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LIGO- T1000468-v2

8/5/2010

Q's of LASTI Violin Modes

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A monolithic quadruple suspension is in the LASTI vacuum chamber and the quality factors of the fused silica suspension fibers have been measured. We compare these measurements to expectations based on theory to help determine if there is any excess loss affecting the suspension fibers.

The theoretical expectation for violin mode Q's is based on two factors; the theoretical mechanical loss and the dissipation dilution factor. The theoretical mechanical loss of fused silica can be calculated from Penn *et al*, Physics Letters A **352** (2006) 3-6. This paper gives this formula for mechanical loss in silica

$$\phi = C_1 (S/V) + C_2 (f / (1 \text{ Hz}))^{C_3} + C_4 \phi_{thermo}$$

where  $\phi$  is the overall mechanical loss,  $S$  is the surface area of the fiber,  $V$  is the volume of the fiber,  $f$  is the frequency of the oscillation,  $\phi_{thermo}$  is the thermoelastic loss in the fiber, and  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  are coefficients that come from fitting mechanical loss data. The Penn paper give values of  $C_1 = 12.1 \cdot 10^{-12}$  m,  $C_2 = 11.8 \cdot 10^{-12}$ ,  $C_3 = 0.77$ , and  $C_4 = 0.61$  for Suprasil 2 fused silica. The distinctions between the different types of fused silica are often lost when the silica is drawn into fibers, so the exact type of silica is likely not important. The only other type of silica where these parameters are available, Suprasil 312, has  $C_1$  and  $C_2$  within a factor of 2 of these numbers,  $C_3$  identical, and no value available for  $C_4$ .

Using the Suprasil 2 numbers, the geometry factors of the LASTI fibers allow for an estimate of  $\phi$ .  $S$  and  $V$  depend on where the bending in the violin mode takes place. FEA models (see Cumming *et al*, Classical and Quantum Gravity **26** (2009) 215012 and Mark Barton) indicate that this bending is in the 800  $\mu\text{m}$  diameter section of the fibers.  $S/V = 4/d$  for fibers, and thus 5000  $1/\mu\text{m}$ . The frequency of the lowest violin mode is about 500 Hz, well above the 3 Hz thermoelastic peak for an 800  $\mu\text{m}$  silica fiber, so thermoelastic damping is ignored from here on. These parameters all together predict

$$\phi(f_0) = 7.33X 10^{-8}$$

$$\phi(f_1) = 6.83 X 10^{-8}$$

for the silica fibers at the first two violin modes.

Dissipation dilution for violin modes has been calculated by Gonzalez and Saulson in Journal of the Acoustical Society of America **96** (1994) 207 as

$$r_n = 2 \delta/l ( 1 + (n \pi)^2 \delta/(2 l) )$$

where  $r_n$  is the dissipation dilution factor,  $\delta$  is  $\sqrt{Y I/T}$ , where  $Y$  is the fiber's Young's modulus,  $I$  is its moment of area, and  $T$  is the tension,  $n$  is the mode number, and  $l$  is the fiber length. Using  $7.27 \cdot 10^9$  Pa for  $Y$ ,  $1/4 \pi (d/2)^4$  for  $I$ , 98 N for  $T$ , and 60 cm for  $l$ , the dissipation dilution factor for the first two violin modes are

$$r_0 = 1 / 77.2$$

$$r_1 = 1 / 74.8$$



The expect violin mode Qs can then be expressed as

$$Q_n = 1 / (r_n \phi(f_n))$$

Using the above values for  $r$  and  $\phi$ , the Q's become

$$Q_0 = 1.3 \cdot 10^9$$

$$Q_1 = 1.0 \cdot 10^9$$

This theoretical value can be compared to the experimentally value on the LASTI suspension measured to be

$$Q_0 = 5.9 \cdot 10^8$$

$$Q_0 = 6.1 \cdot 10^8$$

$$Q_1 = 4.6 \cdot 10^8$$

as shown in the LASTI ILOG July 31, 2010.

The use of a single thickness for the fiber in both the silica mechanical loss (where it enters through  $S/V$ ) and the dissipation dilution factor (where it enters through  $I$ ) is simplistic, the actual fibers have a very large diameter stock region of  $d = 3000 \mu\text{m}$ , a thick region of  $d = 800 \mu\text{m}$ , and a long thin middle region of  $d = 400 \mu\text{m}$ . A true estimate of the mechanical loss and dissipation dilution, and thus the violin mode Q's, requires finite element modeling, which is in progress by Alan Cummings et al.