LIGO Laboratory / LIGO Scientific Collaboration

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Wide-angle scatter from LIGO arm cavities

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Introduction

Initial LIGO beam tube and COS arm cavity baffling was primarily designed to contain small-angle scattering from core optic topography. Such scattered light impinges on beam tube baffles along the arm and on equipment and chamber walls at the far end, is scattered by these surfaces, and is then rescattered into the cavity mode by the same core optic topography. The motion of the scattering points, which generally are not seismically or acoustically isolated, produces phase noise in the cavity mode (Weiss, G950039; Flanagan and Thorne, T940063).

On the other hand, viewing installed L1, H1 and H2 core optics in operation with properly focused CCD cameras reveals a dense constellation of point scatterers on each surface, and a surprising amount of radiation scattered into large view angles. These scattering centers were not foreseen and their source remains unknown. There is evidence they were acquired after polishing and metrology of the bare substrates, perhaps through coating defects or contamination.

There is an ongoing study to understand the sources of optical loss from the installed LIGO optics; the observed recycling gain implies a total round trip arm loss of 140-150 ppm (Kells, T070051). Hiro Yamamoto's assessment of the various loss contributions made a case for a point defect scattering loss of around 10 ppm.

Here we examine the interfering effect of this point-defect light, scattered off nearby vacuum walls and backscattered into the cavity modes. For Advanced LIGO, we conclude that this light will have to be suppressed, through better mirrors or baffling or both.

Version info

v1: This version (v1) is an update to T070089-05 (old DCC numbering system). It fixes a couple of numerical errors in the 05 version (found by Kip Thorne and Eanna Flanagan). In particular, the factor of 2/3pi in the formula at the bottom of p3 was missing, and the factor of 1/sqrt(2) in the formula for R_{sc} was missing. In addition, it appears there were graphical rendering problems in the plots. Finally, the normalization of R_{sc} was changed so that the target for its maximum value is unity.

Scattering Paths

Wide angle

Defects or dust on the cavity mirrors scatter power into wide angles $0.1 < \theta < \pi/2$ (blue rays in Figure 1). We presume the angular distibution is Lambertian (i.e. "flat"), such that the probability of a cavity photon scattering to angle θ off axis is given by $B_L = \dfrac{\alpha\cos\theta}{\pi}$ where α is a constant, nominally the "Lambertian reflectance." This is reasonable for small point defects, and approximately consistent with Kells' measurements out to $\theta \approx \pi/4$. We can approximate α as the total integrated scatter (TIS) or total hemispherical reflectance (THR) of the optic, once the topographic contribution at small angles is excised. The scattered photons fan out to chamber and tube walls at typical distance a, subtending $\Omega_L \approx 2\pi$ steradians.

Each part of the wall can itself be presumed to have a roughly Lambertian backscatter probability per unit solid angle $B_S \approx 0.1\ \mathrm{sr}^{-1}$. Most of the secondary scattered power dissipates harmlessly, but a fraction falls back on the cavity mirror, where it can re-scatter into the cavity mode. To determine the fraction that is recombined into the cavity mode, we rely on a reciprocity relation derived by E. Flanagan and K. Thorne in LIGO Technical note T940063-00-R. They calculate the cross section, $\sigma(z,\vec{y})$, for a cavity mirror to scatter photons arriving from a point $(z,\vec{y})$ back into the cavity mode ($\vec{y}$ is a location in the plans z = constant a distance z from the mirror along the optic axis). They find this cross section is related to the probability $dP/d\Omega(z,\vec{y})$ for the cavity mirror to scatter photons out of the main beam to the point $(z,\vec{y})$:

$$\sigma(z,\vec{y}) = \lambda^2 \frac{dP(z,\vec{y})}{d\Omega}.$$

The probability $dP/d\Omega(z,\vec{y})$ is what we call B_L. To compute the power re-scattered into the cavity mode, the cross section must be multiplied by the scattered light intensity at the cavity mirror. We estimate this intensity, as a fraction of the cavity mode power, as $\alpha \cdot B_S / a^2$. Putting it all together, the fractional power scattered from the main beam, to nearby walls, and re-scattered back into the main beam mode is estimated as

$$\begin{aligned}\frac{P_{sc}}{P_0} &= \left(\frac{\lambda}{a}\right)^2 B_s \int B_L^2 d\Omega \\ &\approx \frac{2}{3\pi}\alpha^2 \cdot \lambda^2 \cdot \left(\frac{B_S}{a^2}\right) \\ &= 2.4 \times 10^{-24} \cdot \left(\frac{\alpha}{10\ \text{ppm}}\right)^2 \cdot \left(\frac{B_S}{0.1\ \text{sr}^{-1}}\right) \cdot \left(\frac{1\ \text{m}}{a}\right)^2\end{aligned}$$

where P_0 is the cavity circulating power. Here we have assigned 10 ppm of loss to wide-angle (point-defect) scatter, as estimated by Hiro Yamamoto in G070240.

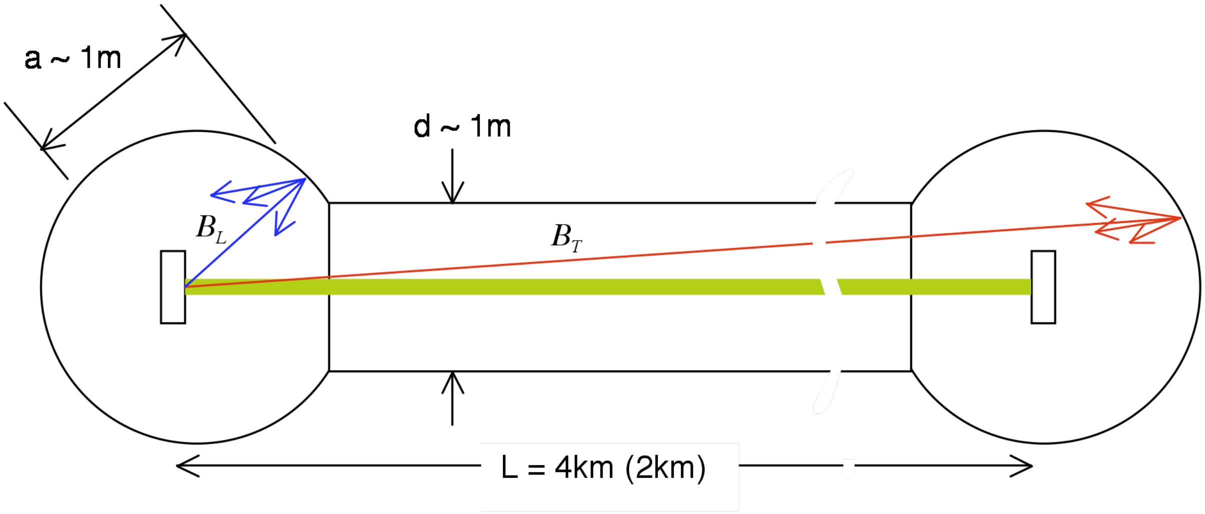

Figure 1: Tube and chamber geometry

Far field (narrow angle)

Power scattered by diffraction from mirror topography at angles $3\times10^{-5} < \theta < 1\times10^{-4}$ will penetrate the beam tube, but fall outside the mirror perimeter, hitting chamber walls and other apparatus behind and around the suspension (red rays in Figure 1). For initial LIGO this component was to be intercepted within the beam manifolds by COS arm cavity baffles. At this writing these baffles have not been erected.

To estimate the power that is scattered into the annulus between the mirror outer diameter and the beam tube inner diameter, we use the one-dimensional PSD fit of the CSIRO mirror data (initial LIGO mirrors) found by Hiro Yamamoto in T070052-02:

$$S_1(f[m^{-1}]) = 7\times10^{-19}\cdot f^{-1.45}[m^3].$$

The fractional power scattered in the annulus is found by the integral:

$$P_a / P_0 = 2\pi D\cdot\left(\frac{4\pi}{\lambda}\right)^2\cdot\int_{f_1}^{f_2} S_1(f)df\,,$$

where D depends on the slope of the spectrum, and is defined in T070052 ($2\pi D = 1.2$ for the CSIRO mirrors). For Advanced LIGO we integrate the annulus 0.17 m < r < 0.5 m, corresponding to f_1 = 0.4 cm^{-1} and f_2 = 1.2 cm^{-1}. This gives: $P_a / P_0 = 2\times10^{-5}$.

This annulus power can then be scattered off the vacuum chamber wall back towards the mirror from which it came, and there re-scattered off the cavity mirror into the cavity mode as described for the wide-angle scatter. Thus the total recombined power can be estimated as:

$$\frac{P_{sc}}{P_0} = \frac{P_a}{P_0} \cdot \left(\frac{B_S}{L^2}\right) \cdot \lambda^2 \cdot B_{COC}$$

$$= 2\times 10^{-22} \cdot \left(\frac{B_S}{0.1\,\mathrm{sr}^{-1}}\right) \cdot \left(\frac{B_{COC}}{1300\,\mathrm{sr}^{-1}}\right)^2$$

Here, B_{COC} is the BRDF of the cavity mirror at small angle ($\theta = 5\times 10^{-5}$ rad), which is dependent on S_1 , and thus comes in as the square.

Thus the small angle scatter is expected to lead to a larger recombined power than the wide angle scatter, by a factor of about 20 for the parameters assumed here. Of course, this is what the arm cavity baffles are designed to reduce. With arm cavity baffles, the most significant path is expected to one which reflects twice off a baffle, and otherwise recombines as above. Thus the recombined power would be the above estimate, multiplied by the square of the baffle reflectivity.

Phase noise from the scattered light

Presuming statistical incoherence (Thorne and Weiss, 1989) the fluctuating field injected into the cavity mode by the scattered light is

$$\delta e_{sc}(f) = \frac{4\pi}{\lambda} \cdot x_{sc}(f) \cdot P_{sc}^{\ 1/2},$$

where P_{sc} is the power recombined into the cavity mode, and $x_{sc}(f)$ is the motion of the scattering surface, relative to the cavity mirror (linear displacement projected onto the scattered ray path).

This can be compared to the field produced by cavity mirror motion equal to the displacement noise of the interferometer:

$$\delta e_{ifo}(f) = \frac{4\pi}{\lambda} \cdot x_{ifo}(f) \cdot P_0^{1/2} \ ,$$

where x_{ifo} is the displacement noise target of the interferometer, and P_0 is the cavity stored power.

On average, half of the phase fluctuations produced by scattered light go into the quadrature that carries the gravitational wave signal, and half into the other quadrature. Taking this into account, the ratio, R_{sc}, of scattered light noise to the interferometer noise floor is thus:

$$R_{sc} = \frac{1}{\sqrt{2}} \frac{x_{sc}(f)}{x_{ifo}(f)} \cdot \left(\frac{P_{sc}}{P_0}\right)^{1/2} .$$

This is a simple expression to use because it doesn't depend on the cavity power or cavity dynamics; all we need is the fractional scattered and recombined power, and the ratio of scattered light path noise to interferometer displacement noise.

Advanced LIGO projection

Figure 2 shows the ratio R_{sc} of scattered light contribution, using the scattered and recombined light power calculated above for wide-angle scattering. For x_{sc}, this calculation uses the accelerometer measurement of the BSC7 motion at LHO, made by Robert Schofield in November 2006 (data shown in the appendix). For x_{ifo}, we use the technical noise limit for Advanced LIGO, as defined in the Advanced LIGO Systems Design, LIGO-T010075-v2, section 4.3. This technical noise limit (shown in Fig. 6 of T010075-v2) is a factor of 10 below the anticipated minimum interferometer strain noise. Thus we aim to keep R_{sc} safely below unity, and for Advanced LIGO we will need to reduce the wide-angle scattered power (either through better mirrors or baffling or both).

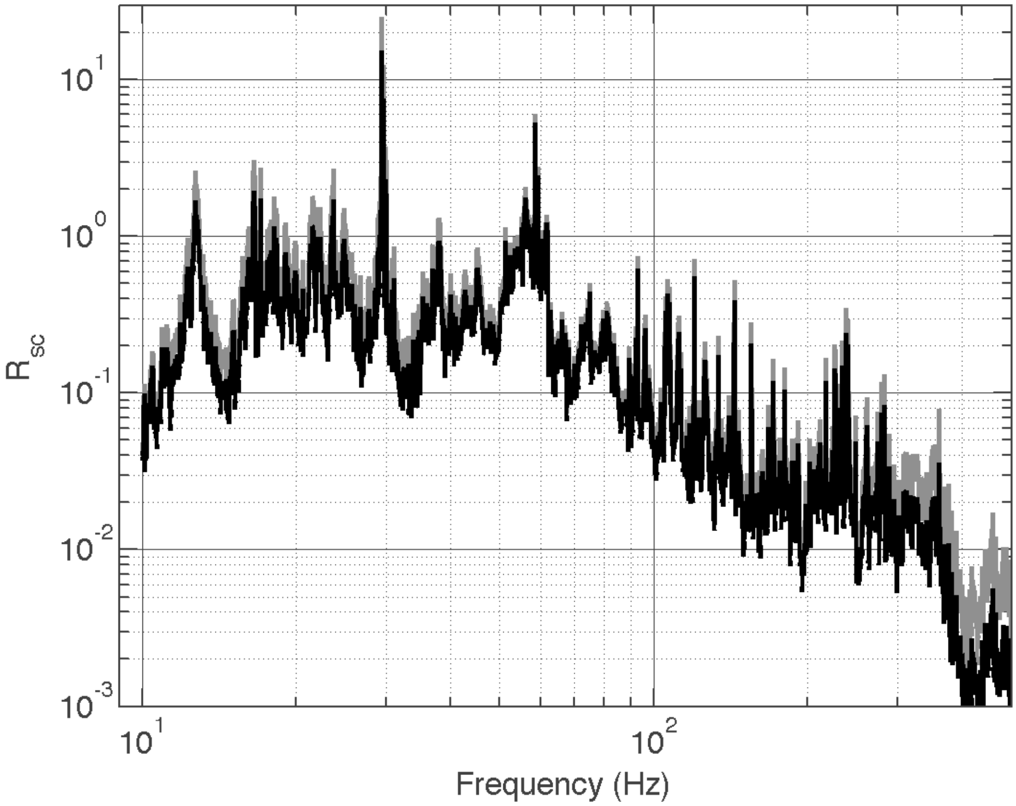

Figure 2. Estimated scattered light noise from wide-angle point defect scattering, relative to the design interferometer noise floor, as defined by the factor R_{sc}. The fractional arm power that is re-scattered into the arm cavity mode is taken here as $4 \times 2.4 \times 10^{-24}$ (factor of 4 for all cavity mirrors). **Black curve:** contribution to Advanced LIGO technical noise limit (see section 4.3 of LIGO-T010075-v2); **Grey curve:** comparison to 1/10th of the Advanced LIGO thermal noise (suspension plus mirror) – this represents a somewhat stricter limit than that given by the total interferometer strain noise. For Advanced LIGO this scattering path needs to be addressed to reduce the scattering noise to $R_{sc} < 1$. The recombined power needs to be reduced by the square of the R_{sc} factor. E.g., reducing the recombined power by a factor of 100 would sufficiently reduce all but the peak at 29 Hz.

Appendix: BSC7 wall motion data

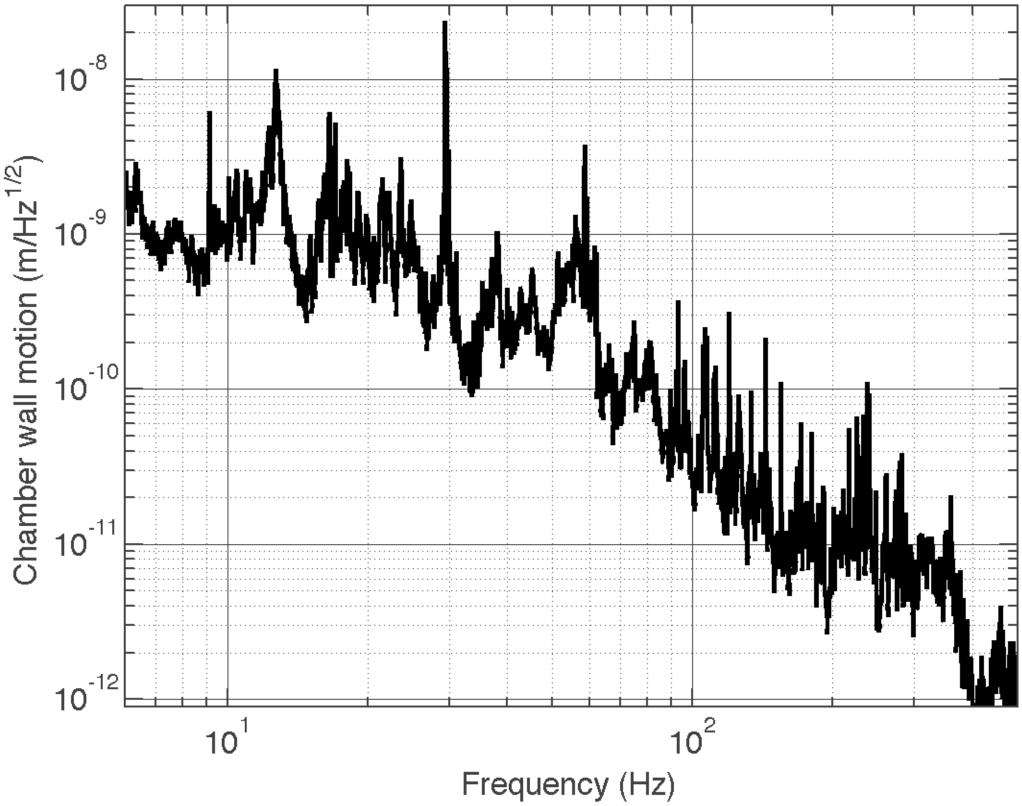

Figure 3. Motion spectrum of the LHO BSC7 chamber wall, measured with an accelerometer mounted at beam height; accelerometer axis is perpendicular to the wall. Measured by Robert Schofield, 17 November 2006 (see LHO iLog entry, 17 Nov 2006, 17:39:30 local).