terms of the parameter space metric. This is defined as the fractional loss in the signal-to-noise ratio when ρ is calculated at a point in parameter space which is slightly different from the point corresponding to the actual signal parameters [36–38]. In our case, we are in principle free to consider any subset of all the possible SFT pairs in calculating the final detection statistic ρ. However, without some control on which SFT pairs are chosen, it seems very hard to get a handle on the parameter space metric for the general cross-correlation statistic ρ defined by (4.4). Our suggestion is the following: Choose a time duration $T_{\max}$ and include only those SFT pairs $\{I, J\}$ for which $|T_I - T_J| \leq T_{\max}$. Thus, $T_{\max}$ can be viewed as the maximum duration over which we choose to maintain strict phase coherence.

If $T_{\max} = T_{\rm obs}$ then we are including all possible pairs, and at the other extreme, if $T_{\max} = 0$ then we are including only self-correlations and time-coincident correlations between different detectors. In the intermediate regime the cross-correlation search is closest in spirit to a semicoherent hierarchical scheme which consists of breaking up the total data available (say from $t = 0$ to $t = T_{\rm obs}$) into shorter segments $[0, T_{\max}], [T_{\max}, 2T_{\max}] \ldots$. One then performs a coherent analysis on each of the segments (using, say, the $\mathcal{F}$-statistic) and combines the results semicoherently [9,11,12]. The pair selection criteria would lead us to choose all possible SFT pairs within each of the segments. Since we have already seen in Sec. V that this is essentially equivalent to the $\mathcal{F}$-statistic, the similarities between the two schemes is obvious. The two schemes are however not exactly identical because this SFT pair selection criteria also includes choosing pairs lying in adjacent data segments (assuming the segments are sufficiently close to each other). Thus, the cross-correlation search with coherence time $T_{\max}$ will be more sensitive than the semicoherent search with coherent segments of duration $T_{\max}$ but the precise improvement depends on the duty cycle of the detectors, i.e. on the gaps between the SFTs and the coherent segments.

With this criteria of choosing pairs, we will see that the resolution depends on $T_{\max}$ the SFT baseline ΔT. To make our results concrete, we will focus on ground-based interferometers by taking the frequency range to be from 50 to 1000 Hz. Given the similarities with the semicoherent and hierarchical schemes discussed above, it is clear that a proper discussion of the metric requires a calculation of the parameter space metric for semicoherent searches. This is a combination of the coherent metric worked out in detail in [34,39], and the semicoherent metric obtained by summing $\mathcal{F}$-statistic segments. Preliminary calculations have been worked out in [9], but a detailed study of its properties is still lacking. We will instead resort to order of magnitude estimates (which, in spite of their approximate nature, have actually turned out to be fairly useful for previous searches; see e.g. [11]).

We can either use the amplitude modulation of the detection statistic $\rho = \sum_\alpha \rho_\alpha$ by which we mean the variation of ρ_α with α, or we can use the frequency modulation reflected in the different frequency bins k and k' used to calculate the cross correlation $\mathcal{Y}_\alpha = \tilde{x}^*_{k,I}\tilde{x}_{k',J}$. Starting with the sky resolution, we identify three factors which could be relevant: the detector beam pattern functions, the detector-pair baseline $\Delta\vec{r}_{IJ}$, and the Doppler information over a duration ΔT and $T_{\max}$; we discuss all of these in turn. The relative importance of these three factors depends on the search parameters.

(i) The expectation value of the cross-correlation statistic varies with the SFT pair index α, and part of this variation is due to the geometrical factor $a_I a_J + b_I b_J$ in (3.11). Since this variation depends on the sky position, it can in principle be used to get sky-position information. The resolution thus obtained is roughly given by the angular scales over which the beam pattern functions vary. Note that this amplitude modulation is due to the rotation of Earth around its axis; this is independent of the signal frequency and gets mostly averaged out if ΔT is comparable or larger than a day.

(ii) The other reason for the variation of the signal-to-noise ratio with α is the $\Delta\vec{r}_{IJ}$ term in (3.11). In the case when the two SFTs are coincident in time ($T_I = T_J$), then $\Delta\vec{r}_{IJ}$ is the separation between the two detectors; for the LIGO Hanford and Livingston observatories, this corresponds to a light travel time of about 10 ms. More generally, the magnitude of $\Delta\vec{r}_{IJ}$ is the distance between the positions of the two (distinct or same) detectors at different times; it could be as much as 2 AU if $T_I - T_J \sim 6$ months. On the other extreme, it could be zero if we are correlating the data with itself (which is what the standard semicoherent methods do); this effect then becomes completely irrelevant. If $\lambda_{\rm gw}$ is the wavelength of the wave we are trying to detect, the sky resolution associated with $\Delta\vec{r}_{IJ}$ is inversely proportional to the frequency:

$$(\Delta\theta)_{\Delta\vec{r}} \approx \frac{\lambda_{\rm gw}}{|\Delta\vec{r}|} = \frac{1}{f \cdot |\Delta\vec{r}|/c}. \tag{7.1}$$

For the Hanford-Livingston pair, this corresponds to about $\mathcal{O}(60°)$ at 100 Hz and about 6° at 1000 Hz.

(iii) The third way of getting sky-position information is through the Doppler shift. This is only useful if the frequency resolution of the SFTs is small enough; the maximum Doppler shift is $f|\vec{v}|/c$, so for the Doppler shift to be important, we must have

$$\Delta T > \frac{\lambda_{\rm gw}}{|\vec{v}|}. \tag{7.2}$$

The magnitude of Earth's orbital velocity in its orbit is $\sim 10^{-4}c$, so (7.2) leads to $\Delta T > 200$ s at 50 Hz and

$\Delta T > 6.67$ s at 1500 Hz. One relevant baseline in this case is the distance traveled by the detector in the duration ΔT. Thus, the sky resolution is (see [11] for further details)

$$(\Delta\theta)_{\rm Doppler} = \frac{\lambda_{\rm gw}}{|\vec{v}|\Delta T}. \tag{7.3}$$

For 1800 s SFTs, this corresponds to $\sim 6°$ at 50 Hz and 0.2° at 1500 Hz. There is finally the baseline corresponding to $T_{\rm max}$, i.e. the distance $d_{\rm max}$ traveled by the detector during $T_{\rm max}$. This leads to

$$(\Delta\theta)_{\rm Doppler} = \frac{\lambda_{\rm gw}}{d_{\rm max}}. \tag{7.4}$$

More generally, the resolution corresponding to $T_{\rm max}$ (for sufficiently large ΔT) is precisely the coherent metric calculated in [34,39].

We see that the first two items above can be viewed as using the amplitude modulation information (dependence of the signal-to-noise ratio on the pair index α), while the third uses the frequency modulation.

Let us now discuss the resolution in spin-down parameters f_k. The spin-down term in $\Delta\Phi_{IJ}$ appears in the combination $f_k(T_I^{k+1} - T_J^{k+1})$. Thus, it is clear that for $T_I \neq T_J$ this leads to a spin-down resolution of

$$(\delta f_k)_{\rm min} = \frac{1}{\max_{I,J}\{|T_I^{k+1} - T_J^{k+1}|\}}. \tag{7.5}$$

Thus, if we were to consider all possible pairs from a given set of SFTs, and if we define the reference time to be in the midpoint of the observation duration, then we would have $\delta f_k \propto T_{\rm obs}^{-(k+1)}$.

We can also consider the frequency resolution $(\delta f)_{\rm sft} = (\Delta T)^{-1}$ of the SFTs themselves. The corresponding resolution in f_k is defined by the smallest change in f_k required to change the frequency by a $(\delta f)_{\rm sft}$ over the full observation time $T_{\rm obs}$. This leads to $\delta f_k = (\delta f)_{\rm sft}/T_{\rm obs}^k$ for $k = 1, 2 \ldots$.

Let us conclude this section by giving a short numerical example for the case when we correlate data from a pair of spatially separated detectors at the same times. We consider frequencies of 100 and 1000 Hz, and two sky positions: one at the celestial equator and one at 45° degrees above it. In each case we consider sources with the optimal orientation $\iota = 0$, without any spin-down parameters, and with $\psi = 0$. The total observation time is taken to be $T_{\rm obs} = 1$ yr and the SFT baseline is $\Delta T = 30$ min. We assume that the two data streams are coming from the LIGO Livingston and Hanford interferometers. For performing the cross correlations, we use

$$Q(t; f, \vec{n}) = \lambda(t; \vec{n})\langle\tilde{\mathcal{G}}(\vec{n})\rangle_{\cos\iota,\psi}, \tag{7.6}$$

where, $\lambda(t; \vec{n})$ is a proportionality constant. We consider essentially identical time segments—same barycentric time—in the two detectors. In a year's worth of observation time there are little over 17 000 such time segments, each of 30 minutes duration. Thus the time-segment indices I, J each, sequentially run over the full observation time. The relevant quantities Q, $\tilde{\mathcal{G}}$, and λ in (7.6) which carry the same indices also do the same over the observation time—thus we may think of each of them as functions of t—the segment time stamp; thus I or J is replaced by t.

For the signal only case, the cross correlation can be written explicitly as

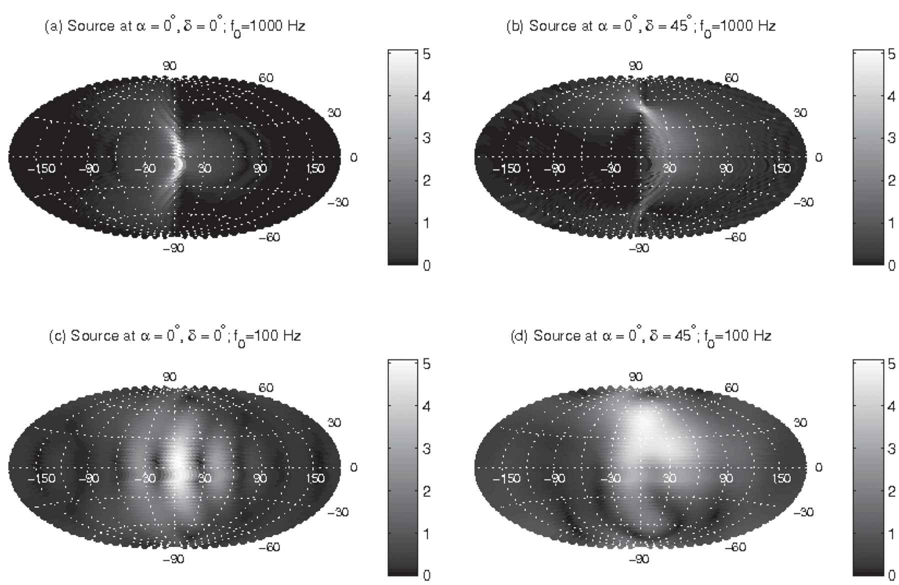

FIG. 1. The point spread functions (PSFs) for sources with frequencies 100 Hz [(c) and (d)] and 1000 Hz [(a) and (b)]. The source is taken at the celestial equator [(a) and (c)] and 45° above the celestial equator [(b) and (d)]. In all the cases, the source orientation is taken to be optimal, i.e., $\iota = \psi = 0$.

$$B(\vec{n},\vec{n}') = \Lambda(\vec{n}) \int_0^{T_{\rm obs}} \langle \tilde{\mathcal{G}}(t;\vec{n})\rangle_{\cos\iota,\psi} h_{(1)}(t;\vec{n}') h_{(2)}(t;\vec{n}') dt, \tag{7.7}$$

where the subscripts in $h_{(1)}$ and $h_{(2)}$ refer to the two distinct detectors we are considering. We have chosen

$$\Lambda^{-1}(\vec{n}) = \frac{1}{\Delta T} \int \lambda^{-1}(t;\vec{n}) dt. \tag{7.8}$$

We choose the proportionality constant λ such that it is inversely proportional to the square of the average total power accessible to the network for a particular direction of the sky in the interval ΔT of the SFTs. Thus we have

$$\lambda^{-1}(t;\vec{n}) = \Delta T \langle \tilde{\mathcal{G}}(t;\vec{n})\rangle^2_{\cos\iota,\psi}. \tag{7.9}$$

This is in the spirit of the normalization scheme adopted in [20]. Figure 1 shows $B(\vec{n},\vec{n}')$ evaluated numerically for point sources at different positions. We note that the maximum value of B is 5. This is the result of the average value of $\tilde{\mathcal{G}}$ we have chosen in defining the filter function together with the fact that we have chosen optimally oriented sources for the numerical computation. The sky resolution is characterized quantitatively by the FWHM (full width at half maximum) of the PSF. From the figures it turns out to be $\simeq 8°$ for $f_0 = 1000$ Hz and $\simeq 80°$ for $f_0 = 100$ Hz. We observe that the agreement between the order of magnitude estimates obtained earlier and the actual values computed from the figure is satisfactory.

VIII. DISCUSSION

Let us summarize the main results of this paper. We have generalized the cross-correlation statistic, traditionally used for the stochastic gravitational wave background searches, to periodic gravitational waves. The features of periodic waves, not present in the stochastic background signals, are nonstationarity and long-term coherence. The nonstationarity may need to be taken into account depending on the frequency resolution, and the long-term coherence implies that we can in principle cross correlate data segments from arbitrary times and arbitrary detectors. In this framework, we can naturally consider a network consisting of an arbitrary number of detectors. Because of the freedom in choosing which data-segment pairs to correlate, the method is very flexible, and these are some of the possibilities:

(i) We can, if we wish, correlate all possible short data segments. If this is done, then we showed that the resulting detection statistic is very close to the $\mathcal{F}$-statistic corresponding to a full matched filter statistic. In this case the parameter space resolution becomes very fine and consequently, while this is ideally the most sensitive method, its computational cost becomes prohibitive for wide parameter space searches.

(ii) At the other extreme, we can choose to correlate only data segments taken from distinct detectors at the same (or very close) times. This is the closest in spirit to the standard directed stochastic background searches using aperture synthesis. In this mode of operation, the search is not computationally intensive, and is very robust against signal uncertainties. However, this also implies poor resolution in parameter space, and thus more expensive follow-ups to verify possible detections and to estimate the signal parameters.

(iii) From the perspective of this paper, the standard semicoherent searches such as PowerFlux, StackSlide, and Hough all correspond to the special case in which we consider only self-correlations [5,6,9–11]. The procedure of considering weighted sums of the cross-correlation power is closest to the PowerFlux method. In fact, many of the lessons learned in the PowerFlux searches should be applicable here with suitable modifications. For example, the estimation of the signal amplitudes developed originally for PowerFlux carries over rather straightforwardly.

(iv) In intermediate regimes when we correlate data segments separated by a maximum coherence time $T_{\rm max} < T_{\rm obs}$, the cross-correlation search is similar to a hierarchical search in which we combine segments of demodulated data. Though, as discussed in Sec. VII, there are differences between the two with the cross-correlation search being somewhat more sensitive.

Conceptually, this method thus provides a unified framework for all the known periodic wave searches, and this might be useful in various calculations and applications. Each of the above modes of operation correspond to tuning the maximum coherence time all the way from small values to the total observation time. The precise value chosen for a specific application depends on the trade-offs between computational cost, sensitivity, and robustness against signal uncertainties. The additional parameter which figures importantly in this trade-off is the length ΔT of the short data segments. It is worth mentioning that there exist other analyses which consider the angular resolution of general multidetector arrays (e.g. [40]); these methods and results might be useful for our purposes as well.

There are a number of open issues for future work. An important question is to get a detailed understanding of the trade-offs mentioned above for various types of searches including all sky searches for isolated GW pulsars, signals from known binary systems or from interesting areas such as the galactic center, etc. This will help us better decide how to best use our computational resources and to maximize our chances of making a detection. Another important issue, which feeds into this optimization problem, is to study the general parameter space metric. To date we only have a proper understanding of the coherent metric, i.e. case (i) above. For the other cases, we have estimates of the parameter space resolution and which are often sufficient for many applications, but a full understanding is still lacking. In addition, it would be interesting to compare the estimation of the amplitude parameters $\{A_+, A_\times\}$ (and ψ) obtained from (6.4) and (6.5) with the maximum likelihood estimators obtained from the $\mathcal{F}$-statistic calculation. In the limit when we consider all possible correlations, we would expect the two estimates to be very close to each other.

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Note added in proof.—Sanjeev Dhurandhar, Badri Krishnan, and John T. Whelan deeply regret the loss of their colleague and coauthor Himan Mukhopadhyay whose promising scientific career was tragically ended by her death as this paper was going to press.

APPENDIX: INCLUDING SELF-CORRELATIONS AND $\mathcal{O}(h_0^2)$ CORRECTIONS

In this section we relax the two assumptions of only looking at $\mathcal{Y}_{IJ}$ for $I \neq J$ and $h \ll n$. We allow self-correlations (which, by themselves, are used in the standard semicoherent searches), and we keep terms of $\mathcal{O}(h_0^2)$ but still neglect $\mathcal{O}(h_0^4)$ terms.

Let us again start from the general statistic ρ defined in (4.4), and let us calculate its mean and standard deviation with the two assumptions relaxed. In general, we have $\mathcal{Y}_{IJ}^* = \mathcal{Y}_{JI}$ so that $\mathcal{Y}_{II}$ is real and so is the corresponding weight u_{II}; $\mathcal{Y}_{II}$ is in fact just the power in a single SFT bin. We will denote $\mathcal{Y}_{II}$ simply by $\mathcal{Y}_I$ and u_{II} by u_I.

The mean is easy to calculate:

$$\langle \mathcal{Y}_{IJ} \rangle := \mu_{IJ} = \frac{1}{2\Delta T} S_n^I \delta_{IJ} + h_0^2 \tilde{\mathcal{G}}_{IJ}. \tag{A1}$$

Thus, the mean is nonzero in the absence of a signal only for the self-correlation terms. In general, ρ will contain self-correlations, and also correlations of distinct pairs. However, we want to be completely general and we do not assume that it contains *all* the possible pairs. This is then the expression for the mean:

$$\langle \rho \rangle := \mu = \frac{1}{\Delta T} \sum_I u_I S_n^I + h_0^2 \sum_\alpha (u_\alpha \tilde{\mathcal{G}}_\alpha + u_\alpha^* \tilde{\mathcal{G}}_\alpha^*). \tag{A2}$$

It is to be understood that the first sum in this equation only contains the self-correlations and the second sum contains all the SFT pairs we have chosen to include, including the self-correlations.

The variance calculation is somewhat more involved. Before looking at the variance of ρ itself, let us look at $\langle \mathcal{Y}_{IJ} \mathcal{Y}_{KL} \rangle$. Note that for the pure noise terms

$$\langle \tilde{n}_I^* \tilde{n}_J \tilde{n}_K^* \tilde{n}_L \rangle = 2\delta_{I(J}\delta_{L)K} \langle |\tilde{n}_I|^2 \rangle \langle |\tilde{n}_K|^2 \rangle. \tag{A3}$$

Here, we use the notation that indices within parentheses are symmetrized over: $X_{(IJ)} = (X_{IJ} + X_{JI})/2$. This also covers the $I = J = K = L$ case, so there is no need to consider that separately.

Consider now the signal. In general, the terms in $\langle \mathcal{Y}_{IJ} \mathcal{Y}_{KL} \rangle$ with odd powers of h will vanish because the noise is assumed to have zero mean. Thus, schematically, we will have

$$\langle \mathcal{Y}_{IJ} \mathcal{Y}_{KL} \rangle = A + Bh_0^2 + Ch_0^4. \tag{A4}$$

Let us ignore the h_0^4 terms and focus only on the second order terms. The reader can convince herself that we only need to keep the following terms in $\mathcal{Y}_{IJ} \mathcal{Y}_{KL}$:

$$\tilde{h}_I^* \tilde{h}_J \tilde{n}_K^* \tilde{n}_L + \tilde{h}_K^* \tilde{h}_J \tilde{n}_I^* \tilde{n}_L + \tilde{h}_K^* \tilde{h}_L \tilde{n}_I^* \tilde{n}_J + \tilde{h}_I^* \tilde{h}_L \tilde{n}_K^* \tilde{n}_J. \tag{A5}$$

Putting together (A3) and (A5), we end up with

$$\langle \mathcal{Y}_{IJ} \mathcal{Y}_{KL} \rangle = \frac{1}{2(\Delta T)^2} \delta_{I(J}\delta_{L)K} S_n^{(I)} S_n^{(K)} + \frac{h_0^2}{\Delta T} [\tilde{\mathcal{G}}_{I(J}\delta_{L)K} S_n^{(K)} + \delta_{I(J}\tilde{\mathcal{G}}_{L)K} S_n^{(I)}]. \tag{A6}$$

We are now ready to look at the variance of ρ. Let us define $\rho_\alpha = u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*$, so that $\rho = \sum_\alpha \rho_\alpha$. Then, we have

$$\mathrm{Var}(\rho) = \sum_\alpha \mathrm{Var}(\rho_\alpha) + \sum_{\alpha,\beta(\alpha\neq\beta)} \mathrm{Cov}(\rho_\alpha, \rho_\beta). \tag{A7}$$

Let us start with the variances

$$\mathrm{Var}(\rho_{IJ}) = \langle \rho_{IJ}^2 \rangle - \mu_{IJ}^2. \tag{A8}$$

For $I \neq J$, $\mu_{IJ} = \mathcal{O}(h_0^2)$ so that μ_{IJ}^2 can be ignored. Thus, in this case we get

$$\sigma_\alpha^2 = \mathrm{Var}(\rho_{IJ}) = 2|u_{IJ}|^2 \left\{ \frac{S_n^{(I)} S_n^{(J)}}{4\Delta T^2} + \frac{h_0^2}{2\Delta T} (\tilde{\mathcal{G}}_I S_n^{(J)} + \tilde{\mathcal{G}}_J S_n^{(I)}) \right\}. \tag{A9}$$

For the $I = J$ case, we can no longer ignore the μ_α term. Keeping terms up to $\mathcal{O}(h_0^2)$ we end up with

$$\sigma_I^2 = \mathrm{Var}(\rho_I) = 4u_I^2 \left\{ \left(\frac{S_n^{(I)}}{2\Delta T} \right)^2 + \frac{h_0^2}{\Delta T} \tilde{\mathcal{G}}_I S_n^{(I)} \right\}. \tag{A10}$$

Turning now to the covariances, first note that if I, J, K, L are all distinct, then up to $\mathcal{O}(h_0^4)$ terms, $\mathrm{Cov}(\rho_{IJ}, \rho_{KL}) = 0$; thus we need at least one pair of matching indices to get a nonzero result. Using (A6) the expressions for all the nonzero cases are the following ($I \neq J$) ignoring, as always, the $\mathcal{O}(h_0^4)$ terms:

$$\langle \mathcal{Y}_{II} \mathcal{Y}_{IJ} \rangle = \frac{h_0^2}{2\Delta T}(\tilde{\mathcal{G}}_{IJ} + \tilde{\mathcal{G}}_{JI}) S_n^{(I)}, \tag{A11a}$$

$$\langle \mathcal{Y}_{II} \mathcal{Y}_{JI} \rangle = \frac{h_0^2}{\Delta T} \tilde{\mathcal{G}}_{IJ} S_n^{(I)}, \tag{A11b}$$

$$\langle \mathcal{Y}_{II} \mathcal{Y}_{JJ} \rangle = \frac{S_n^{(I)} S_n^{(J)}}{4\Delta T^2} + \frac{h_0^2}{2\Delta T}(\tilde{\mathcal{G}}_I S_n^{(J)} + \tilde{\mathcal{G}}_J S_n^{(I)}). \tag{A11c}$$

It turns out that the only nonzero covariance is

$$\mathrm{Cov}(\rho_I, \rho_{IJ}) = \frac{h_0^2}{\Delta T} u_I S_n^{(I)} (u_{IJ} \tilde{\mathcal{G}}_{IJ}^* + u_{IJ}^* \tilde{\mathcal{G}}_{IJ}). \tag{A12}$$

We are almost done now. Substituting the results of (A9), (A10), and (A12), in (A7) we get

$$\sigma^2 = 2\sum_\alpha |u_\alpha|^2 \sigma_{(0),\alpha}^2 + \frac{h_0^2}{\Delta T}\Big\{\sum_I 4u_I^2 \tilde{\mathcal{G}}_I S_n^{(I)} + \sum_{\alpha, I\neq J} |u_{IJ}|^2 (\tilde{\mathcal{G}}_I S_n^{(J)} + \tilde{\mathcal{G}}_J S_n^{(I)}) + \sum u_I (u_{IJ} \tilde{\mathcal{G}}_{IJ}^* + u_{IJ}^* \tilde{\mathcal{G}}_{IJ}) S_n^{(I)}\Big\}. \tag{A13}$$

Here we have defined the variances in the absence of a signal:

$$\sigma_{(0),I} = \frac{(S_n^{(I)})^2}{2\Delta T^2}, \tag{A14a}$$

$$\sigma_{(0),IJ} = \frac{S_n^{(I)} S_n^{(J)}}{4\Delta T^2}, \qquad I \neq J. \tag{A14b}$$

It is convenient to write (A13) in the abbreviated form,

$$\sigma^2 = \sigma_{(0)}^2 + h_0^2 \sigma_{(1)}^2, \tag{A15}$$

where the definitions of σ_0 and $\sigma_{(1)}$ are obvious from (A13).

We are finally ready to derive the equation for the sensitivity, i.e. the analogs of (4.9) and (4.15). Equation (4.7) for the threshold is unchanged as long as we use $\sigma_{(0)}$ instead of σ in that equation.[7] Equation (4.8) for the false dismissal rate becomes

$$\gamma = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\rho_{\rm th} - \mu}{\sqrt{2}\sigma}\right). \tag{A16}$$

Keeping terms linear in h_0^2, we get

$$\begin{aligned}\mathrm{erfc}^{-1}(2\gamma) &= \frac{\rho_{\rm th} - \mu}{\sqrt{2}\sigma_{(0)}}\left(1 - \frac{h_0^2 \sigma_{(1)}^2}{2\sigma_{(0)}^2}\right) \\ &= \mathrm{erfc}^{-1}(2\alpha) - \frac{h_0^2}{\sqrt{2}\sigma_{(0)}}\sum_\alpha (u_\alpha \tilde{\mathcal{G}}_\alpha + u_\alpha^* \tilde{\mathcal{G}}_\alpha^*) \\ &\quad - \frac{h_0^2 \sigma_{(1)}^2}{2\sigma_{(0)}^2}\,\mathrm{erfc}^{-1}(2\alpha).\end{aligned} \tag{A17}$$

Solving for h_0 leads to the generalization of (4.9):

$$h_0^2 = 2\mathcal{S}\left(\frac{\sqrt{\sum_\alpha |u_\alpha|^2 \sigma_{(0),\alpha}^2}}{\sum_\alpha (u_\alpha \tilde{\mathcal{G}}_\alpha + u_\alpha^* \tilde{\mathcal{G}}_\alpha^*) + \sigma_{(1)}^2\,\mathrm{erfc}^{-1}(2\alpha)/\sqrt{2}\sigma_{(0)}}\right). \tag{A18}$$

Finding the optimal weights is now not as straightforward as before. However, we note that, when $\sigma_{(1)}$ is ignored, then the optimal weights are again given by (4.10) except that now it holds also for the self-correlations. In the general case when we do not ignore $\sigma_{(1)}$, it is simpler to continue using the optimal weights derived earlier in (4.10), and to substitute it in (A18) to derive the corresponding sensitivity.

[7]The mean of ρ is now no longer necessarily zero in the absence of a signal (see (A2). But this only leads to an additive correction to the threshold $\rho_{\rm th}$, and we assume this correction has been made.

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