Gravitational Waves

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REU Lecture

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Outline

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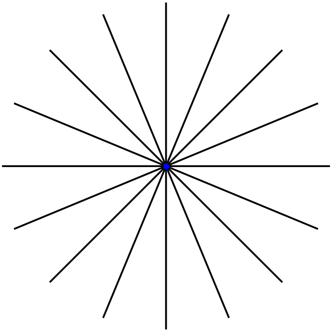
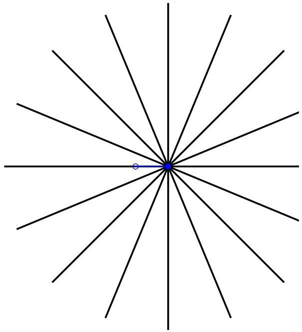
Action at a Distance

- Newtonian gravity: mass generates gravitational field
- Lines of force point towards object



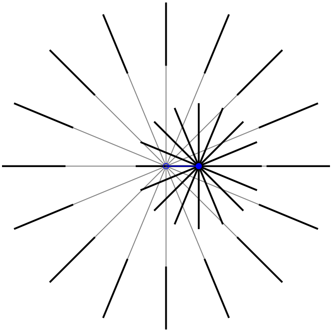
Issues with Causality

- Move object; Newton says: lines point to new location
- Relativity says: can't communicate faster than light to avoid paradoxes
- You could send me supraluminal messages via grav field



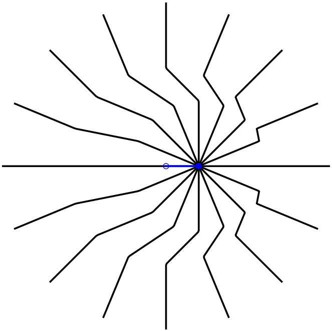
Gravitational Speed Limit

- If I'm 10 light years away, I can't know you moved the object 6 years ago
- Far away, gravitational field lines have to point to old location of the object



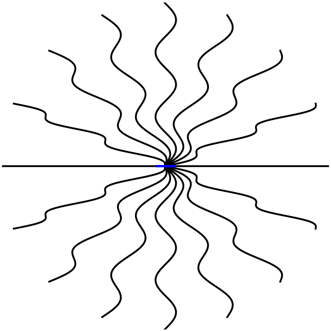
Gravitational Shock Wave

- Sudden motion (acceleration) of object generates gravitational shock wave expanding at speed of light



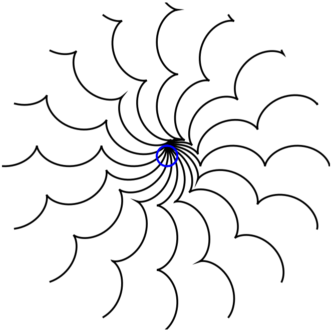
Ripples in the Gravitational Field

- Move object back & forth $\longrightarrow$ gravitational wave
- Same argument applies to electricity:
  - can derive magnetism as relativistic effect
  - accelerating charges generate electromagnetic waves propagating @ speed of light



Gravitational Wave from Orbiting Mass?

- Move around in a circle
- Still get grav wave pattern, but looks a bit funny
- Time to move beyond simple pseudo-Newtonian picture



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The Equivalence Principle

- Funny thing about (Newtonian) gravitational forces: always proportional to an object's mass, something in a gravitational field undergoes the same acceleration, no matter what it is
- Fictitious forces (e.g., centrifugal force) in non-inertial (accelerating, rotating, etc) reference frames behave the same way
- In Einstein's general relativity, gravity is something like a fictitious force which only manifests itself because the reference frame is non-inertial
- The catch: **NO** (globally) inertial reference frames!

A Thought Experiment

- In a freely falling elevator: Can you tell you're not in space?
- You, the elevator, and anything you drop are accelerating downwards at $9.8\,\mathrm{m/s^2} \longrightarrow$ no relative acceleration

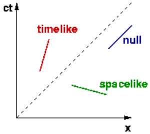
A Thought Experiment

- In a freely falling elevator: Can you tell you're not in space?
- You, the elevator, and anything you drop are accelerating downwards at $9.8\,\mathrm{m/s^2} \longrightarrow$ no relative acceleration
- Actually, you can tell if the elevator is big enough:
  - Top of elevator farther from Earth $\rightarrow$ grav field weaker $\rightarrow$ stuff accelerates less $\Longrightarrow$ accelerates up in elevator frame
  - Bottom of elevator closer to Earth $\rightarrow$ grav field stronger $\rightarrow$ stuff accelerates more $\Longrightarrow$ down in elevator frame
  - stuff @ sides accel inward bc lines to ctr of $\oplus$ converge
- This relative acceleration is measurable manifestation of gravity: **tidal force**

Spacetime Geometry

- Recall in special relativity, speed of light c same for all inertial observers
- Given pair of events, different observers measure different Δx, Δy, Δz & even Δt, but all agree on

$$(\Delta s)^2 = -c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$



- If $(\Delta s)^2 = 0$, have lightlike or null-sep events
- If $(\Delta s)^2 > 0$, have spacelike-separated events
- If $(\Delta s)^2 < 0$, have timelike-separated events

Notational Simplifications

- Work in units where $c = 1$ (defines what we mean by measuring time in meters and distance in (light-)seconds)
- Four-vector $\{x^\alpha\} = \{x^0, x^1, x^2, x^3\} = \{t, x, y, z\}$
- Einstein summation convention: implied sum over repeated indices so for example
  $g_{\alpha\beta} V^\alpha V^\beta$ means $\sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} g_{\alpha\beta} V^\alpha V^\beta$
  & $g_{ij} V^i V^j$ means $\sum_{i=1}^{3} \sum_{j=1}^{3} g_{ij} V^i V^j$
- So $(\Delta s)^2 = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta$ where $\{\eta_{\alpha\beta}\} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

General Relativity in a Nutshell

- In GR, talk about infinitesimal separations $\Delta \to d$
- Geometry described by

$$(ds)^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

$g_{\alpha\beta}(\{x^\gamma\})$ in general is not the flat Minkowski metric $\eta_{\alpha\beta}$

- You can always choose coördinates so that **at one point** $g_{\alpha\beta} = 0$ & $\frac{\partial g_{\alpha\beta}}{\partial x^\gamma} = 0$ (equivalence principle)
- Cannot get rid of $\frac{\partial^2 g_{\alpha\beta}}{\partial x^\gamma \partial x^\delta}$, even at a point (tidal effects)
- Einstein's equations describe how $\frac{\partial^2 g_{\alpha\beta}}{\partial x^\gamma \partial x^\delta}$ determined by density of matter and energy

Outline

Gravitational Wave as Metric Perturbation

- Full GR complicated (choice of coörds, global struct, etc)
- Far from source, much simpler:
  - $\approx$ a plane wave
  - GW $h_{\alpha\beta}$ is a small perturbation on top of flat metric $\eta_{\alpha\beta}$
    $g_{\alpha\beta}=\eta_{\alpha\beta}+h_{\alpha\beta}$
  - Can choose coörds to leave only two polarization states;
    E.g. Plane wave propagating in z direction

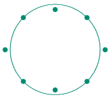
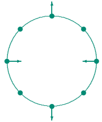
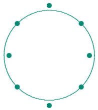
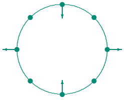
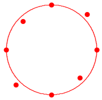
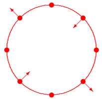
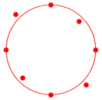
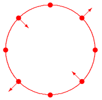
$$\{h_{\alpha\beta}\}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i2\pi f(z-t)}$$

h_+ and $h_\times$ are amplitudes of “plus” and “cross” pol states.

$$\overleftrightarrow{h} = \left[h_+ \overleftrightarrow{e}_+ + h_\times \overleftrightarrow{e}_\times\right] e^{i2\pi f(\hat{k}\cdot\vec{r}-t)}$$

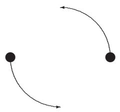
Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization				
Cross (×) Polarization				

Gravitational Wave Generation

- Generated by moving/oscillating mass distribution
- Classic example: orbiting binary system



(e.g., Binary Pulsar 1913+16
– Observed energy loss agrees w/GW prediction)

The Polarization Basis

- wave propagating along $\hat{k}$;
  construct $\overleftrightarrow{e}_{+,\times}$ from $\perp$ unit vectors $\hat{\ell}$ & $\hat{m}$:

$$\overleftrightarrow{e}_{+} = \hat{\ell} \otimes \hat{\ell} - \hat{m} \otimes \hat{m} \qquad \overleftrightarrow{e}_{\times} = \hat{\ell} \otimes \hat{m} + \hat{m} \otimes \hat{\ell}$$

- arbitrary choice of $\hat{\ell}$ within plane $\perp$ $\hat{k}$ (fixes $\hat{m} = \hat{k} \times \hat{\ell}$)
  Free to choose polarization basis convenient to situation

Example: Linear polarization

- Consider binary system seen edge on:
  masses seen going back & forth in one direction; call that $\hat{\ell}$
- In that pol basis, $h_\times = 0$ and only h_+
  linear polarization

$$h_+ = A \cos \Phi(t)$$
$$h_\times = 0$$

with $|A_+| > |A_\times|$

Example: Circular polarization

- Consider binary system seen face on:
  masses seen going in circle
- In any pol basis, h_+ & $h_\times$ have same amp; out of phase
  circular polarization

$$h_+ = A\cos\Phi(t)$$
$$h_\times = A\sin\Phi(t)$$

Example: Elliptical polarization

- General case: binary system seen at an angle: masses seen going around an ellipse; long axis of that ellipse picks preferred direction $\hat{\ell}$ for pol basis
- In that pol basis, h_+ & $h_\times$ out of phase; h_+ has greater amp **elliptical polarization**

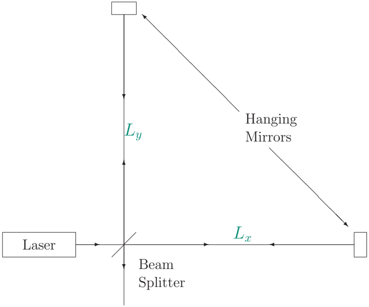
$$h_+ = A_+ \cos\Phi(t)$$
$$h_\times = A_\times \sin\Phi(t)$$

with $|A_+| > |A_\times|$

Outline

Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in

$$L_x - L_y = \sqrt{g_{11} L_0^2} - \sqrt{g_{22} L_0^2}$$
$$= \sqrt{(1+h_{11})L_0^2} - \sqrt{(1+h_{22})L_0^2}$$
$$\approx L_0 \frac{h_{11}-h_{22}}{2} \sim L_0 h_+$$

- More gen, $(L_1 - L_2)/L_0 = \overleftrightarrow{h} : \overleftrightarrow{d}$ with "response tensor"

$$\overleftrightarrow{d} = \frac{\hat{n}_1 \otimes \hat{n}_1 - \hat{n}_2 \otimes \hat{n}_2}{2}$$

Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

LIGO Livingston (La.)

GEO-600 (Germany)

Virgo (Italy)

Summary

- Relativistic causality implies gravitational waves
- General Relativity describes gravity as geometry
- Far from source, GWs are plane waves w/2 pol states
- GW detectors measure fluctuations in distances