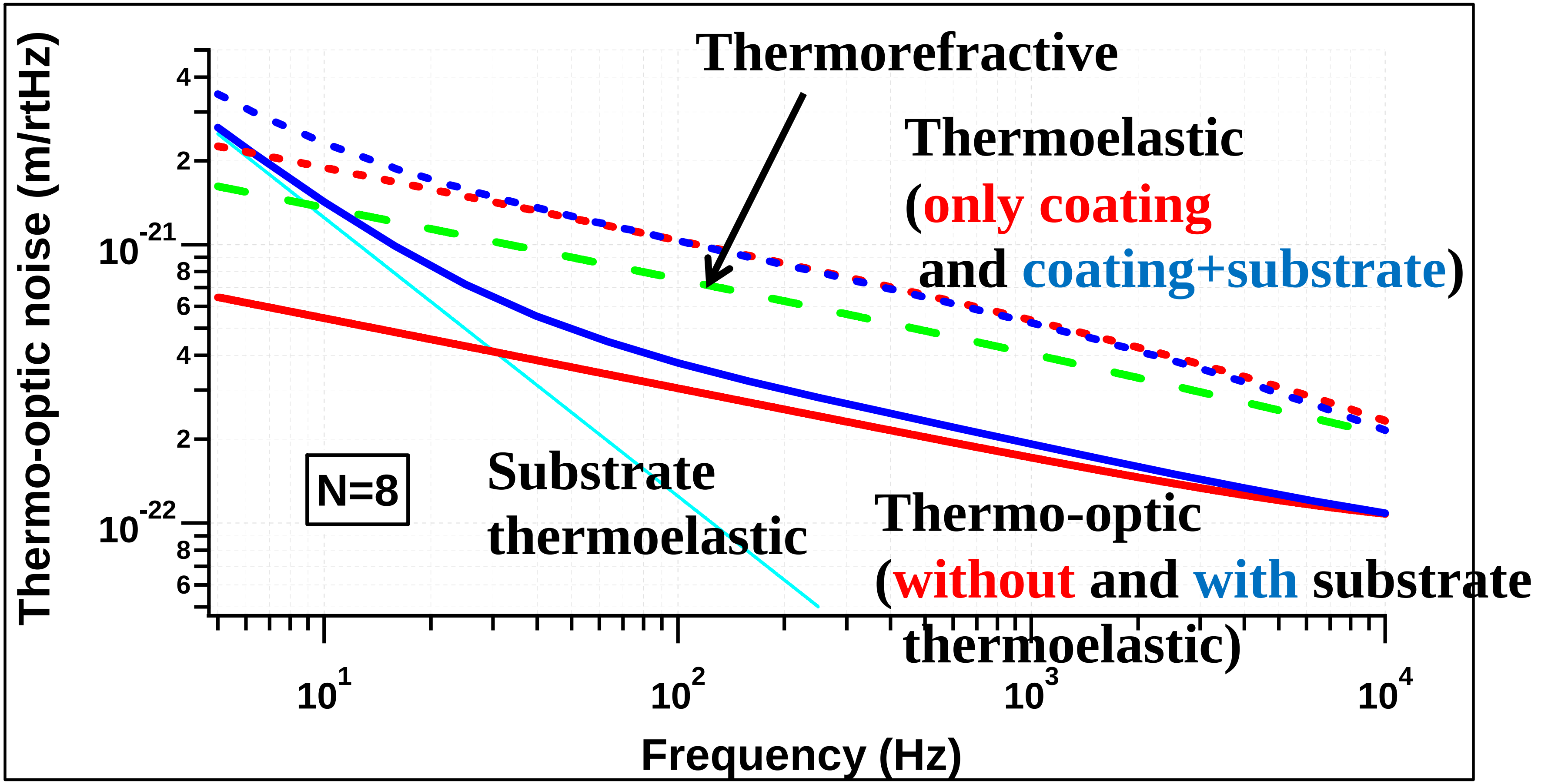
4-4. Thermo-optic noise with substrate thermoelastic noise (advanced LIGO front mirror)



Small thermo-optic noise (thermoelastic and thermorefractive noises cancel each other well).

Correlation is not so strong.

4-4. Thermo-optic noise with substrate thermoelastic noise (advanced LIGO front mirror)



Small thermo-optic noise (thermoelastic and thermorefractive noises cancel each other well).

Our result includes substrate thermoelastic noise.