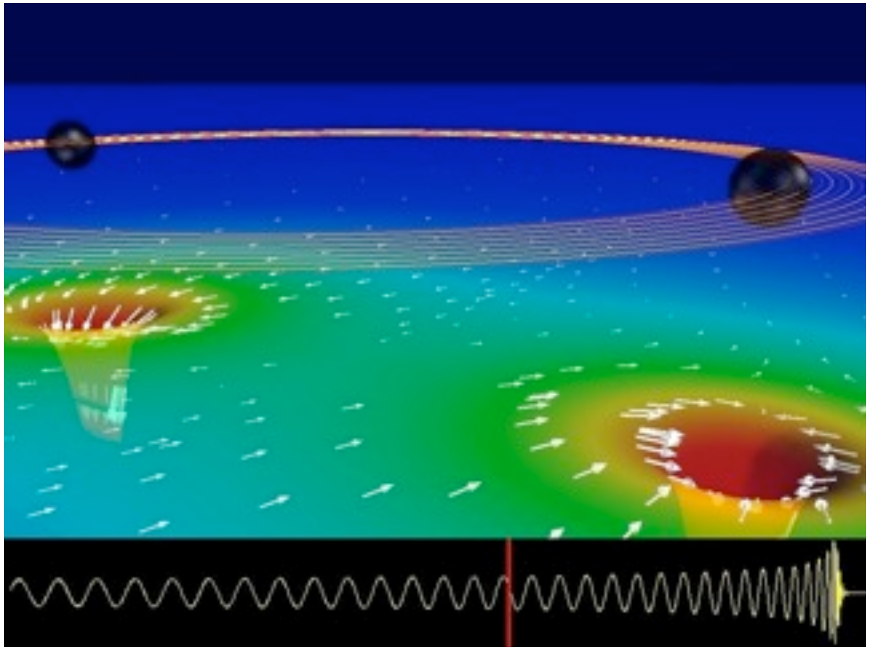
Testing General Relativity with Gravitational-Wave Observations

B.S. Sathyaprakash
LIGO, Caltech, August 3, 2010

Goal of the talk

To show that gravitational-wave observations of compact binaries offer the best possible tests of general relativity, indeed any metric theory of gravity, beyond the solar system tests and binary pulsar tests.



A metric theory of gravity

- Tests of the equivalence principle have confirmed that the only possible theories of gravity are the so-called metric theories
- A metric theory of gravity is one in which
  - there exists a symmetric metric tensor
  - test bodies follow geodesics of this metric
  - in local Lorentz frames, non-gravitational laws of physics are those of special relativity
  - All non-gravitational fields couple in the same manner to a single gravitational field - that is "universal coupling"
    - Metric is a property of the spacetime
- The only gravitational field that enters the equations of motion is the metric
  - Other fields (scalar, vector, etc.) may generate the spacetime curvature associated with the metric but they cannot directly influence the equations of motion

Will, LRR

Parametrized post-Newtonian formalism

- In slow-motion, weak-field limit all metric theories of gravity have the same structure
  - Can be written as an expansion about the Minkowski metric in terms of dimensionless gravitational potentials of varying degrees of smallness
- Potentials are constructed from the matter variables
- The only way that one metric theory differs from another is in the numerical values of the coefficients that appear in front of the metric potentials
  - Current PPN formalism has 10 parameters

Will, LRR

Why a compact binary?

- Black holes and neutron stars are the most compact objects
  - Surface potential energy of a test particle is equal to its rest mass energy

$$\frac{GmM}{R} \sim mc^2$$

- Binaries comprising of neutron stars and/or black holes are termed compact binaries
- Being the most compact objects, they are also the most luminous sources of gravitational radiation
  - The luminosity of a binary could increase a million times in the course of its evolution through a detector's sensitivity band
  - The luminosity of a binary black hole (no matter how small or large) outshines the luminosity in all visible matter in the Universe

BBH Signals as Testbeds for GR

- Gravity gets ultra-strong during a BBH merger compared to any observations in the solar system or in binary pulsars
  - In the solar system: $\varphi/c^2 \sim 10^{-6}$
  - In a binary pulsar it is still very small: $\varphi/c^2 \sim 10^{-4}$
  - Near a black hole $\varphi/c^2 \sim 1$
  - Merging binary black holes are the best systems for strong-field tests of GR
- Dissipative predictions of gravity are not even tested at the 1PN level
  - In binary black holes even $(v/c)^7$ PN terms might not be adequate for high-SNR (~100) events

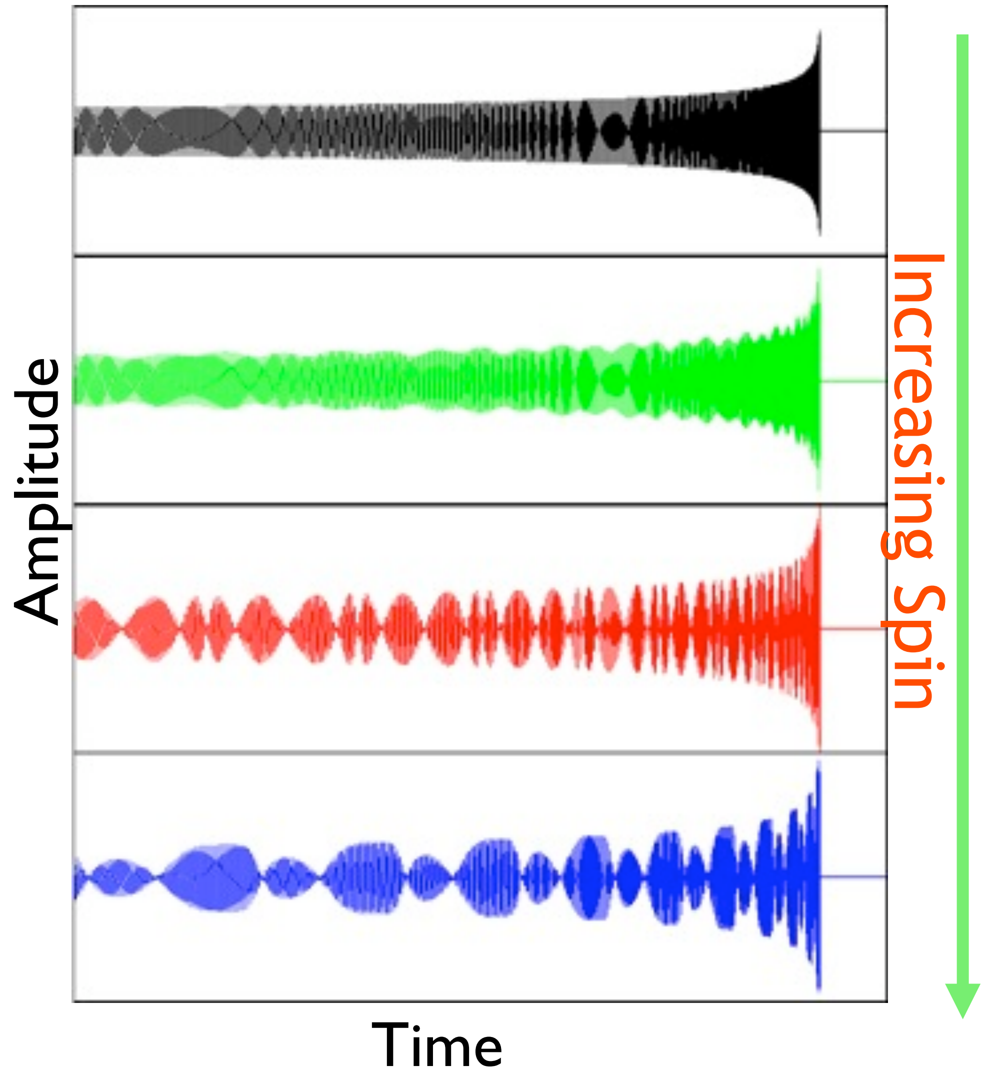
Compact binaries: theoretically the best studied sources

- In general relativity the two-body problem has no known exact analytic solution
  - It is an "ill-posed" problem (B. Carter)
- Approximate methods have been used to understand the dynamics: post-Newtonian (PN) approximation
  - The binary evolves by emitting gravitational-waves whose amplitude and frequency both grow with time - a chirp
  - Coalescence results in a single deformed black hole which emits "ringdown" signals with characteristic frequency and damping time
- Progress in analytical and numerical relativity over the last decade has led to a good understanding of the merger dynamics

Black hole binary waveforms

- Late-time dynamics of compact binaries is highly relativistic, dictated by non-linear general relativistic effects
- Post-Newtonian theory, which is used to model the evolution, is now known to $O(v^7)$
- The shape and strength of the emitted radiation depend on many parameters of the binary: masses, spins, distance, orientation, sky location, ...

$$h(t) = 4\eta \frac{M}{D} \frac{M}{r(t)} \cos 2\varphi(t)$$



Structure of the full post-Newtonian (PN) waveform

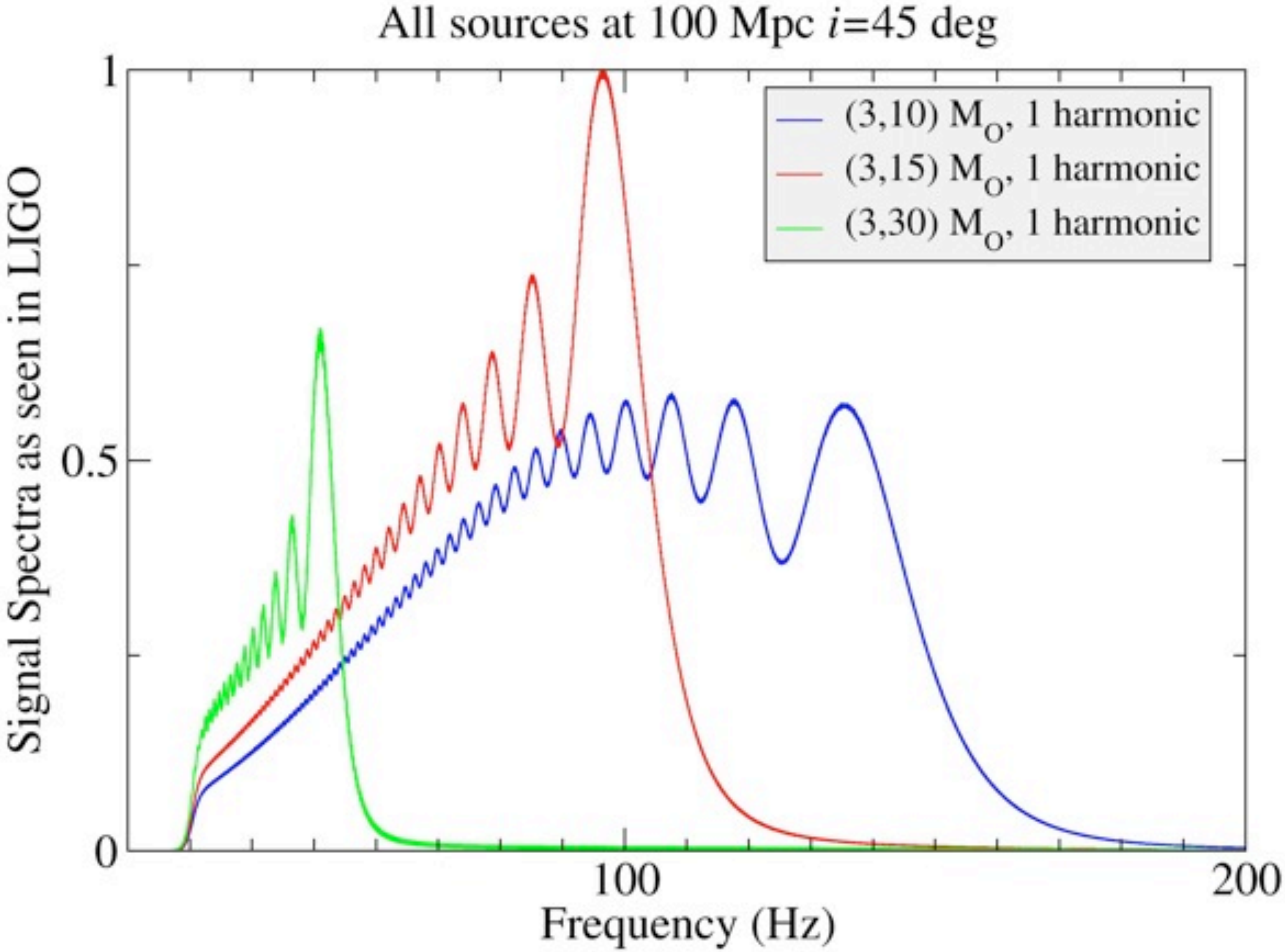
- Radiation is emitted not just at twice the orbital frequency but at all other harmonics too

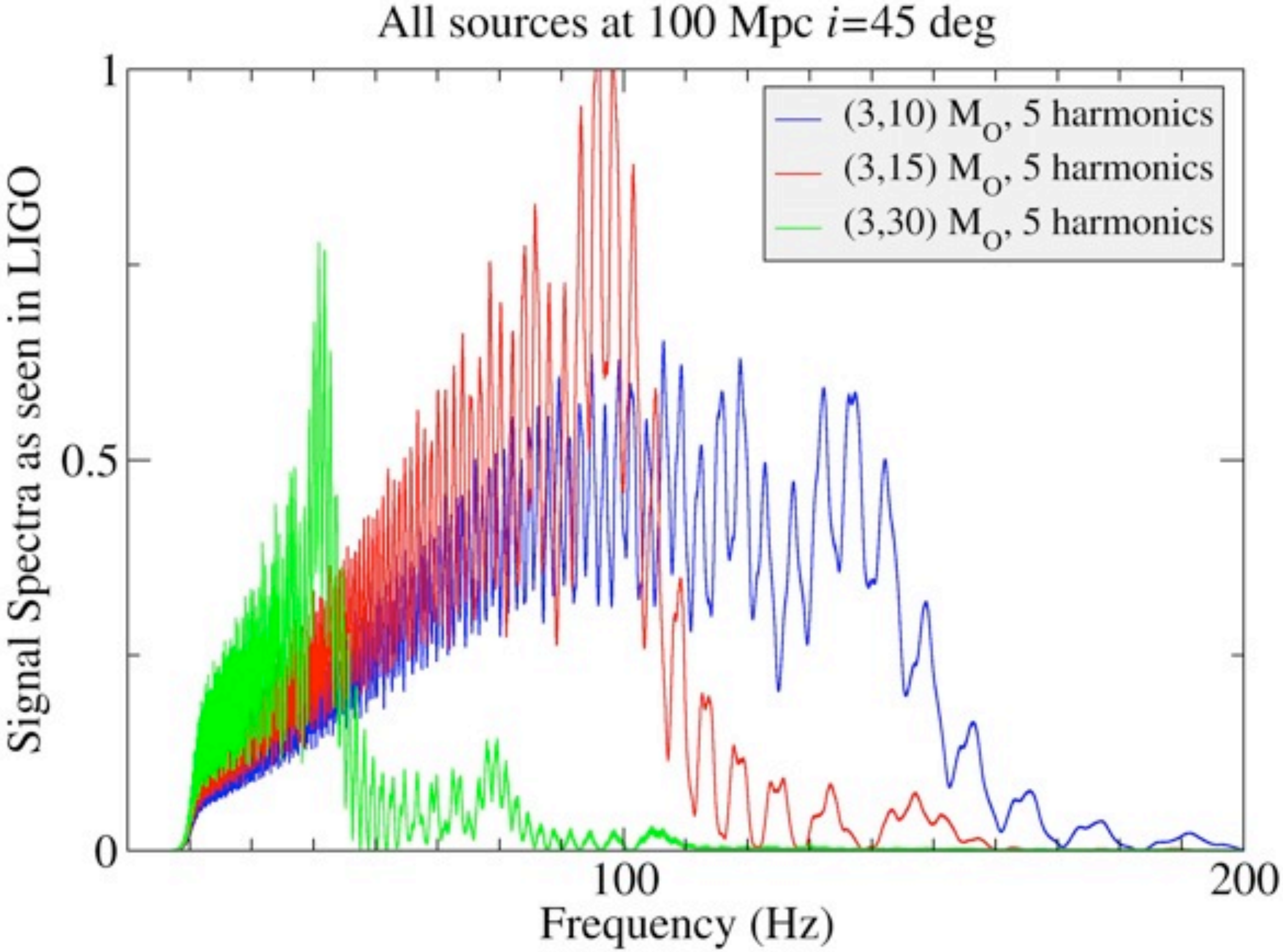
$$h(t) = \frac{2M\eta}{D_{\rm L}} \sum_{k=1}^{7} \sum_{n=0}^{5} A_{(k,n/2)} \cos\left[k\Psi(t) + \phi_{(k,\,n/2)}\right] x^{\frac{n}{2}+1}(t)$$

- This is the "full" waveform (FWF). The waveform corresponding to n=0 is called the restricted PN waveform (RFW)
- These amplitude corrections have a lot of additional structure
- Increased mass reach of detectors
- Greatly improved parameter estimation accuracies

Blanchet, Damour, Iyer, Jaranowski, Schaefer, Will, Wiseman

Andrade, Arun, Buonanno, Gopakumar, Joguet, Esposito-Farase, Faye, Kidder, Nissanke, Ohashi, Owen, Ponsot, Qusaillah, Tagoshi …

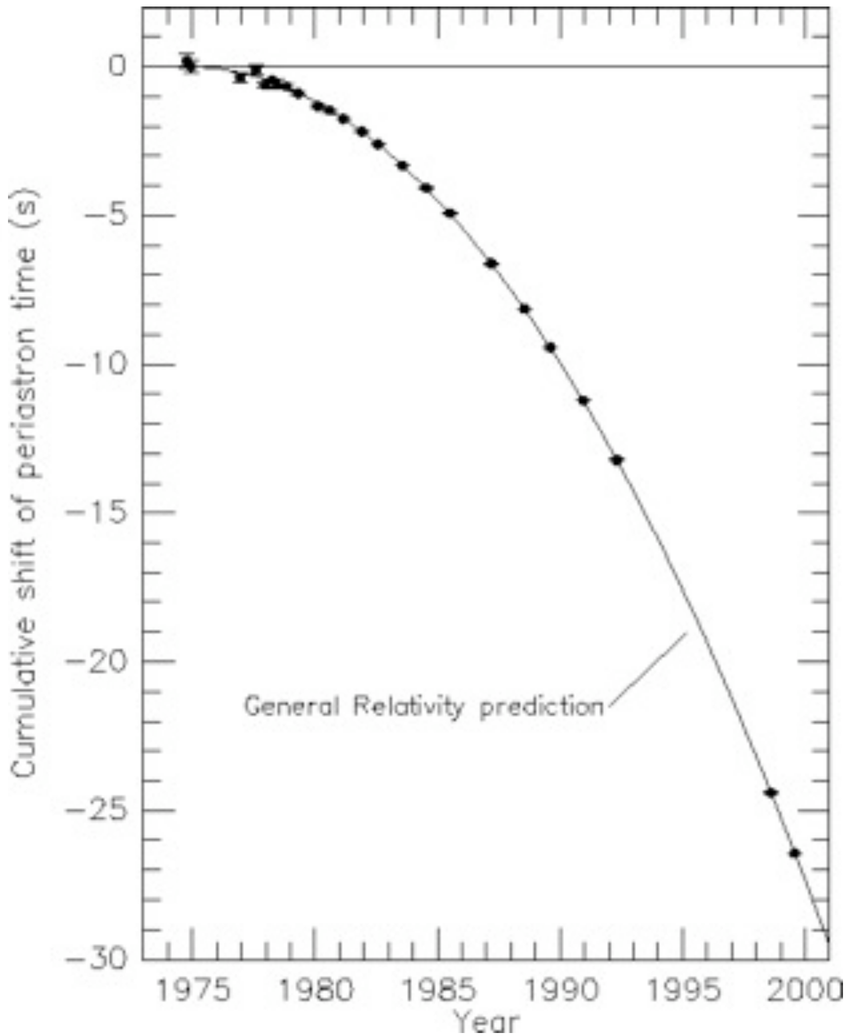

All sources at 100 Mpc i=45 deg
(3,10) M_O, 1 harmonic
(3,15) M_O, 1 harmonic
(3,30) M_O, 1 harmonic
Signal Spectra as seen in LIGO
Frequency (Hz)
0
0.5
1
100
200


All sources at 100 Mpc i=45 deg
(3,10) M_O, 5 harmonics
(3,15) M_O, 5 harmonics
(3,30) M_O, 5 harmonics
Signal Spectra as seen in LIGO
Frequency (Hz)
1
0.5
0
100
200

Testing GR with binary radio pulsars

Hulse-Taylor Binary: A persistent source of Gravitational Waves

- In 1974 Hulse and Taylor observed the first binary pulsar
  - Two neutron stars - each 1.4 $M_\odot$
  - Period ~ 7.5 Hrs, eccentricity 0.62
- Einstein's gravity predicts the binary should emit gravitational radiation
  - The stars spiral in toward each other, causes a decrease in the orbital period
  - Observed decrease in period - about 10 micro seconds per year - is in agreement with Einstein's theory to fraction of a percent



Taylor and Weisberg, 2000, Will Living Review

How does a binary pulsar help test GR?

- Non-orbital parameters
  - position of the pulsar on the sky; period of the pulsar and its rate of change
- Five Keplerian parameters, e.g.
  - Eccentricity e
  - Orbital period P_b
  - Semi-major axis projected along the line of sight $a_p \sin i$
- Five post-Keplerian parameters
  - Average rate of periastron advance $\langle d\omega/dt \rangle$
  - Amplitude of delays in arrival of pulses γ
  - Rate of change of orbital period dP_b/dt
  - “range” and “shape” of the Shapiro time delay

Measured effects depend only on the two masses of the binary

- Average rate of periastron advance

$$\langle \dot{\omega} \rangle = \frac{6\pi f_b (2\pi M f_b)^{2/3}}{(1-e^2)}$$

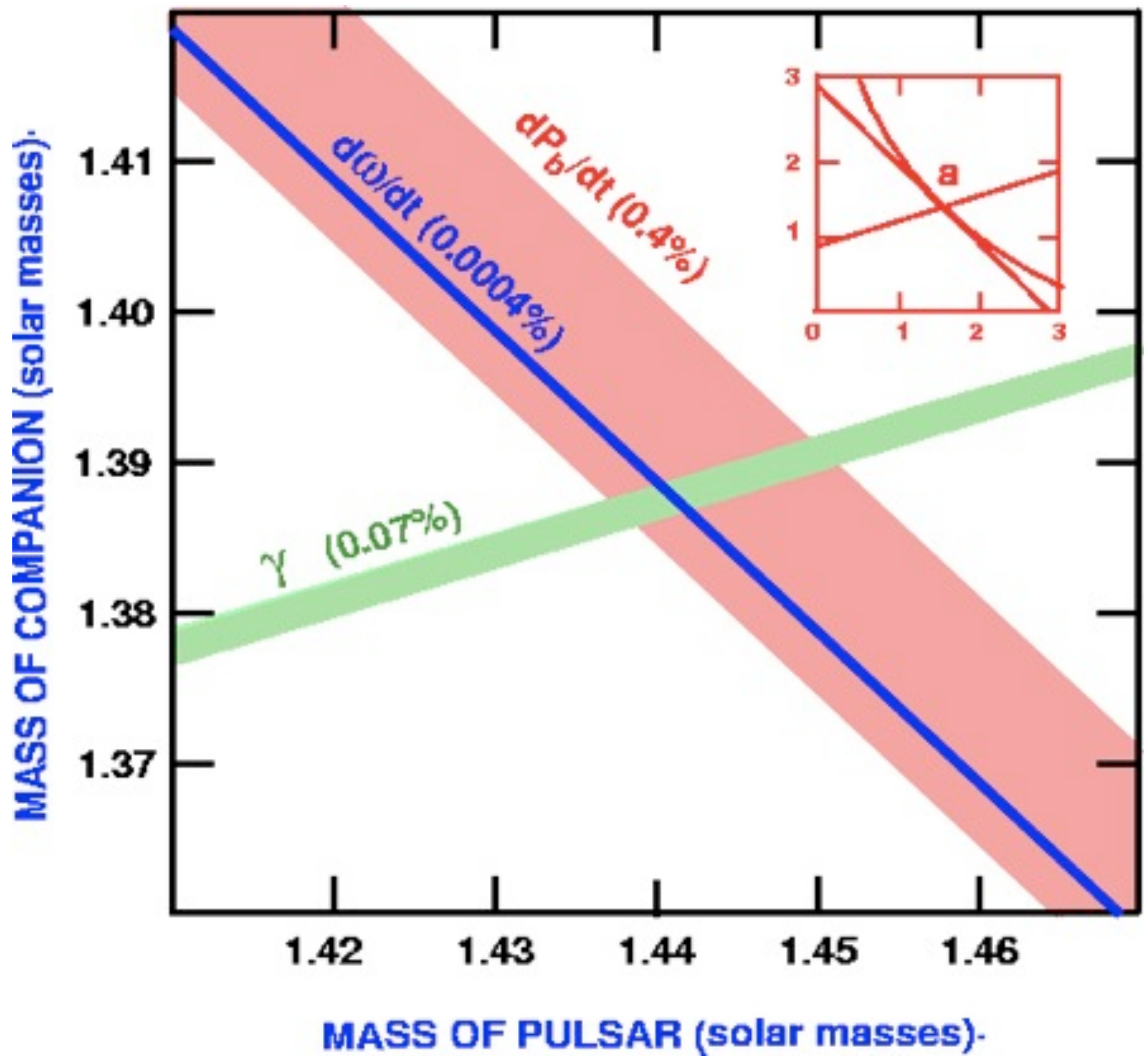
- Amplitude of delays in arrival times

$$\gamma = \frac{(2\pi M f_b)^{2/3}}{2\pi f_b} \frac{e m_2}{M} \left(1 + \frac{m_2}{M}\right)$$

- Rate of change of the orbital period

$$\dot{P}_b = -\frac{192}{5} (2\pi \mathcal{M} f_b)^{5/3} F(e)$$

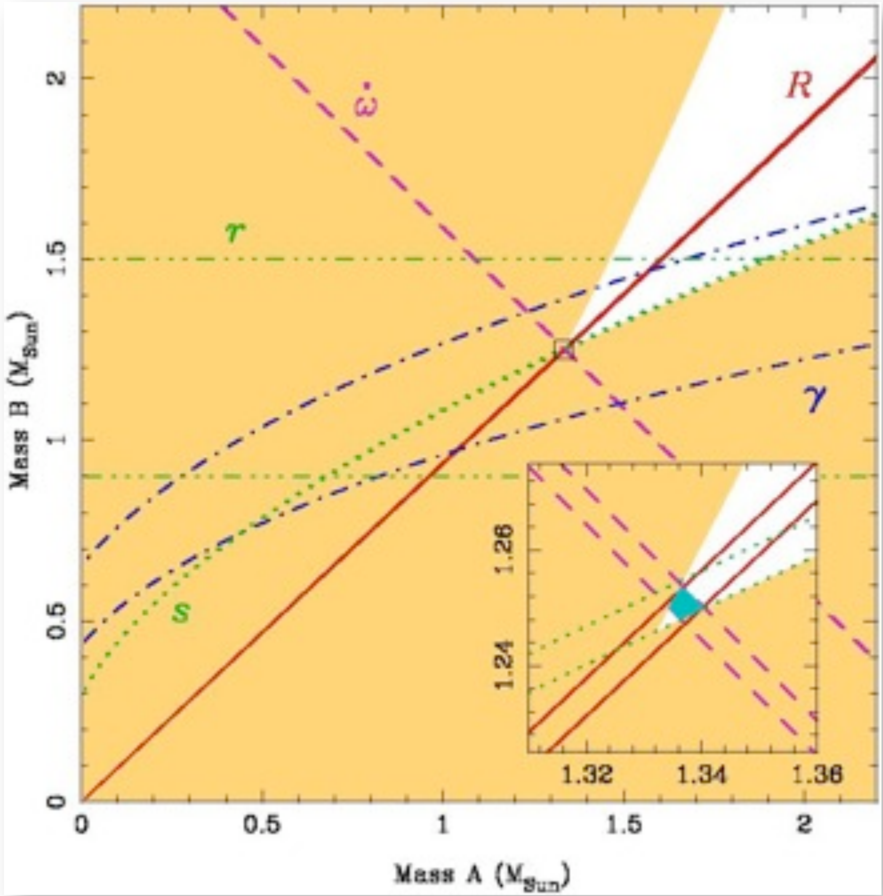
Test of GR in PSR 1913+16



Binary pulsar J0737-3039

- J0737-3039 is the fastest binary known to date
  - Strongly relativistic, P_b=2.5 Hrs
  - Mildly eccentric, *e*=0.088
  - Highly inclined (*i* > 87 deg)
- The most relativistic
  - Greatest periastron advance: *dω/dt:* 16.8 degrees per year (almost entirely general relativistic effect), compared to relativistic part of Mercury's perihelion advance of 42 seconds of arc per century
  - Orbit is shrinking by a few millimeters each year due to gravitational radiation reaction

Burgay et al Nature 2003



Future tests of GR with GW observations

Qualitative Tests

- Polarization states
  - Are there polarizations other than those predicted by GR
    - No concrete proposals yet but some work within the LIGO-Virgo collaboration
- Quasi-normal modes
  - Is the inspiral phase followed by a quasi-normal mode?
    - No concrete evaluations yet
  - Are the different quasi-normal modes consistent with each other?
    - Berti, Cardosa, Will: In the context of LISA, Kamaretsos et al (this talk)
- Is the geometry of the merged object that of a Kerr black hole? (Ryan)
  - Many evaluations in the context of LISA

Quantitative Tests

- Is the phasing of the waveform consistent with General Relativity
  - Can we measure the different post-Newtonian terms and to what accuracy?
    - Detailed study in the case of non-spinning BBH on a quasi-circular orbit (Mishra et al)
    - Effect of spin is important: Neglecting them could lead to erroneous conclusion that GR is wrong while it is not
- Is the signal from the merger phase consistent with the predictions of numerical relativity simulations?
  - Are the parameters of the system from the inspiral, merger and ringdown phases consistent with one another?

Do gravitational waves travel at the speed of light?

- Coincident observation of a supermassive black hole binary and the associated gravitational radiation can be used to constrain the speed of gravitational waves:
- If Δt is the time difference in the arrival times of GW and EM radiation and D is the distance to the source then the fractional difference in the speeds is

$$\frac{\Delta v}{c} = \frac{\Delta t}{D/c} \simeq 10^{-14} \left(\frac{\Delta t}{1\mathrm{sec}}\right) \left(\frac{D}{1\mathrm{Mpc}}\right)$$

- It is important to study what the EM signatures of massive BBH mergers are
- Can be used to set limits on the mass of the graviton slightly better than the current limits.

Will (1994, 98)

Massive graviton causes dispersion

- A massive graviton induces dispersion in the waves

$$\frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{E^2}, \quad v_g/c \approx 1 - \tfrac{1}{2}(c/\lambda_g f)^2, \text{ where } \lambda_g = h/m_g c$$

- Arrival times are altered due to a massive graviton - frequency-dependent effect
- One can test for the presence of this term by including an extra term in our templates

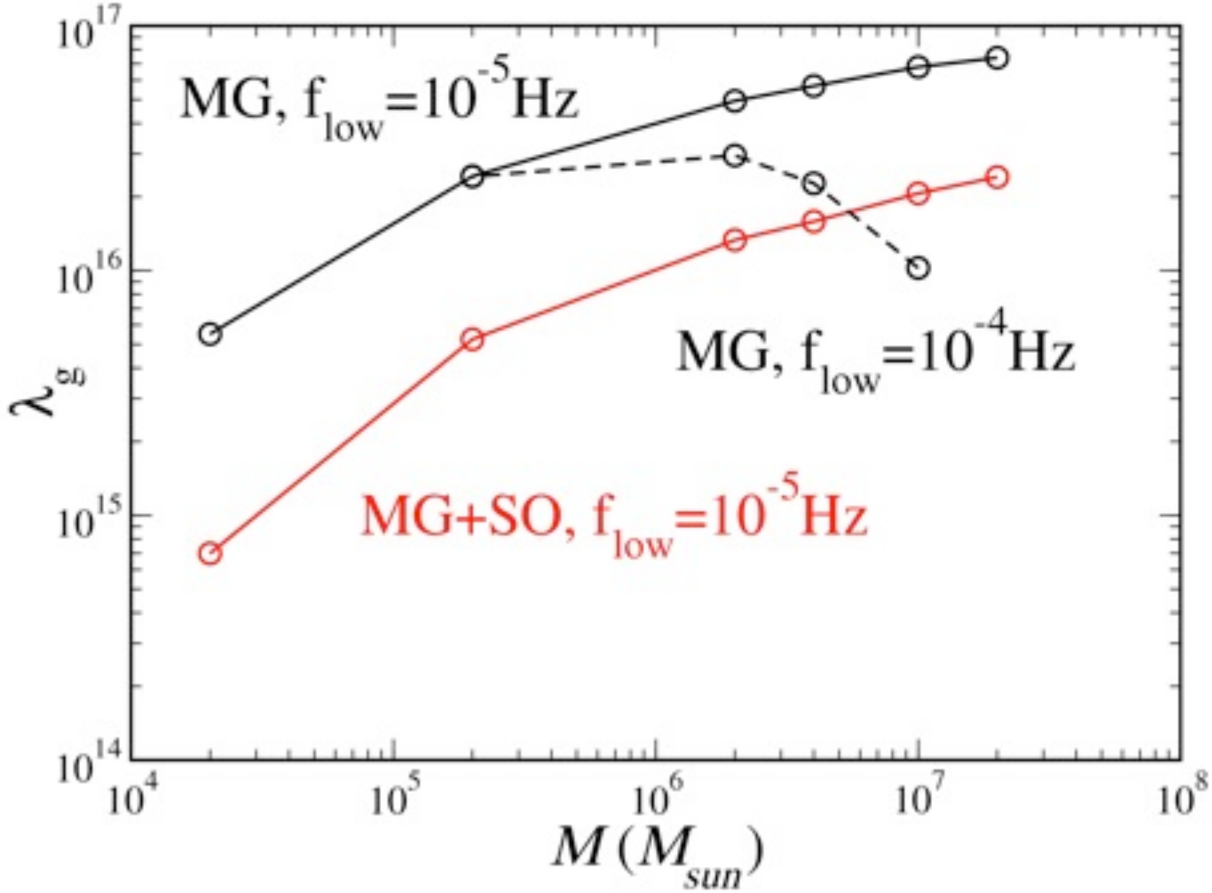
$$t_a = (1+Z)\left[t_e + \frac{D}{2\lambda_g^2 f_e^2}\right] \qquad \Delta\psi_k(f) = \frac{k}{2}\Delta\psi(2f/k) = -\frac{k^2}{4}\pi D/f_e\lambda_g^2$$

Will (1994, 98)

Bound on λ_g as a function of total mass

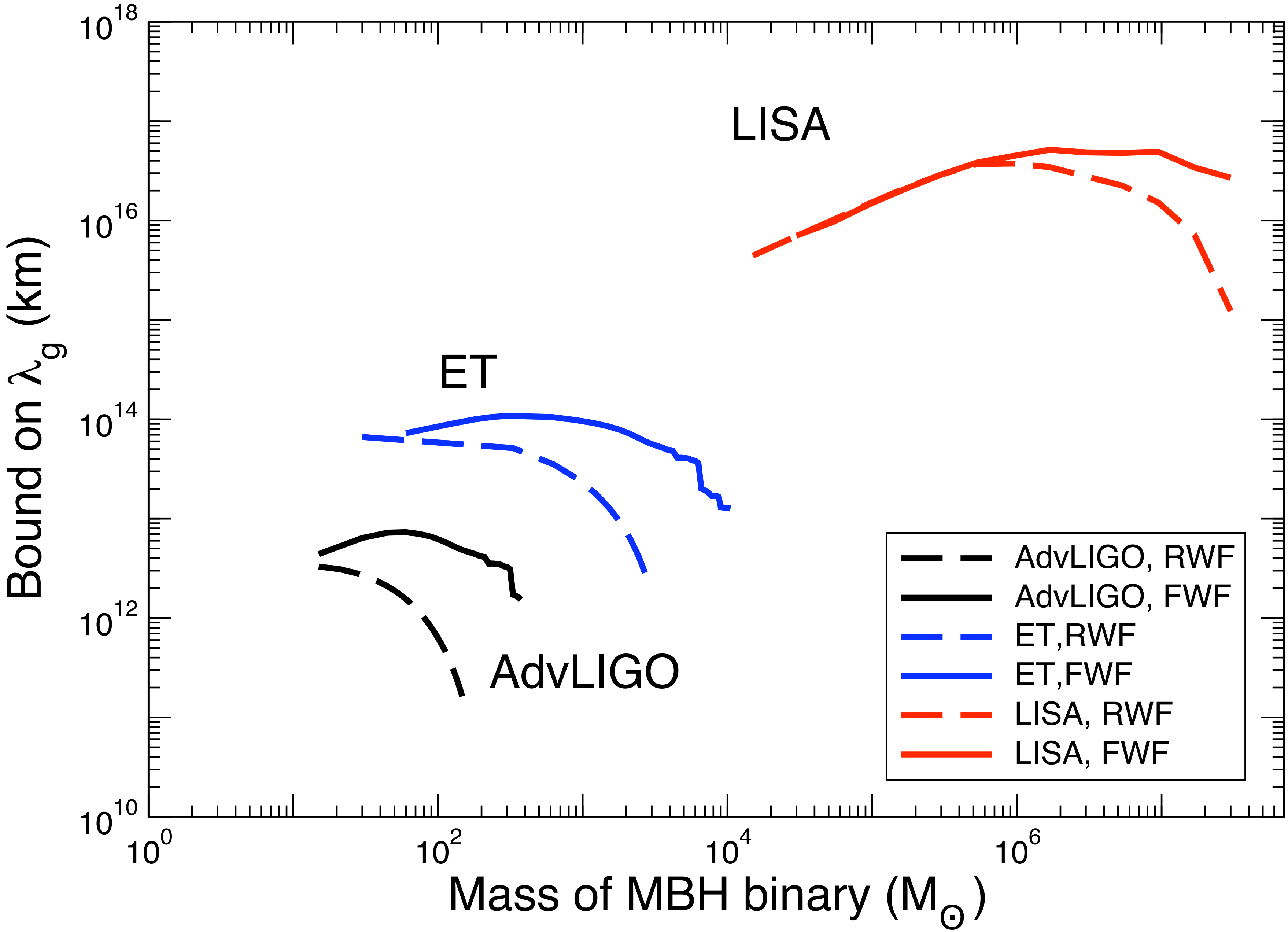
- Limits based on GW observations will be five orders-of-magnitude better than solar system limits

- Still not as good as (model-dependent) limits based on dynamics of galaxy clusters



Berti, Buonanno and Will (2006)

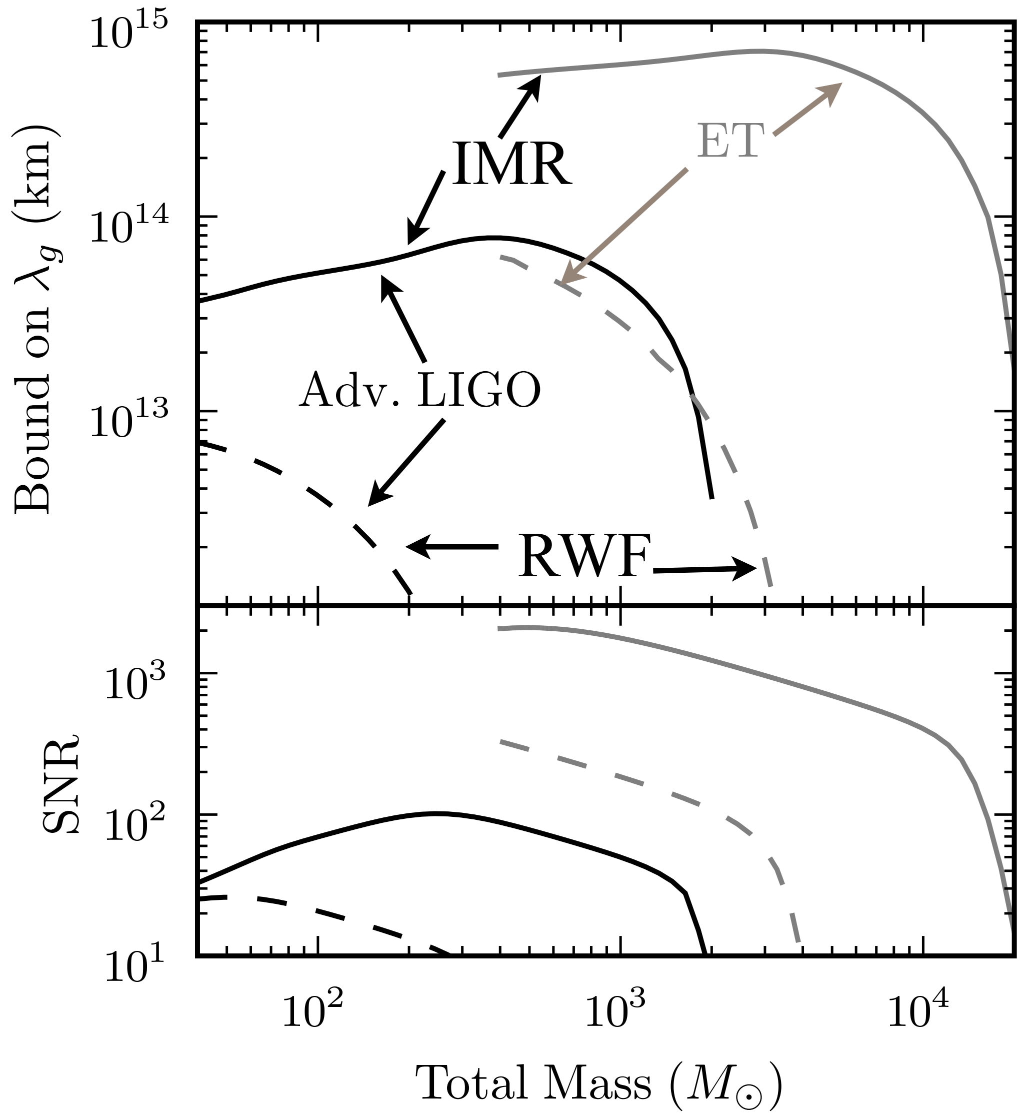
Arun and Will (2009)



Improving bounds with IMR Signals

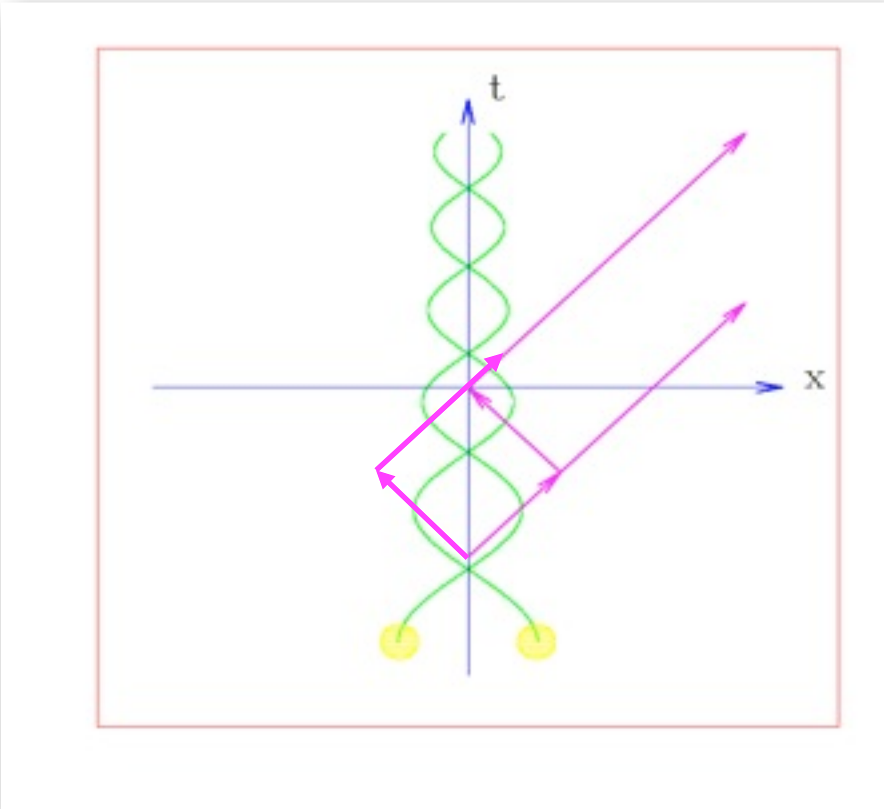
Keppel and Ajith (2010)

- By including the merger and ringdown part of the coalescence it is possible to improve the bound on graviton wavelength
- Equal mass compact binaries assumed to be at 1 Gpc
- ET can achieve 2 to 3 orders of magnitude better bound than the best possible model-independent bounds



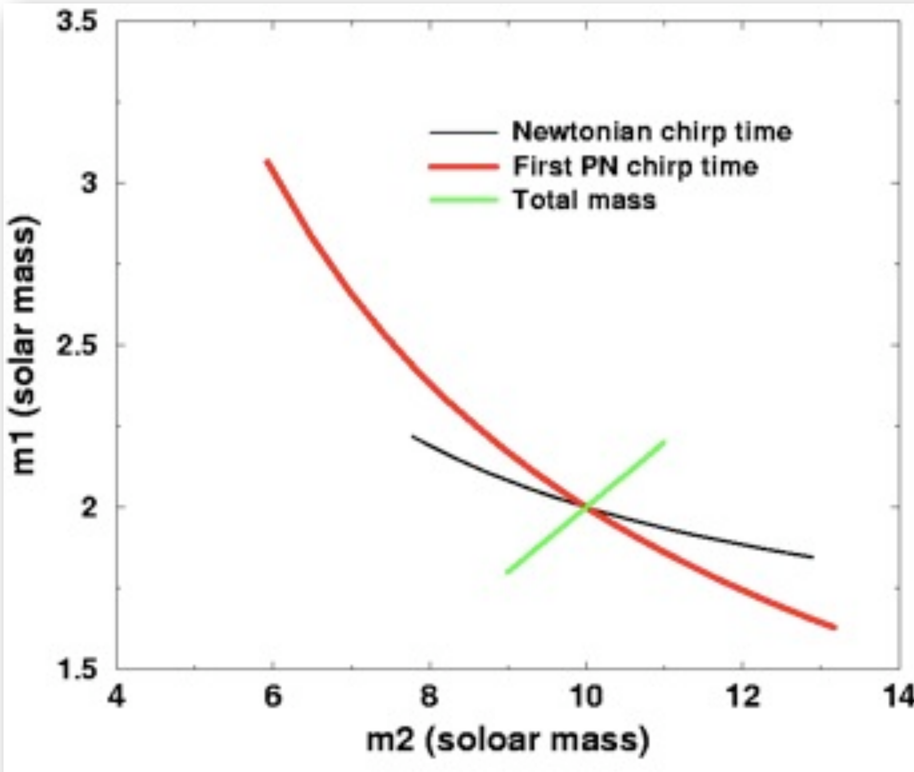
Testing the tail effect

Gravitational wave tails



Blanchet and Schaefer (1994)

Testing the presence of tails



Blanchet and Sathyaprakash (1995)

Testing general relativity with post-Newtonian theory

Post-Newtonian expansion of orbital phase of a binary contains terms which all depend on the two masses of the binary

$$H(f) = \frac{\mathcal{A}(M, \nu, \text{angles})}{D_L} f^{-7/6} \exp\left[-i\psi(f)\right]$$

$$\psi(f) = 2\pi f t_C + \varphi_C + \sum_k \psi_k f^{(k-5)/3}$$

$$\psi_k = \frac{3}{128}(\pi M)^{(k-5)/3} \alpha_k(\nu)$$

$$\alpha_0 = 1, \quad \alpha_1 = 0, \quad \alpha_2 = \frac{3715}{756} + \frac{55}{9}\nu, \ldots$$

Testing general relativity with post-Newtonian theory

- Post-Newtonian expansion of orbital phase of a binary contains terms which all depend on the two masses of the binary

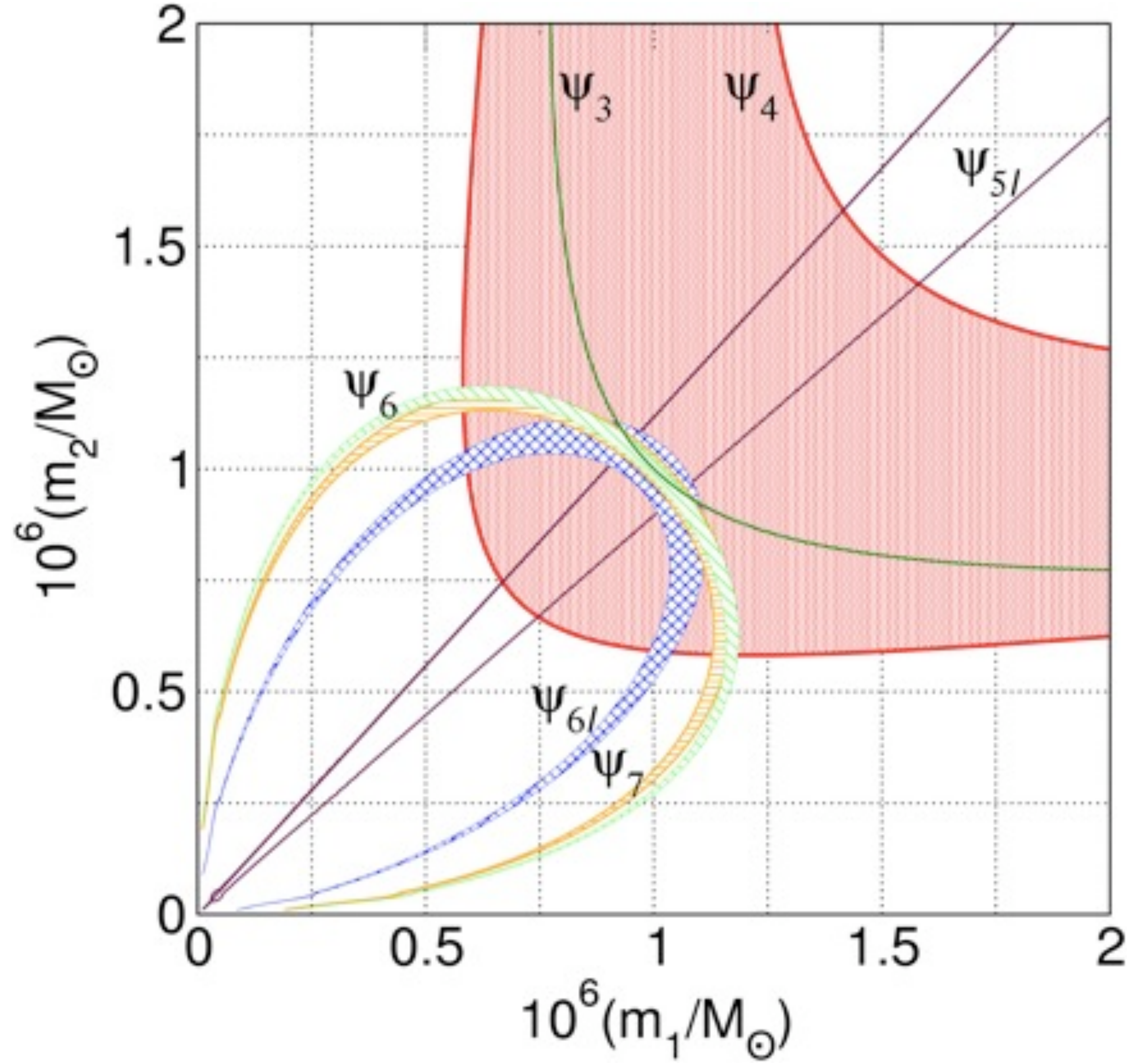
$$\psi_k = \frac{3}{128}(\pi M)^{(k-5)/3}\alpha_k(\nu)$$

- Different terms arise because of different physical effects
- Measuring any two of these will fix the masses
- Other parameters will have to consistent with the first two

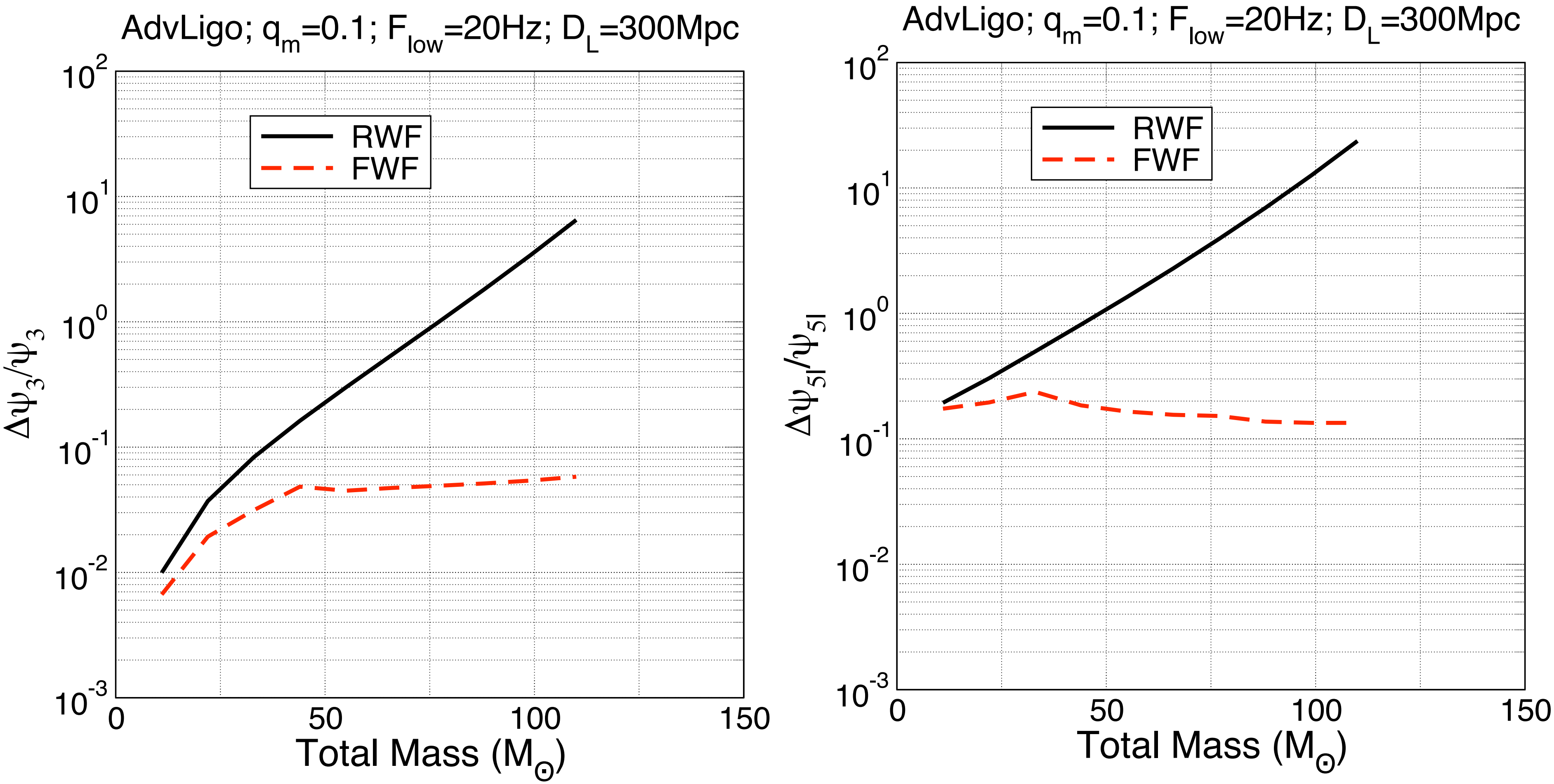
Arun, Iyer, Qusailah, Sathyaprakash (2006a, b)

Testing post-Newtonian theory

Arun, Iyer, Qusailah, Sathyaprakash (2006a, b)

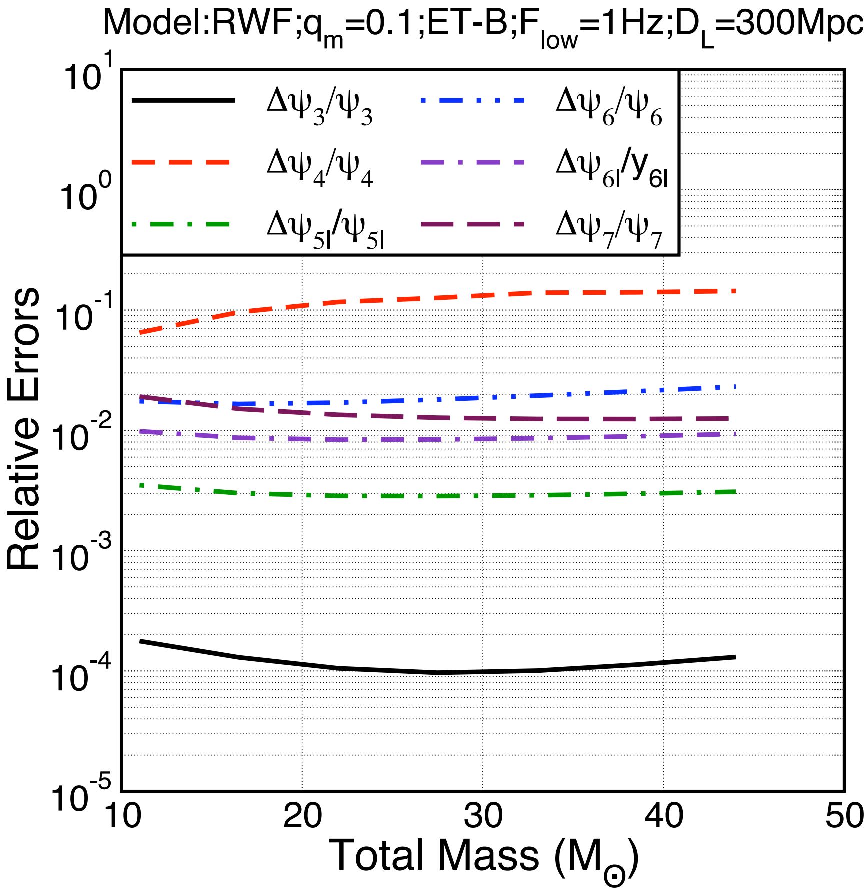


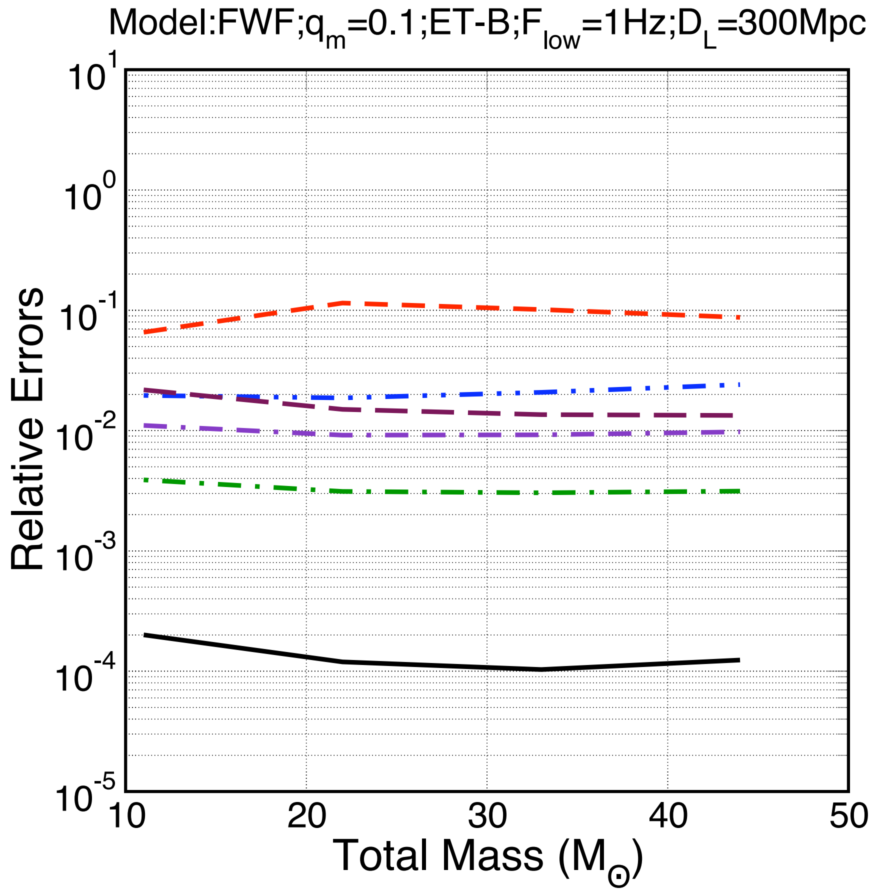
Confirming the presence of tail- and log-terms with Advanced LIGO



Arun, Mishra, Iyer, Sathyaprakash (2010)

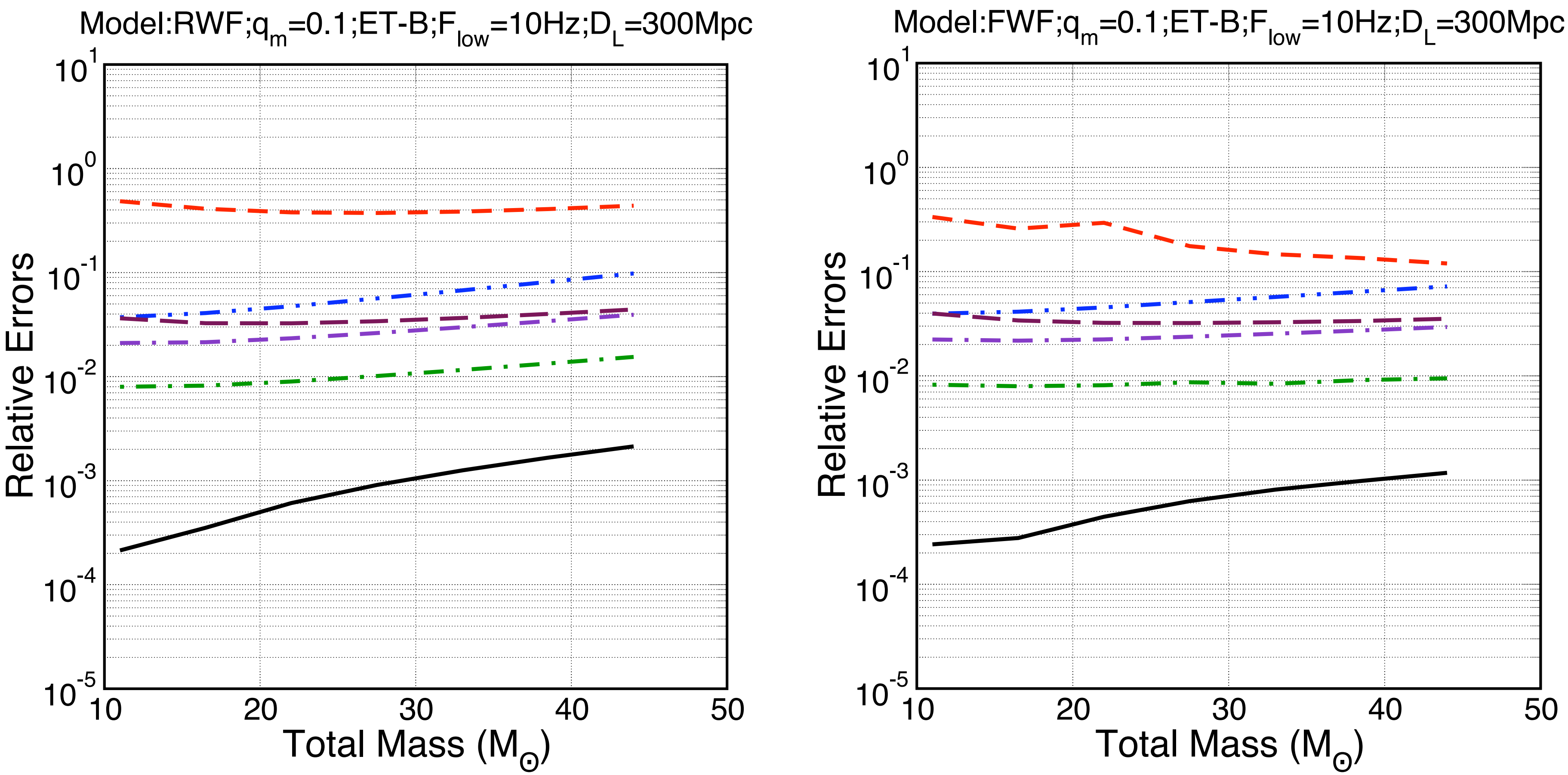
PN parameter accuracies with ET
1 Hz lower cutoff





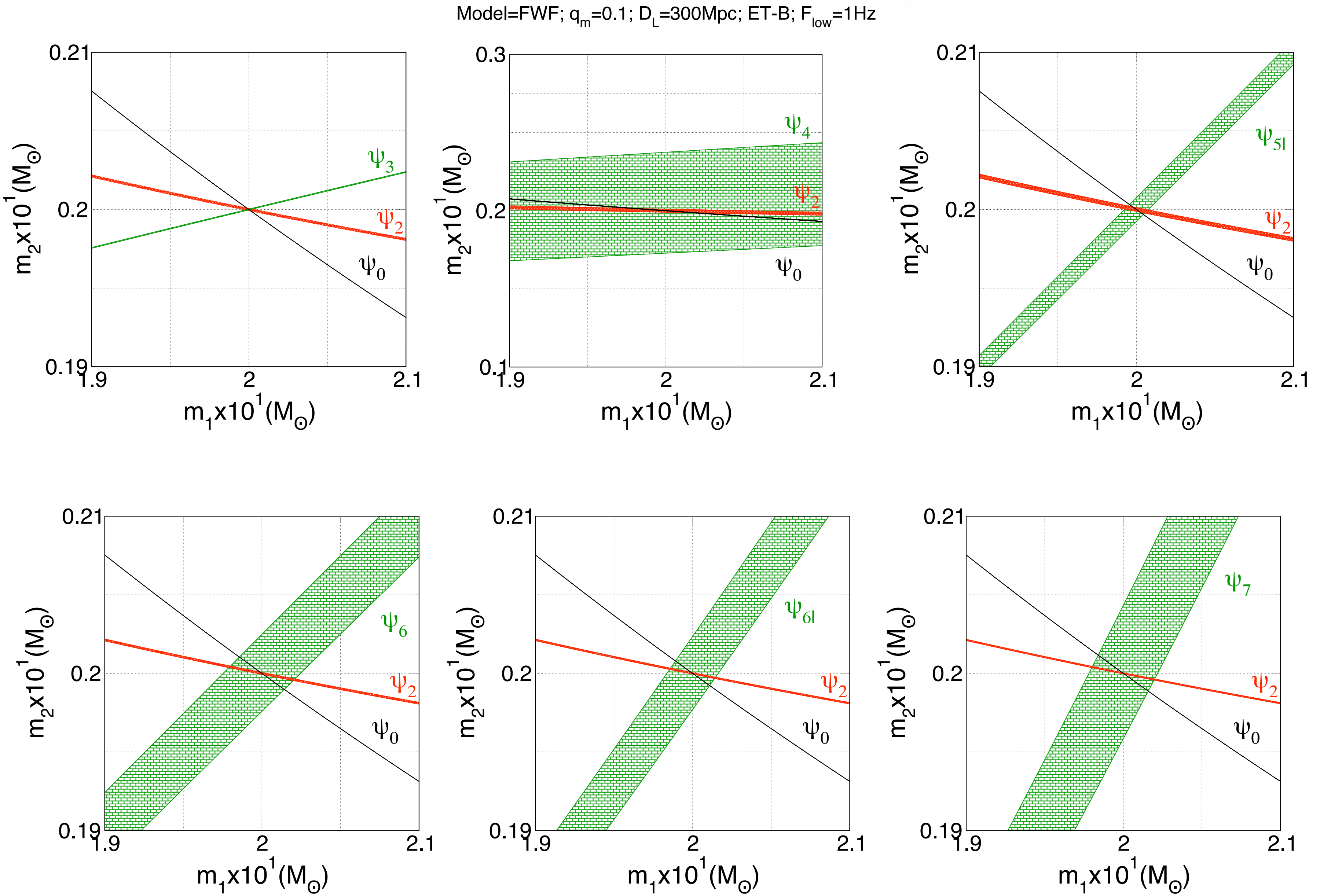
Arun, Mishra, Iyer, Sathyaprakash (2010)

PN parameter accuracies with ET 10 Hz lower cutoff



Arun, Mishra, Iyer, Sathyaprakash (2010)

Test as seen in the plane of component masses



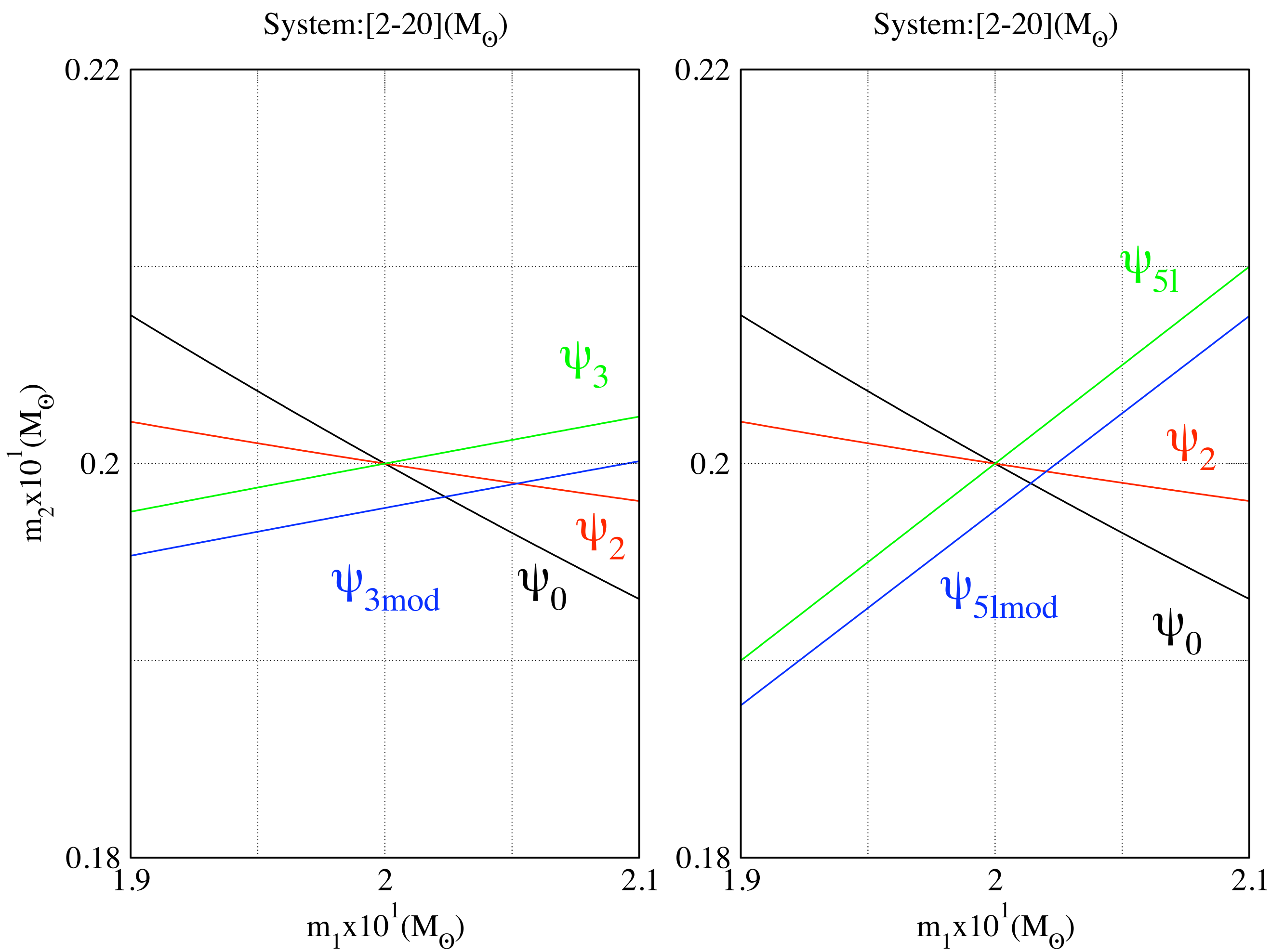
Power of a PN Test

- Suppose the GR k^{th} PN coefficient is $q_k(m_1,m_2)$ while the true k^{th} PN coefficient is $p_k(m_1,m_2)$
- The "measured value of the k^{th} PN coefficient is, say, p_0
- The curve $q_k(m_1,m_2)= p_0$ in the (m_1,m_2) plane will not pass through the masses determined from the other parameters

Arun, Mishra, Iyer, Sathyaprakash (2010)

Power of the PPN test

Effect of changing the coefficients ψ_3 and ψ_{51} by 1% on the test.

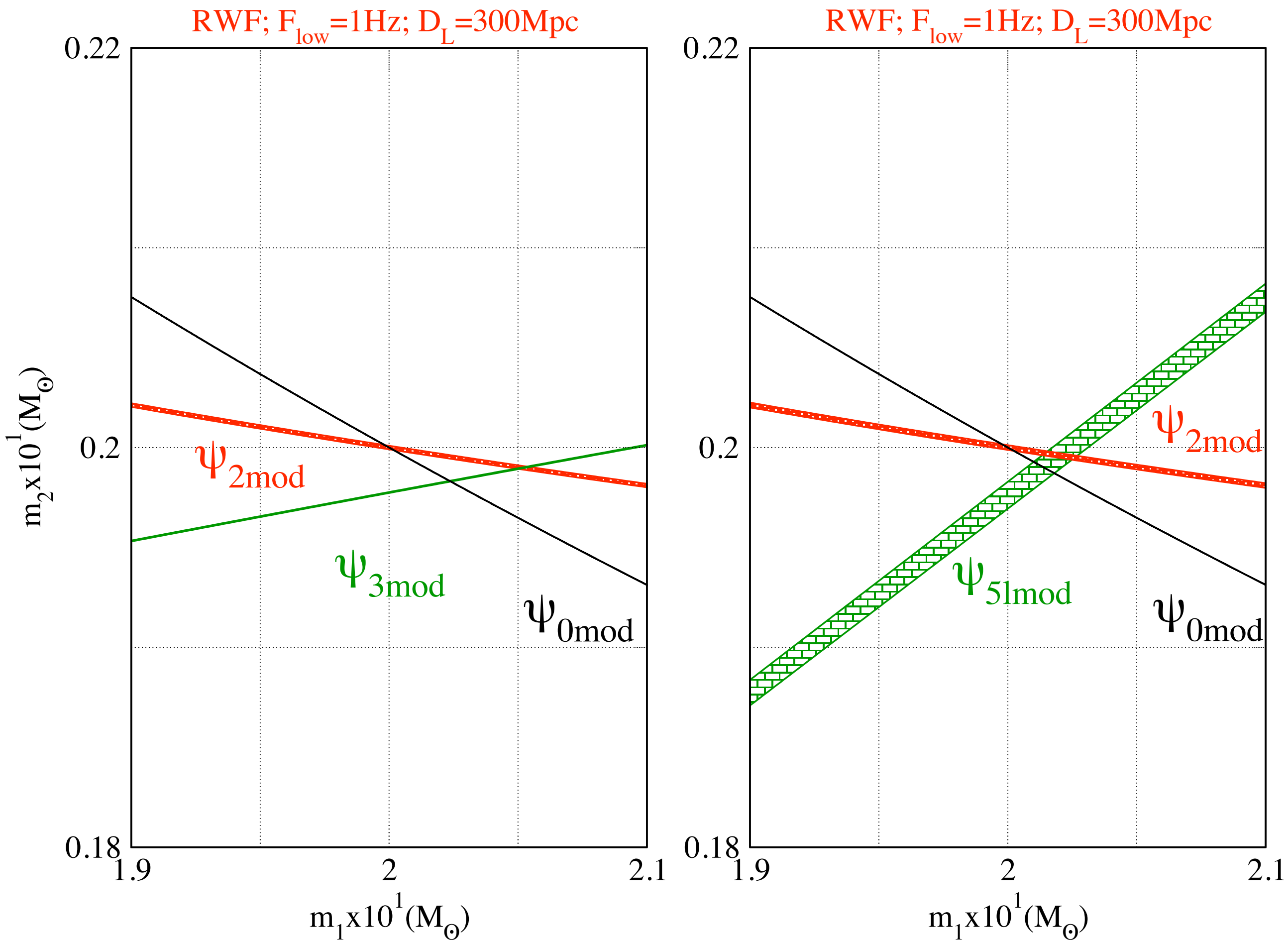


NOTE: Blue curve in the plot corresponds to the new ψ_k

Arun, Mishra, Iyer, Sathyaprakash (2010)

Efficacy of the PPN Test

Effect of changing the coefficients ψ_3 and ψ_{51} by 1% on the test.



NOTE: Reference System: (2-20) ($M_\odot$)

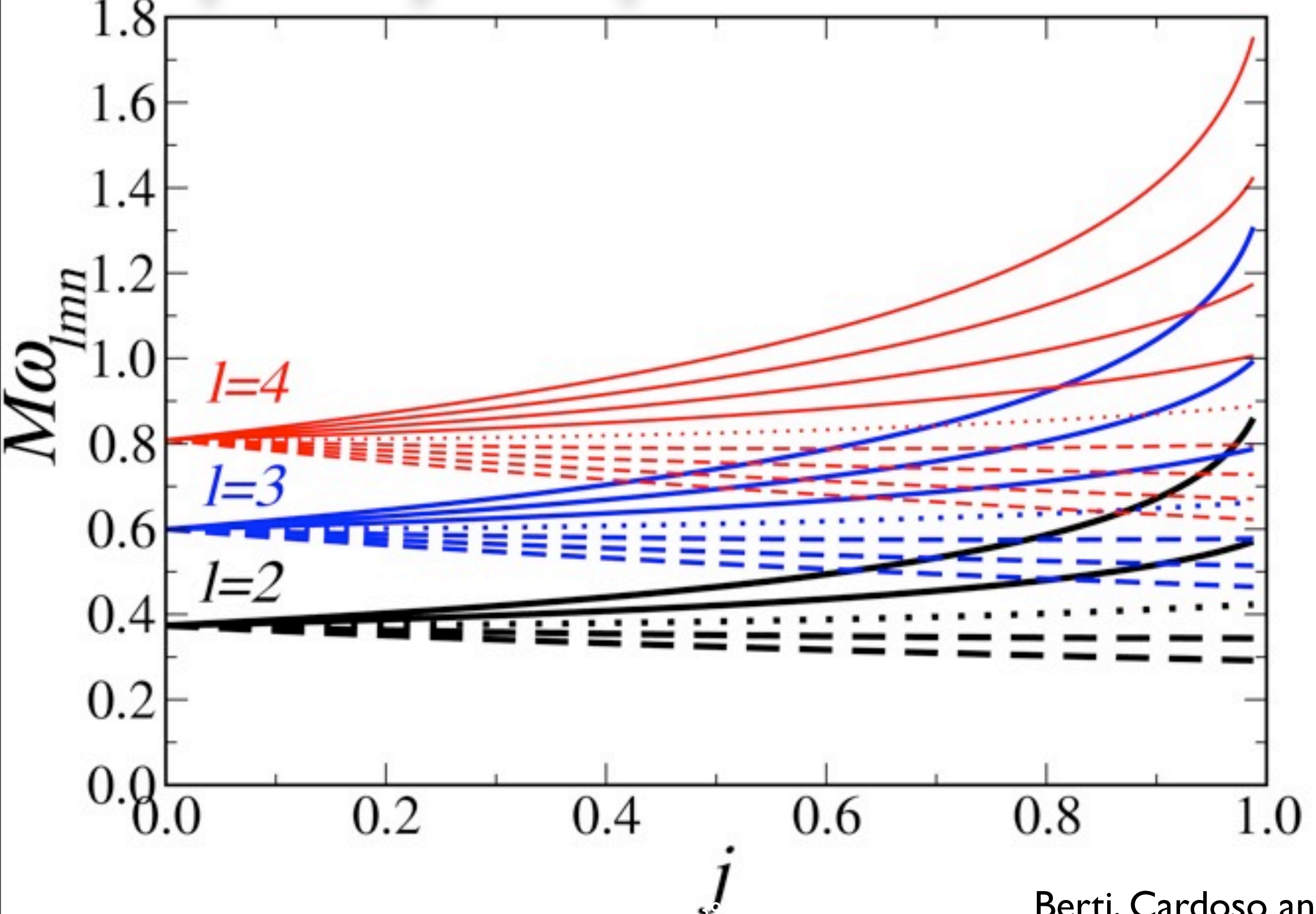
Arun, Mishra, Iyer, Sathyaprakash (2010)

Black Hole Quasi-Normal Modes And Tests of GR

Black hole quasi-normal modes

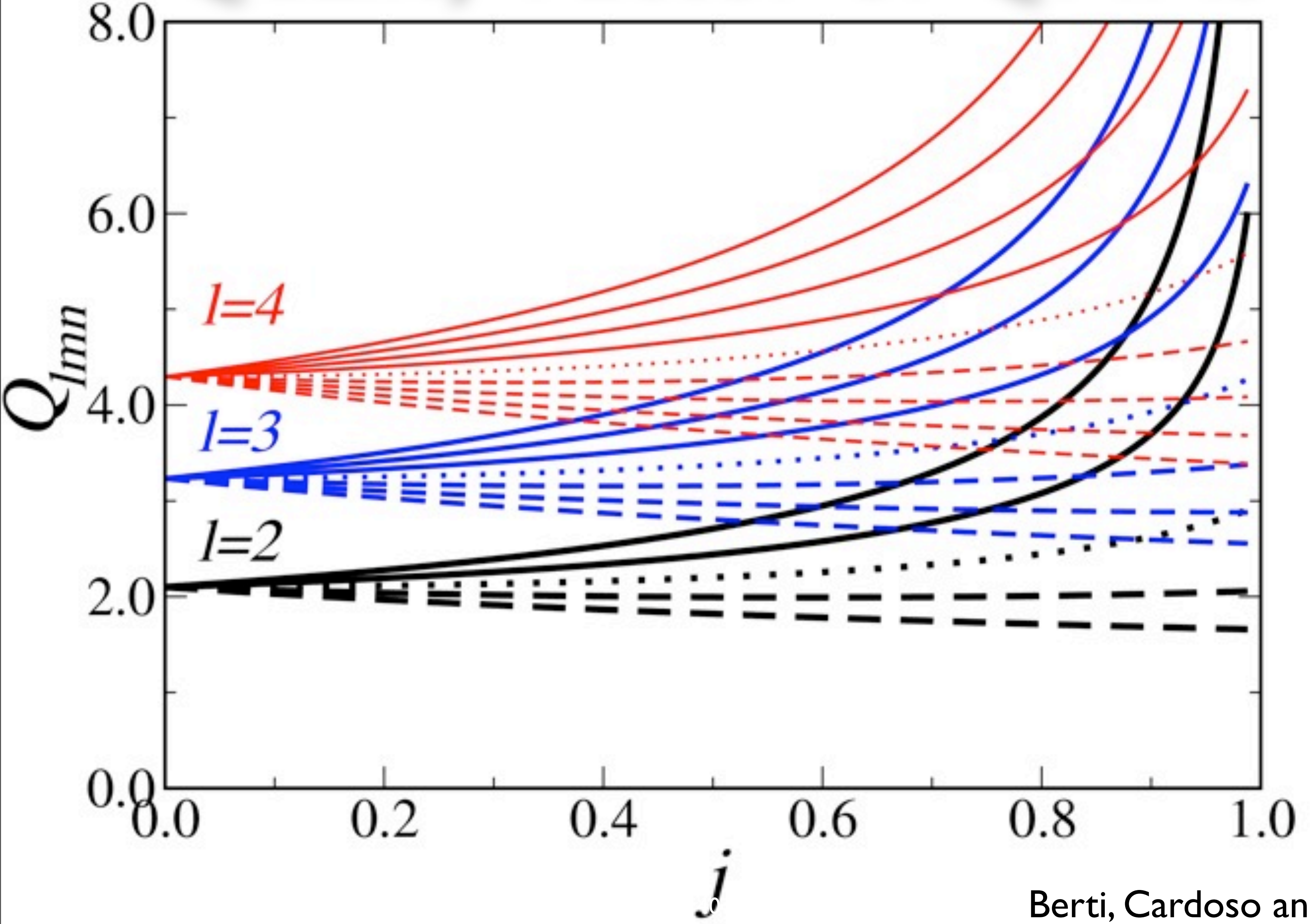
- Damped sinusoids with characteristic frequencies and decay times
  - In general relativity frequencies f_{lmn} and decay times t_{lmn} all depend only on the mass M and spin q of the black hole
- Measuring two or modes unambiguously, would severely constrain general relativity
  - If modes depend on other parameters (e.g., the structure of the central object), then test of the consistency between different mode frequencies and damping times would fail

Frequency of quasi normal modes



Berti, Cardoso and Will

Quality Factor of QNMs



Berti, Cardoso and Will

Tests with QNM

- Studying QNM from NR simulations at various mass ratios: 1:1, 1:2, 1:4, 1:8, final spins from -0.8 to +0.8
  - It is not too difficult to generate the QNM only part of the merger signal
  - Can carry out a wide exploration of the parameter space
- What is the relative energy in the various ringdown modes?
  - Are there at least two modes containing enough energy so that their damping times and frequencies can be measured with good (i.e. at least 10% accuracy)?
  - 33 seems to contain contain enough energy compared to 22 modes; should be possible to extract the total mass and spin magnitude
  - Measuring the relative amplitudes of the different modes can shed light on the binary progenitor, namely the total mass and its mass ratio
  - Polarization of ringdown modes can measure the spin axis of merged BH

Kamaretsos, Hannam, Husa, Sathyaprakash, 2010

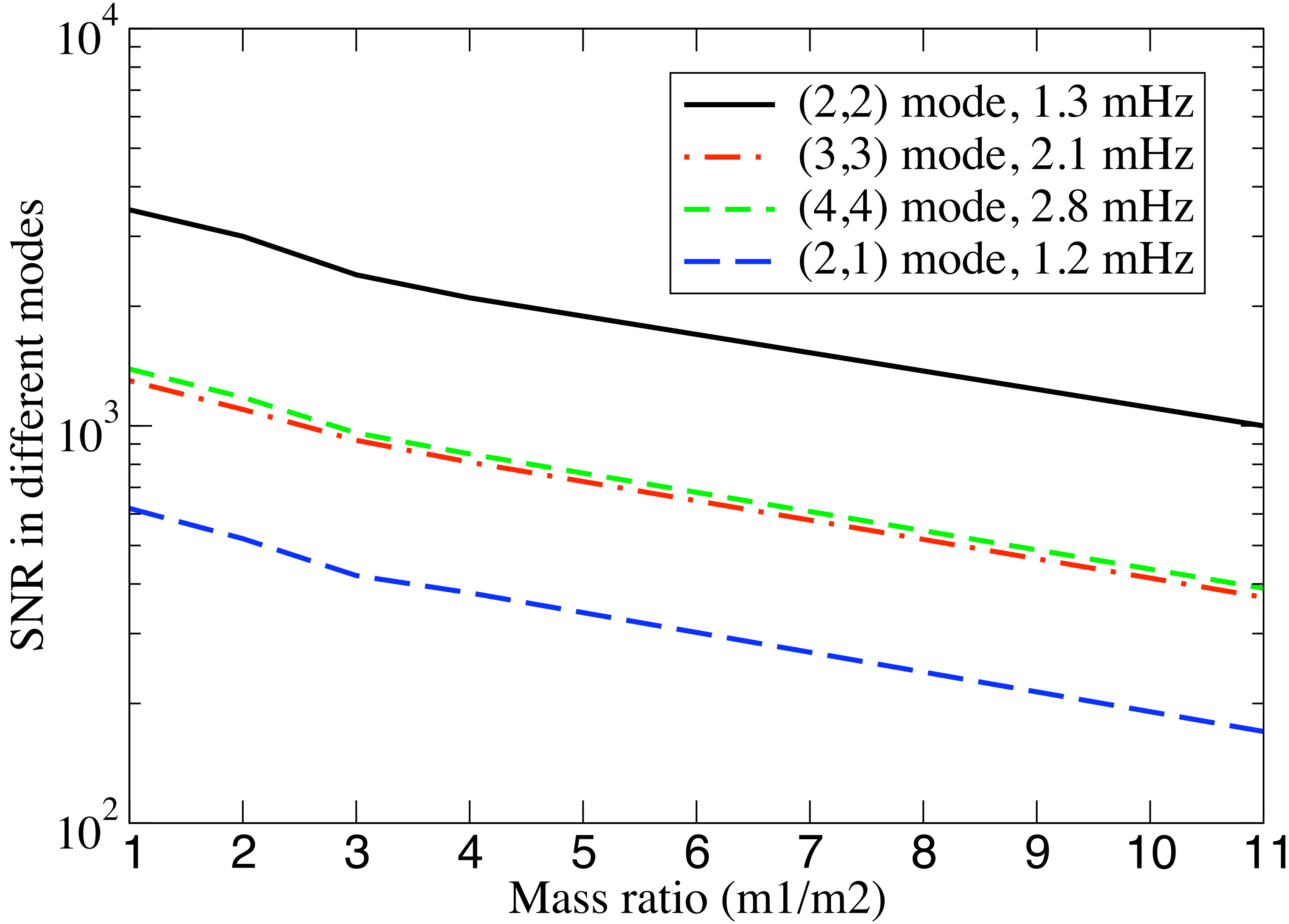
Emitted energy and relative amplitudes of different quasi-normal modes

Table 1: For different mass ratios (q=1, 2, 3, 4, 11), we show the final spin of the black hole, percent of energy in the radiation, amplitude of (2,1), (3,3), (4,4) modes relative to (2,2) mode.

q	j	% total energy	A_{21}/A_{22}	A_{33}/A_{22}	A_{44}/A_{22}
1	0.69	4.9	0.04	0.00	0.05
2	0.62	3.8	0.05	0.13	0.06
3	0.54	2.8	0.07	0.21	0.08
4	0.47	2.2	0.08	0.25	0.09
11	0.25	0.7	0.14	0.31	0.14

Kamaretsos, Hannam, Husa, Sathyaprakash, 2010

LISA SNRs for different QNMs



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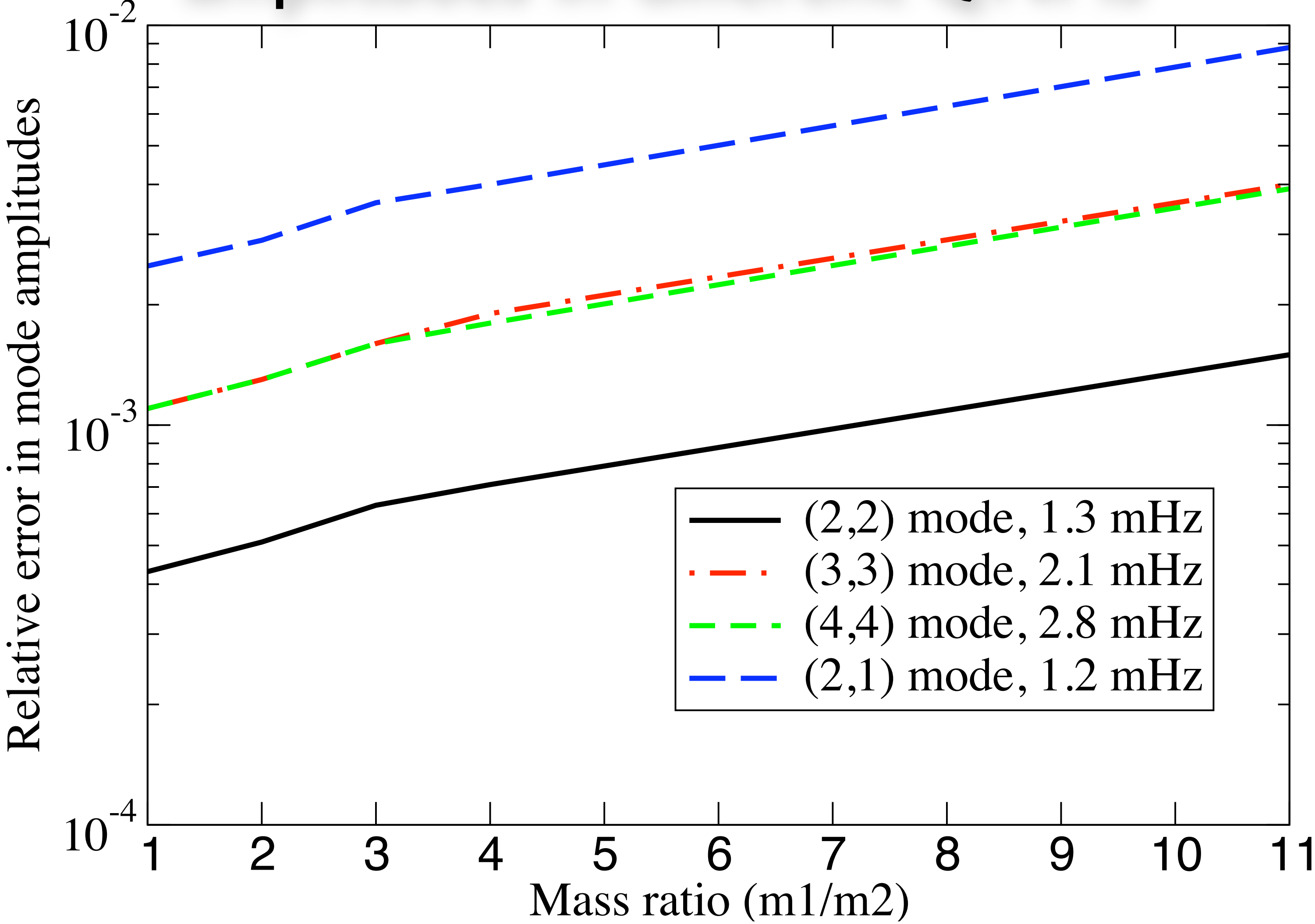
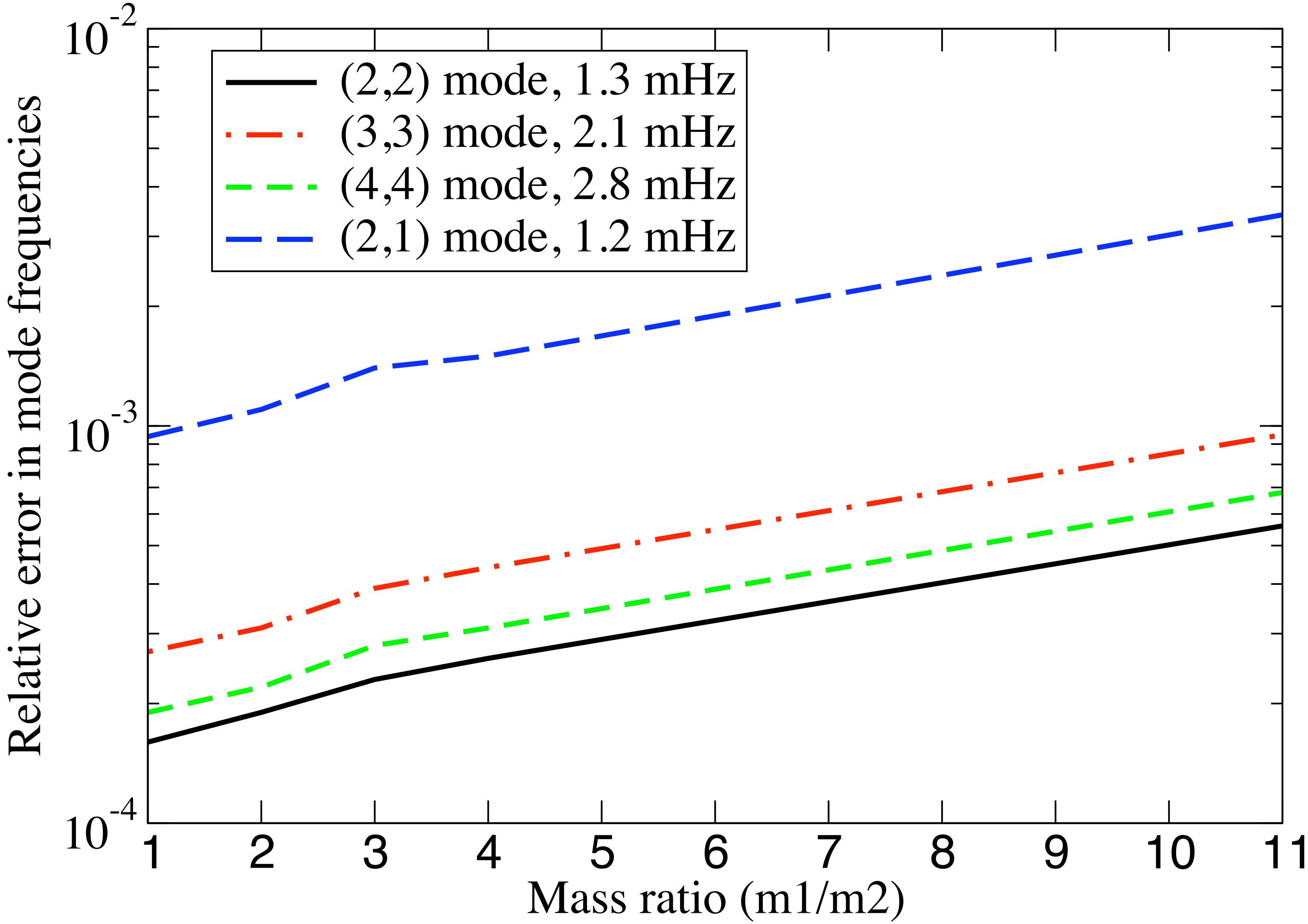
LISA measurement accuracies of amplitudes in different QNMs



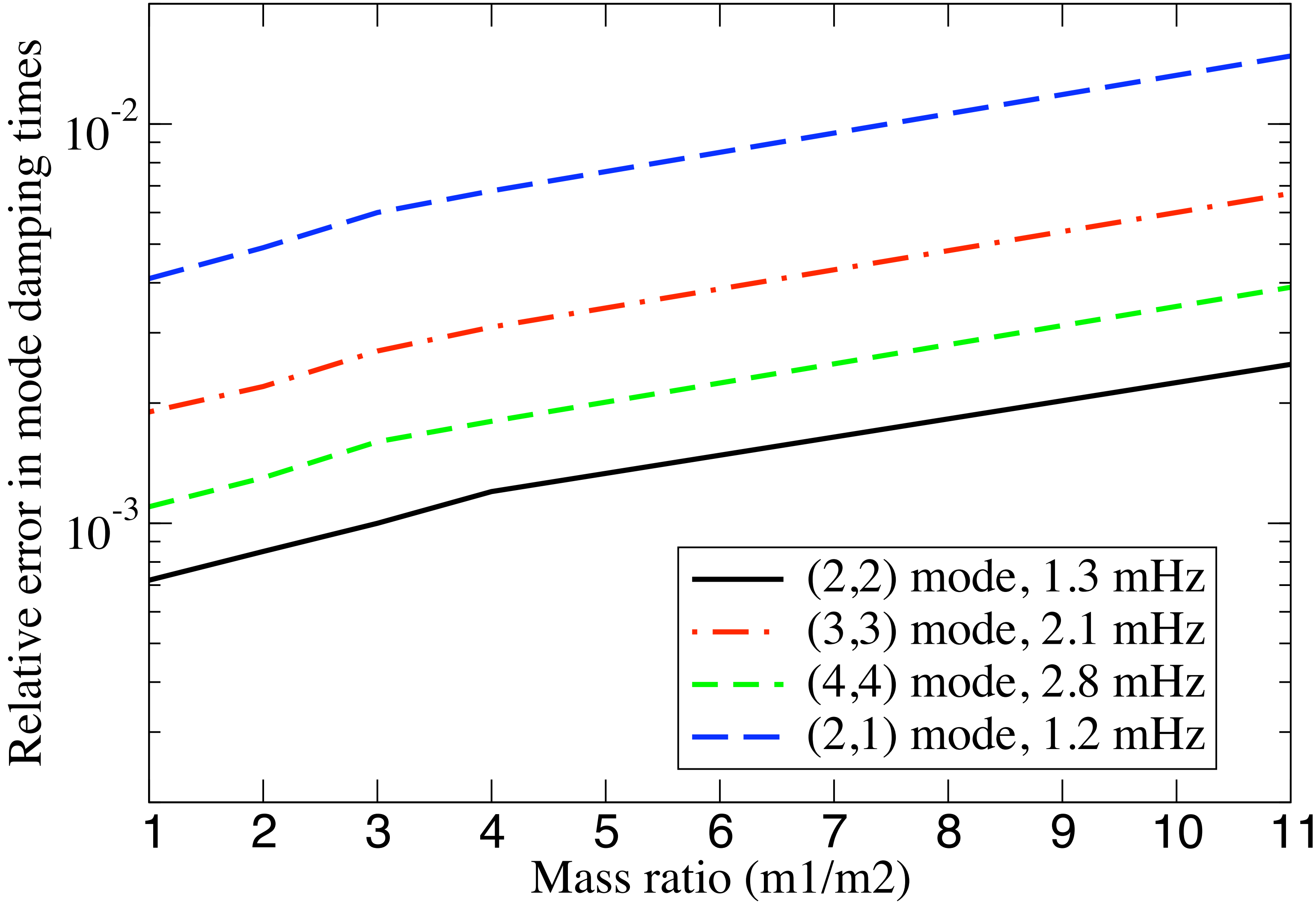
Kamaretsos, Hannam, Husa, Sathyaprakash, 2010

LISA measurement accuracies of mode frequencies



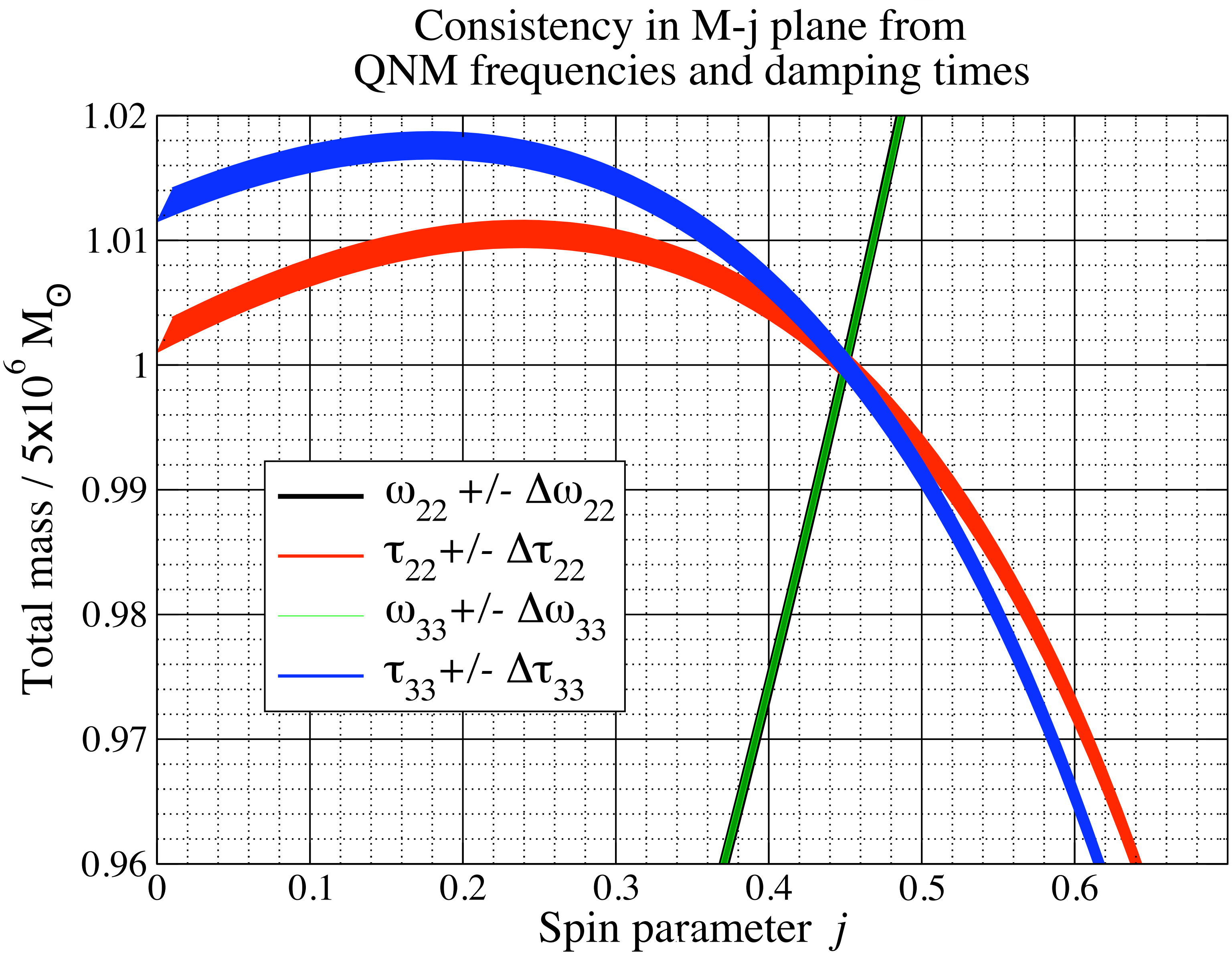
Kamaretsos, Hannam, Husa, Sathyaprakash, 2010

LISA measurement accuracies damping times

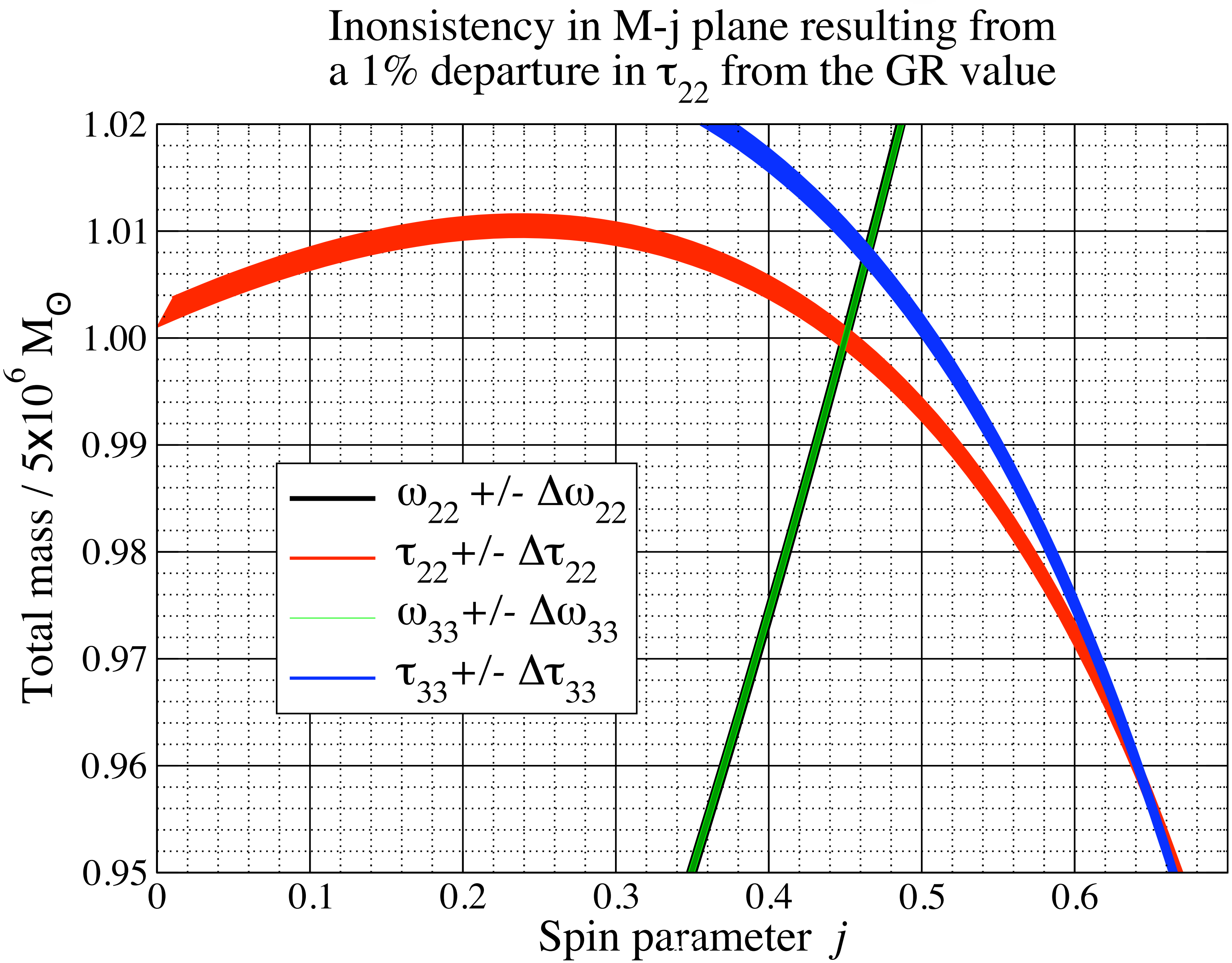


Kamaretsos, Hannam, Husa, Sathyaprakash, 2010

How can QNMs help test GR



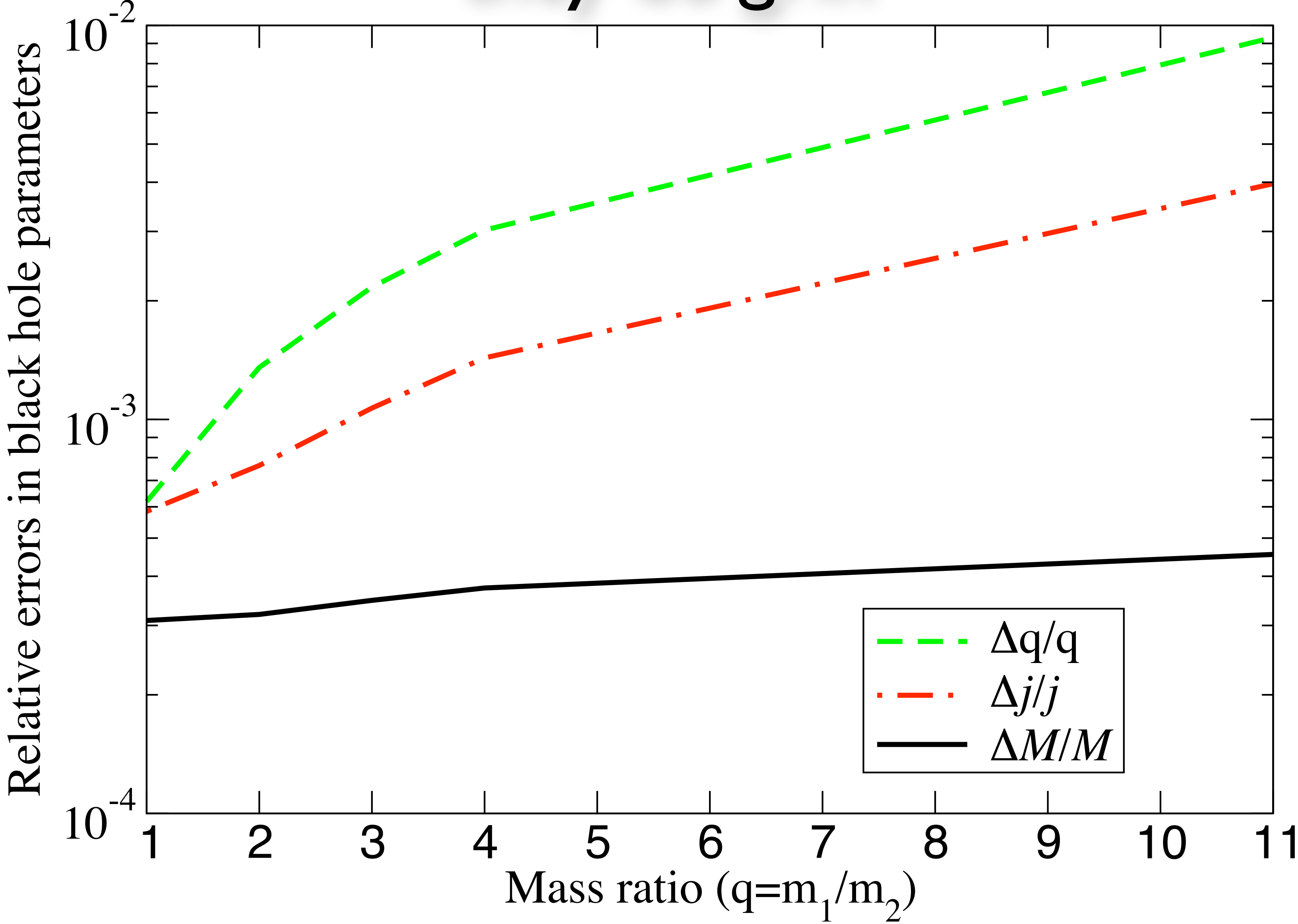
How can QNMs help test GR



Black holes ain't no hair but they do grin

- Black hole no hair theorems don't apply to deformed black holes
- From the ringdown signals it should in principle be possible to infer the nature of the perturber
- In the case of binary mergers it should be possible to measure the masses and spins of the component stars that resulted in the final black hole

Black holes ain't no hair but they do grin



Conclusions

- Gravitational-wave observations offer new tests of general relativity in the dissipative strongly non-linear regime
  - Advanced LIGO can already test tails of gravitational waves and the presence of the log-term in the PN expansion
  - Einstein Telescope will measure all known PN coefficients (except one at 2PN order) to a good accuracy
- Black hole quasi-normal modes will be very useful in testing GR
  - Consistency between different mode frequencies and damping times can be used to constrain GR
  - Ringdown modes can be used to measure component masses of progenitor binary and test predictions of numerical relativity