Simple Calculation of Active Platform Performance

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1 Summary

We derive the performance equations for an active isolation system. We consider ground translation, ground tilt, and sensor noise for a system with sensor blending and sensor correction. We assume a plant and sensor with no cross coupling, except for low-frequency tilt-horizontal coupling.

We see that, at low frequency, the transmission of ground translation is approximately equal to the displacement sensor blending filter, for systems with feedback only. When sensor correction is added, the low-frequency performance becomes the product of the displacement sensor blend filter and the complement of the high-pass sensor correction signal.

Noise from the differential vertical GS-13s used to measure tilt is also a major limitation. This noise can cause erroneous tilt of the table, which causes excess horizontal motion.

2 Ground Translation

We first consider the translation performance of a platform with active feedback and sensor correction. We describe the platform translation as x_p and the relevant platform tilt at t_p. The inputs are the ground motion for translation, g_x, and tilt, g_t, and noise of the various sensors: feedback displacement sensors, $n_{disp,x}$; feedback geophones, $n_{GS13,x}$; and ground based sensor sensor correction STS-2, $n_{STS,x}$.

We make a simple model of the system as shown below in figure 1. The controller is K_x, the plant is P. There are several inputs to the plant, including ground translation, ground tilt, and commands from the controller. We distinguish the coupling of these to the translation output by defining $P_{x \leftarrow g_x}$, $P_{x \leftarrow g_t}$, and $P_{x \leftarrow K}$.

There are three sensors which are relevant, the feedback inertial sensor, S_{GS13}, the feedback displacement sensor, S_{disp}, and the ground-based sensor correction inertial sensor, $S_{STS,x}$. The feedback inertial sensor for the Advanced LIGO HAM, and for stage 2 of the Advanced LIGO BSC is a GS-13, hence the name. Also, this document was written before we switched out the STS-2s for the Trillium 240s. The sensor blocks are meant to represent the path from the motion input to the sensor through the DAC, hence the units of S_{GS13} would be counts/meter. In order to include DAC noise in this model, it would need to be represented in terms of equivalent platform motion.

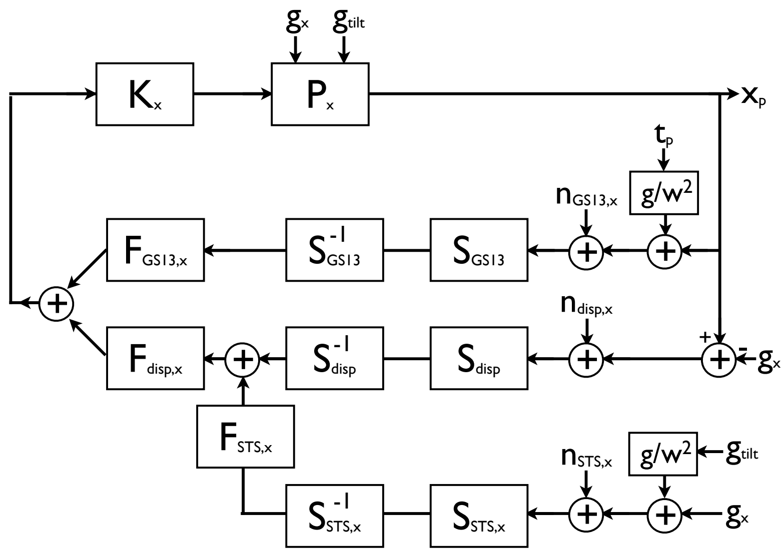

Figure 1: Full diagram of the translation control loops

In this analysis we next invert the sensor response with S_{disp}^{-1}, etc. so that the signals from the 3 sensors are in matching units, namely meters (this is actually nanometers at the observatories, so we can see the signals on the dataviewer screens). This allows us to remove the sensor blocks from the analysis, as shown in figure 2. If the slightly freewheeling nature of this transform is troubling, one can image that the sensor and sensor inversion blocks are replaced by a block C with a flat transfer function equal to 1 count per meter. It is useful to have a full physical picture when one goes to put this into hardware, but it will not be important in the rest of the performance analysis. The next blocks are the complementary filter blocks for the inertial and displacement sensor, $F_{GS13,x}$ and $F_{disp,x}$, and the high-pass filter block for the sensor correction path, $F_{STS,x}$. The filters for the feedback sensors are truly complementary, hence $F_{GS13,x} + F_{disp,x} = 1$. The sensor correction filter, $F_{STS,x}$ becomes 1 at high frequency, but does not have a complement in this diagram. Never-the-less, we will see its complement later in the analysis.

We begin looking at figure 2 and writing the loop gain, G_x, which we will abbreviate as G for the time being. We can start at the output, and express it in terms of loop elements and itself.

$$G = P_{x \leftarrow K} K_x (F_{GS13,x} + F_{disp,x}) \tag{1}$$

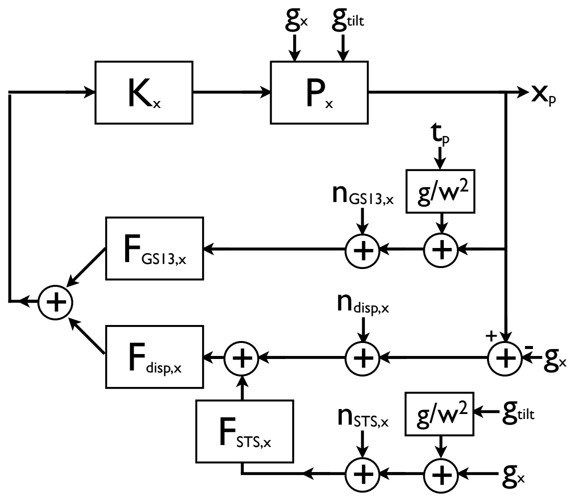

Figure 2: Simplified diagram of the translation control loops

Which immediately simplifies to

$$G = P_{x \leftarrow K} K_x \tag{2}$$

because the filters for the inertial sensor and displacement sensor are complementary. It is important to note that there are no minus signs in the loop, although there are some where the displacement sensor sees ground motion.

We can work out the coupling of ground translation to platform translation by starting at x_p and working backwards.

$$x_p = P_{x \leftarrow g_x} \cdot g_x + P_{x \leftarrow K} \cdot K_x \cdot \left(F_{GS13} \cdot x_p + F_{disp,x} \cdot (x_p - g_x) + F_{disp,x} \cdot F_{STS,x} \cdot g_x \right) \tag{3}$$

We can rewrite this as

$$x_p = P_{x \leftarrow g_x} \cdot g_x + G \cdot x_p - G \cdot F_{disp,x} \cdot g_x + G \cdot F_{disp,x} \cdot F_{STS,x} \cdot g_x. \tag{4}$$

Additional algebra yields

$$x_p \cdot (1 - G) = P_{x \leftarrow g_x} \cdot g_x - G \cdot F_{disp,x} \cdot g_x + G \cdot F_{disp,x} \cdot F_{STS,x} \cdot g_x. \tag{5}$$

We now write x_p in terms of ground translation

$$x_p = \frac{P_{x \leftarrow g_x}}{1 - G} g_x + \frac{-G}{1 - G} F_{disp,x} \cdot g_x + \frac{G}{1 - G} F_{disp,x} \cdot F_{STS,x} \cdot g_x. \tag{6}$$

At frequencies below 1 Hz we can make a few approximations. The loop gain, G is large, typically more than 100. The plant transfer function from ground translation to platform translation is approximately 1. With the damping loops on, it is never larger than about 3. In this case, the first term, the direct plant coupling, can be ignored, and $G/(1-G)$ is approximately -1, so equation 6 becomes:

$$x_p \approx F_{disp,x} \cdot g_x - F_{disp,x} \cdot F_{STS,x} \cdot g_x \tag{7}$$

If there is no sensor correction running (and no sensor noise), we see that the isolation performance is simply the displacement sensor blend filter. With the sensor correction, we get more performance, and we can express equation 7 as

$$x_p \approx F_{disp,x} \cdot (1 - F_{STS,x}) \cdot g_x = F_{disp,x} \cdot F_{STS,x,comp} \cdot g_x \tag{8}$$

where $F_{STS,x,comp}$ is the complementary filter to the high-pass ground based sensor correction filter. Hua pointed this out in his thesis.

The coupling of all the inputs can be worked out in a similar fashion. However, we just straight to the conclusion by remembering that the coupling of an input to an output is the forward path divided by $1-G$. For the platform motion, this becomes:

$$\begin{aligned} x_p = \; & \frac{P_{x \leftarrow g_x}}{1-G} g_x \\ & + \frac{P_{x \leftarrow g_{tilt}}}{1-G} g_{tilt} \\ & + \frac{G}{1-G} F_{GS13,x} \cdot n_{GS13,x} \\ & + \frac{G}{1-G} F_{GS13,x} \cdot \frac{g}{\omega^2} \cdot t_p \\ & + \frac{G}{1-G} F_{disp,x} \cdot n_{disp,x} \\ & + \frac{-G}{1-G} F_{disp,x} \cdot g_x \\ & + \frac{G}{1-G} F_{disp,x} \cdot F_{STS,x} \cdot n_{STS,x} \\ & + \frac{G}{1-G} F_{disp,x} \cdot F_{STS,x} \cdot g_x \\ & + \frac{G}{1-G} F_{disp,x} \cdot F_{STS,x} \cdot \frac{g}{\omega^2} \cdot g_{tilt} \end{aligned} \tag{9}$$

The term for platform tilt, t_p, can be quite important. In the next section we derive the relationship between the platform tilt and ground tilt and sensor noise.

3 Tilt

The tilt loops are amenable to the same analysis. Figure 3 shows the simplified paths for the tilt coupling.

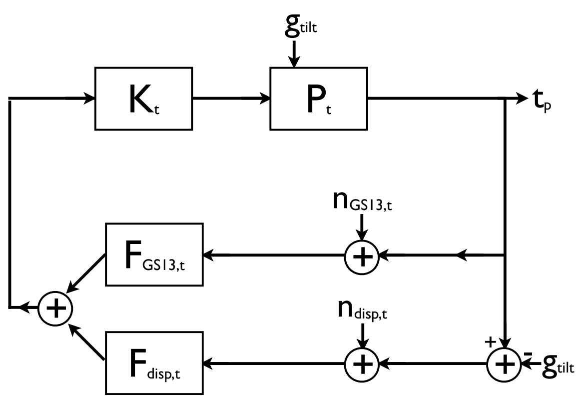

Figure 3: Simplified diagram of the tilt control loops

$$
\begin{aligned}
t_p = \; & \frac{P_{t \leftarrow g_{tilt}}}{1 - G_t} g_{tilt} \\
& + \frac{G_t}{1 - G_t} F_{GS13,t} \cdot n_{GS13,t} \\
& + \frac{G_t}{1 - G_t} F_{disp,t} \cdot n_{disp,t} \\
& + \frac{-G_t}{1 - G_t} F_{disp,t} \cdot g_{tilt}
\end{aligned}
\tag{10}
$$

Again, we see that when the loop gain is large, the tilt of the platform is approximately

$$
\begin{aligned}
t_p \approx \; & - F_{GS13,t} \cdot n_{GS13,t} \\
& - F_{disp,t} \cdot n_{disp,t} \\
& + F_{disp,t} \cdot g_{tilt}
\end{aligned}
\tag{11}
$$

4 Coupling tilt to translation

It is useful to write the expressions for how all the tilt sources influence the horizontal motion of the cg. We begin by looking at equation 9 and keeping the sources related to tilt. We also use G_x for the closed loop gain for the x loop, to distinguish it from G_t.

$$\begin{aligned} x_p = & \frac{P_{x \leftarrow g_{tilt}}}{1 - G_x} g_{tilt} \\ & + \frac{G_x}{1 - G_x} F_{disp,x} \cdot F_{STS,x} \cdot \frac{g}{\omega^2} \cdot g_{tilt} \\ & + \frac{G_x}{1 - G_x} F_{GS13,x} \cdot \frac{g}{\omega^2} \cdot t_p \end{aligned} \tag{12}$$

We expand the platform tilt, t_p, from equation 10 to yield

$$\begin{aligned} x_p = & \frac{P_{x \leftarrow g_{tilt}}}{1 - G_x} g_{tilt} \\ & + \frac{G_x}{1 - G_x} F_{disp,x} \cdot F_{STS,x} \cdot \frac{g}{\omega^2} \cdot g_{tilt} \\ & + \frac{G_x}{1 - G_x} F_{GS13,x} \cdot \frac{g}{\omega^2} \cdot \frac{G_t}{1 - G_t} \cdot ... \\ & \left(\frac{P_{t \leftarrow g_{tilt}}}{G_t} g_{tilt} + F_{GS13,t} \cdot n_{GS13,t} + F_{disp,t} \cdot n_{disp,t} - F_{disp,t} \cdot g_{tilt} \right) \end{aligned} \tag{13}$$

When the loop gains are large, this becomes

$$\begin{aligned} x_p \approx & - F_{disp,x} \cdot F_{STS,x} \cdot \frac{g}{\omega^2} \cdot g_{tilt} \\ & + F_{GS13,x} \cdot \frac{g}{\omega^2} \cdot \left(F_{GS13,t} \cdot n_{GS13,t} + F_{disp,t} \cdot n_{disp,t} - F_{disp,t} \cdot g_{tilt} \right) \end{aligned} \tag{14}$$