GEOMETRICAL FACTORS IN THE SEARCH FOR GRAVITATIONAL WAVES FROM BINARY INSPIRAL

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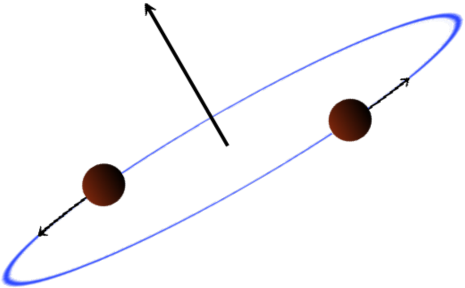
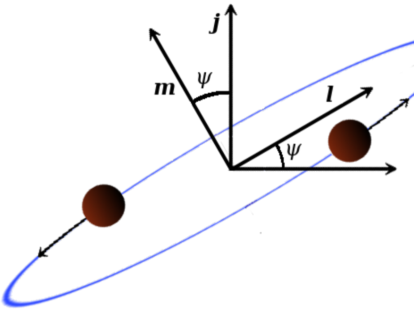
Overview

Binary Systems



- Two compact, massive objects (black holes, neutron stars) orbit one another.
- System radiates energy as gravitational waves, objects spiral inwards (inspiral).
- Orbital frequency increases as system loses energy.

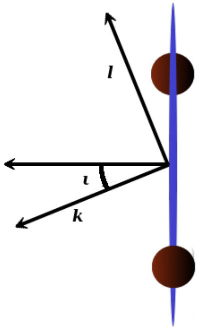
Orientation of Orbital Plane



Orientation defined by two angles:

1. Polarization angle ψ
2. Inclination angle ι

Orientation of Orbital Plane



Orientation defined by two angles:

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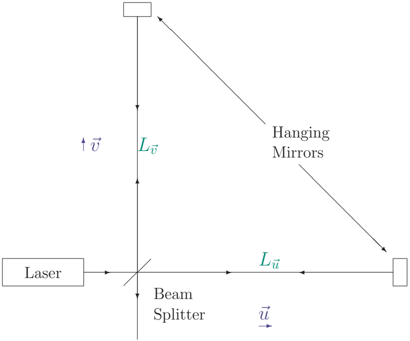
Propagating Gravitational Waves

- GW from single, distant source can be treated as a plane wave.
- Propagation direction defined by unit vector $\vec{k}$, pointing from source to observer.
- Wave has metric perturbation tensor

$$\boldsymbol{h} = h_+ \boldsymbol{e}_+ + h_\times \boldsymbol{e}_\times$$

- Matrices $\vec{e}_+$ and $\vec{e}_\times$ form a *polarization basis*.

Interferometer Response



Laser interferometer measures strain h, given by

$$h = \frac{L_{\vec{u}} - L_{\vec{v}}}{L_0} = h_{ab} d^{ab},$$

in terms of metric perturbation h and detector response tensor d.

Interferometer Response

Rewriting in terms of polarization basis,

$$h = (h_+ e_{+ab} + h_\times e_{\times ab}) d^{ab}$$

$$= h_+ F_+ + h_\times F_\times$$

With *antenna pattern factors*

$$F_+ \equiv F_+(\psi, \iota, \text{sky position}, \text{detector}) = e_{+ab} d^{ab}$$

$$F_\times \equiv F_\times(\psi, \iota, \text{sky position}, \text{detector}) = e_{\times ab} d^{ab}$$

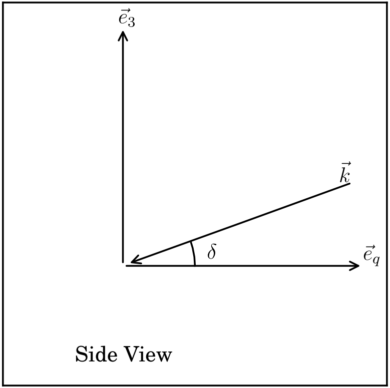
Detectors in two locations:

Livingston, Louisiana

Hanford, Washington

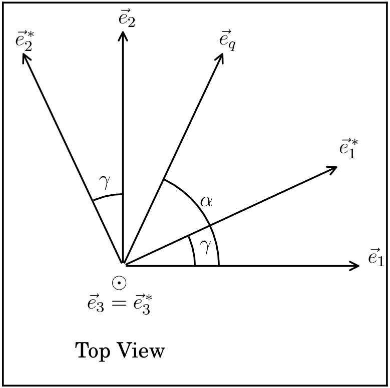
Equatorial Coordinates: Earth-Fixed and Inertial

- Earth-fixed, latitude λ, longitude β, correspond to $\{\vec{e_1}^*, \vec{e_2}^*, \vec{e_3}^*\}$ (Cartesian, rotates with Earth).
- Intertial declination δ, right ascention α, correspond to $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ (Stationary).



Equatorial Coordinates: Earth-Fixed and Inertial

- Greenwich sidereal time (GST, γ) measures angle between meridian at Greenwich, England ($\vec{e_1}^*$), and vernal equinox ($\vec{e}_1$).
- Local hour angle (LHA) measures angle from source meridian ($\vec{e}_q$) to observer meridian (Not shown in figure).

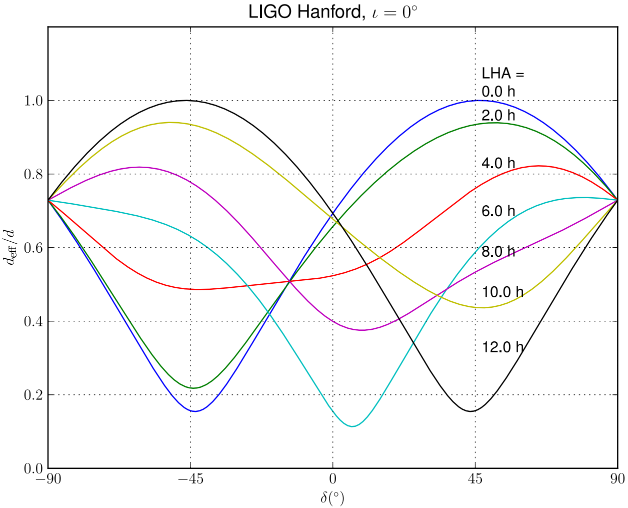


Threshold Distance

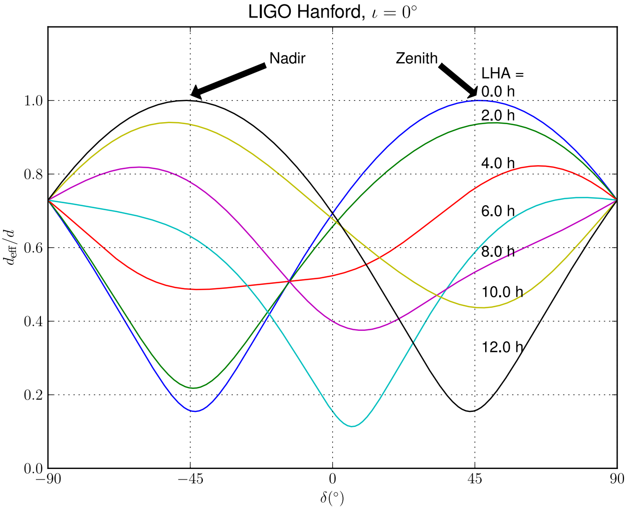
- For binary source at distance d, GW signal depends on sky position and orbital plane orientation.
- Source at distance d produces same signal as optimally located/oriented source at *effective distance* d_{eff}.
- $$\frac{d}{d_{\text{eff}}} = \sqrt{F_+^2 \frac{(1+\cos^2\iota)^2}{4} + F_\times^2 \cos^2\iota}$$

  gives threshold at which detector can see optimally located/oriented sources.

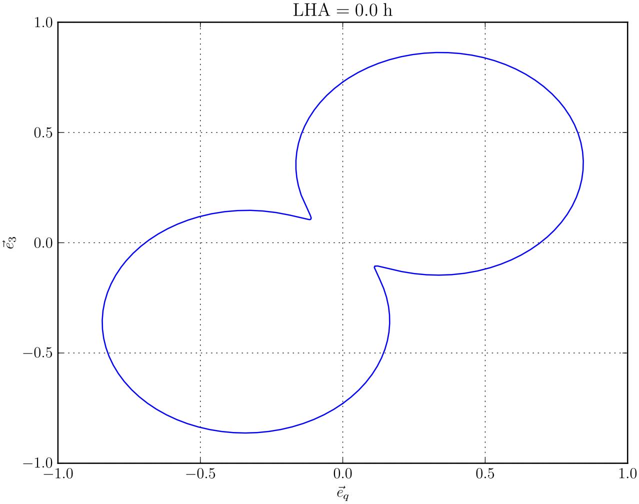
Threshold Distance vs. Source Declination



Threshold Distance vs. Source Declination



Cross-section of Surface d/d_{eff} for LIGO Hanford



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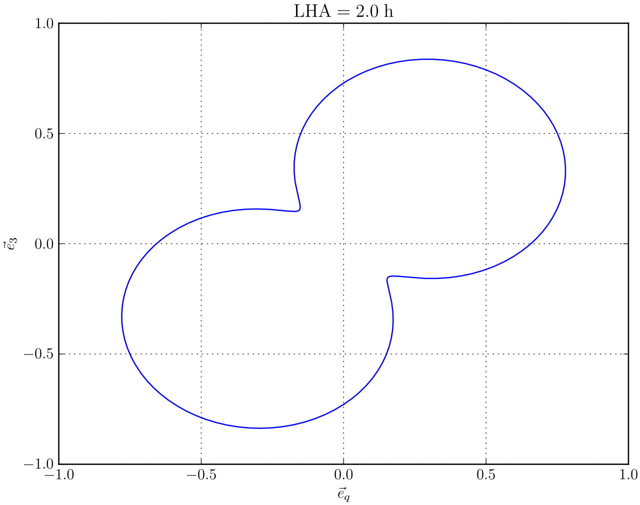

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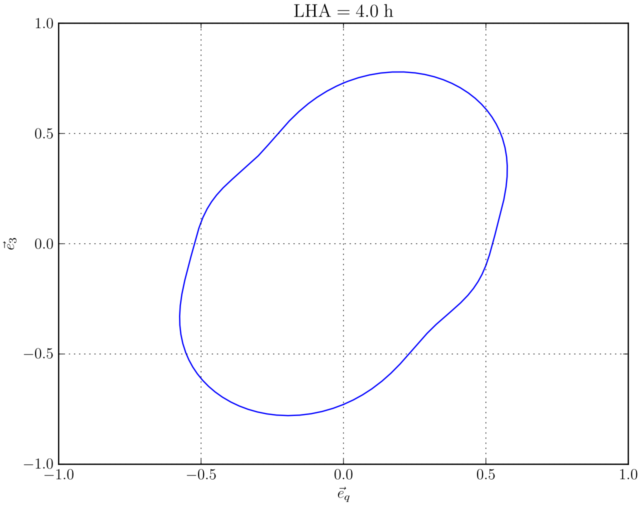

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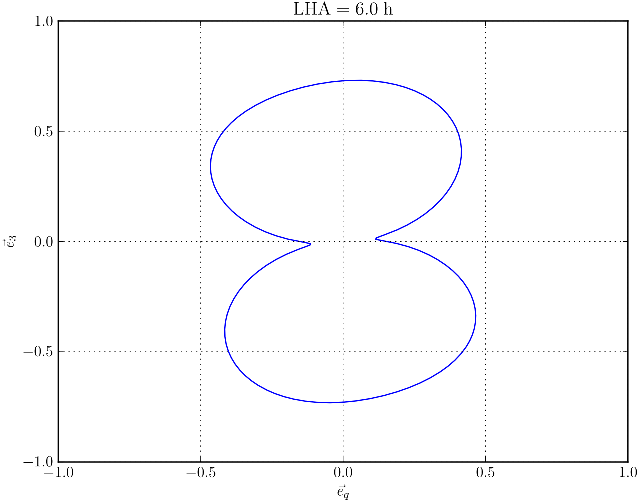

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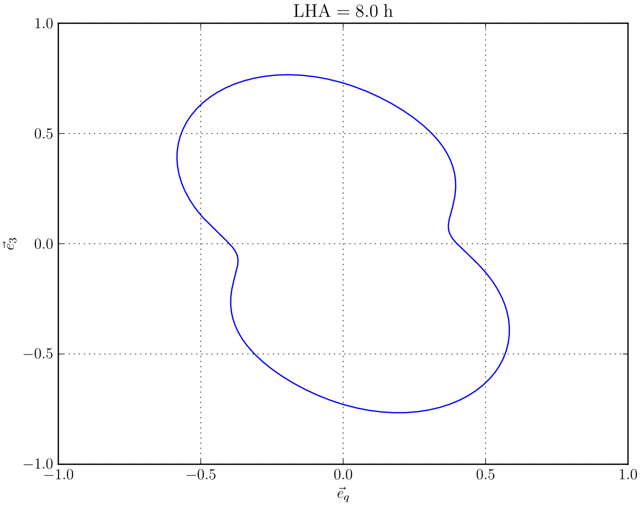

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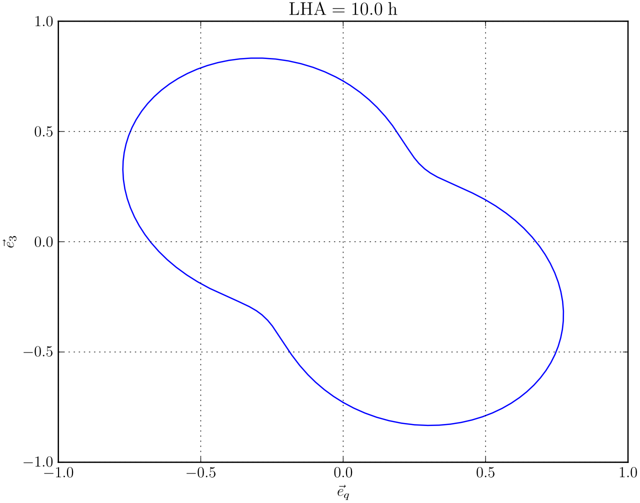

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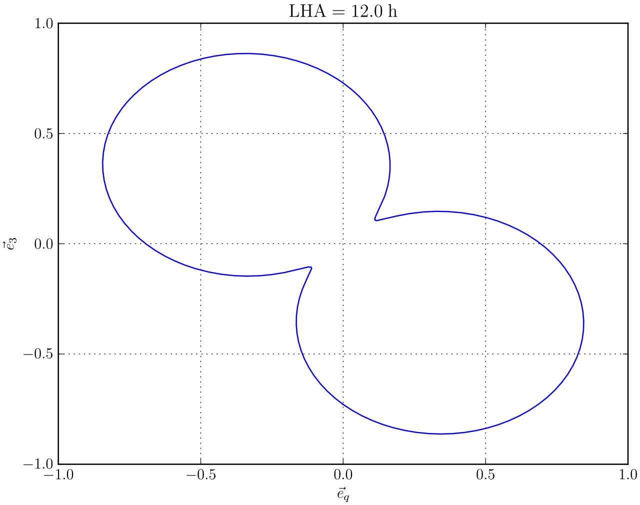

Summary and Outlook

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- Binary inspiral a GW source
- GW signal seen at detector depends on location, orientation of binary
- Signal from source at distance d same as optimal source at distance d_{eff}

Outlook

- Calculate and plot other parameterizations of d/d_{eff}
- 3D plots of surface d/d_{eff}

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Acknowledgments

Mentor: John T. Whelan

Duncan A. Brown

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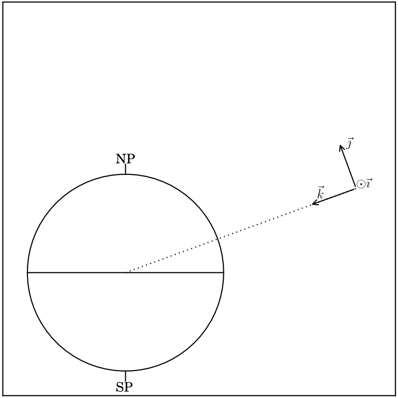
Polarization Bases

- $\vec{\ell}$ chosen such that $\vec{\ell} \perp \vec{k}$, and $\vec{m} = \vec{k} \times \vec{\ell}$.
- Polarization basis can be written in terms of $\vec{\ell}$, $\vec{m}$:

$$e_{+ab} = \ell_a \ell_b - m_a m_b$$

$$e_{\times ab} = \ell_a m_b - m_a \ell_b$$

- Reference basis of $\vec{i}$, $\vec{j}$, and $\vec{k}$, convenient for analysis.



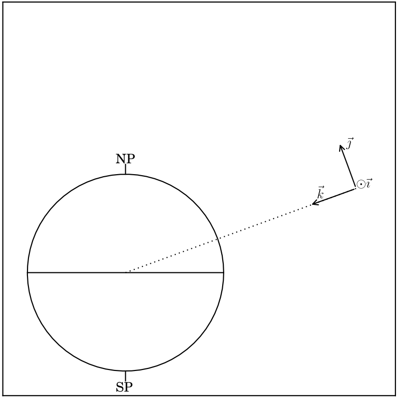
Polarization Bases

- In terms of $\vec{i}$ and $\vec{j}$, reference polarization basis written

$$\varepsilon_{+ab} = i_a i_b - j_a j_b$$

$$\varepsilon_{\times ab} = i_a j_b - i_a j_b$$

- Since $\vec{i}, \vec{j} \perp \vec{k}$, $\vec{i}, \vec{j}$ coplanar with $\vec{\ell}, \vec{m}$.

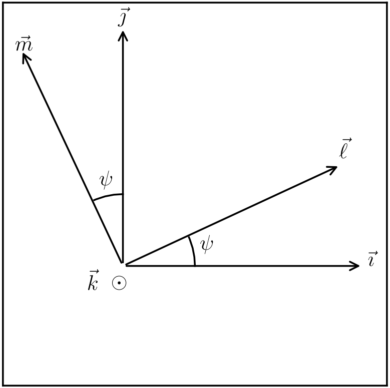


Polarization Bases

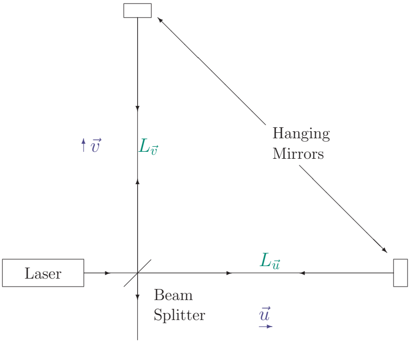
- Related to source basis by polarization angle ψ:

$$e_{+ab} = \varepsilon_{+ab} \cos 2\psi + \varepsilon_{\times ab} \sin 2\psi$$

$$e_{\times ab} = -\varepsilon_{+ab} \sin 2\psi + \varepsilon_{\times ab} \cos 2$$



Definition of Response Tensor



$$h = \frac{L_{\vec{u}} - L_{\vec{v}}}{L_0}$$

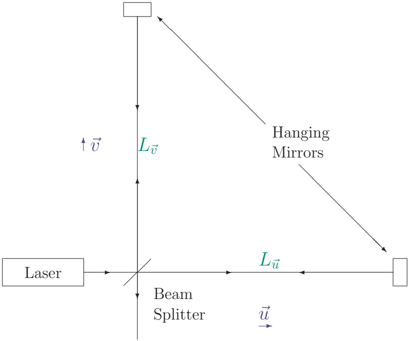
Arm lengths given by

$$L_{\vec{u}} = L_0(1 + \frac{1}{2}u^a h_{ab} u^b),$$

$$L_{\vec{v}} = L_0(1 + \frac{1}{2}v^a h_{ab} v^b),$$

where h_{ab} are components of perturbation tensor.

Definition of Response Tensor



Detector response tensor