LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

*LIGO Laboratory / LIGO Scientific Collaboration*

LIGO- E080170-00-Z *Advanced LIGO* 04/16/08

# Effect of BS wedge on mode-matching in Advanced LIGO

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# 1 Introduction

## 1.1 Purpose

The purpose of this document is to describe the effect of beam splitter (BS) wedge on the signal loss. Here we assume that the signal loss is equivalent to the mode mismatch between the arm and the recycling cavity.

## 1.2 Scope

Beam splitter (BS) in Advanced LIGO will have a wedge to separate the ghost beams from the main IFO beam. This BS wedge acts as a prism introducing unsymmetrical magnification as the beam passes through the BS from HR side to the AR side as compared to the beam passing through from AR side to the HR side. In this note, we describe the effect of beam expanding/reducing due to beam splitter wedge.

## 1.3 Definitions

## 1.4 Acronyms

BS: Beam Splitter

ROC: Radius of curvature

### 1.4.1 LIGO Documents

### 1.4.2 Non-LIGO Documents

# 2 General description

As part of the e-LIGO IO upgrade, FI has to be put between MMT1 and MMT2. The reflected beam from the PRM travels back to the FI and is rejected by the thin film polarizer (TFP) and propagates towards the ISCT1 table. The rejected beam is used to measure the power through refl diode and is used for the WFS3 and WFS4. WFS3 is used to detect the RM degree of freedom by using the non-resonant (NR) sideband while WFS4 is used to detect a linear combination of common ITM and ETM degrees of freedom.

However, as the reflected beam is rejected by the TFP, it never sees MMT1, Due to this, the beam size as it reaches the periscope is out of Rayleigh range and is expending. In initial LIGO, the FI was in between SM1 and MMT1 and therefore would see MMT1 ROC. The beam was 'collimated' as it reaches the ISCT1 table. This situation demanded re-optimizing the ISCT1 table.

# 3 Geometry and ABCD Matrix of BS

The geometry of the BS is shown in Fig. 1 as described by Hiro Yamamoto in his presentation.

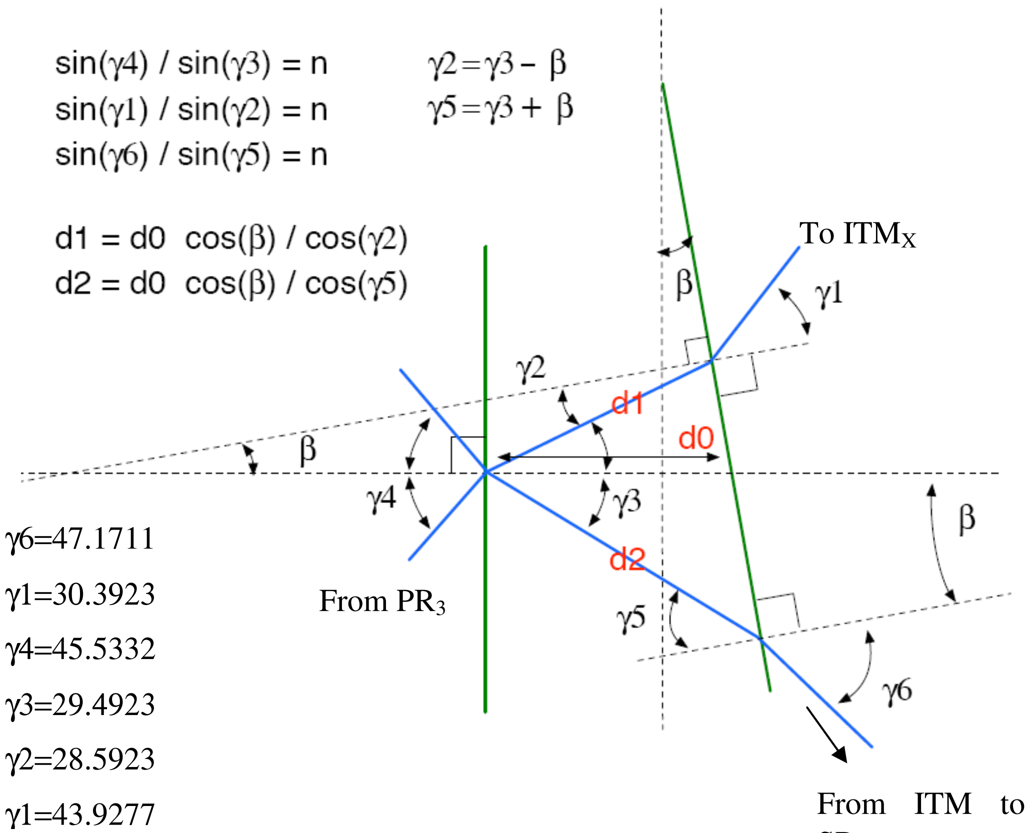


Fig. 1: Geometry of the BS wedge angle.

Based upon this, we can calculate the ABCD matrices of BS when the beam passes through different directions.

$$\text{ABCD matrix from } PR_3 \text{ to ITMx} = \begin{bmatrix} 1.0192 & 0.0378 \\ 0 & 0.9811 \end{bmatrix}$$

$$\text{ABCD matrix from ITMx to } PR_3 = \begin{bmatrix} 0.9811 & 0.0378 \\ 0 & 1.0192 \end{bmatrix}$$

ABCD matrix from ITMx to $SR_3$ = $\begin{bmatrix} 0.9608 & 0.0750 \\ 0 & 1.0408 \end{bmatrix}$

ABCD matrix from $SR_3$ to ITMx = $\begin{bmatrix} 1.0408 & 0.0750 \\ 0 & 0.9608 \end{bmatrix}$

ABCD matrix from ITMy to $SR_3$ = $\begin{bmatrix} 1.0211 & 0.0370 \\ 0 & 0.9793 \end{bmatrix}$

The above matrices are for the case of 0.9 degree wedge angle. Note that, a negative wedge angle can be used to model the situation where the wedge angle is on the RC side instead of the TM side.

## 3.1 Wedge Angle Effect in PRC

For the case of PRC, we model the following path:

$PR_3 \rightarrow BS \rightarrow ITMx \rightarrow BS \rightarrow PR_3 \rightarrow PR_2$

Note that we are using a simple modal model where the RC mode is determined by propagating the beam coming out from the ITMx to the PRM and then back to ITMx. Then equating the two complex q values we determine the Eigen mode of the recycling cavity. This method completely neglects the effect of coupled cavities.

Note that the following parameters are being used.

R_ITM = 1971 ; R_ETM = 2191; L_C = 3994.75; n=1.44963;

t_ITM_eff = 0.330*n+0.005; % ITM = 0.2 and CP = 0.13 according to COC documents. Mike has ITM=0.1

t_ITM_m = 0.330/n + 0.005;

dIBSx = 4.8218; t_BS = 0.0729; dB3 = 20.0091; % Critical distances and ROCs

R_3 = 34.75; d32 = 16.5226; R_2 = -2.346; d21 = 15.7604; R_1 = 8.2196;

(All numbers are in meter)

The results are presented in the following figures.

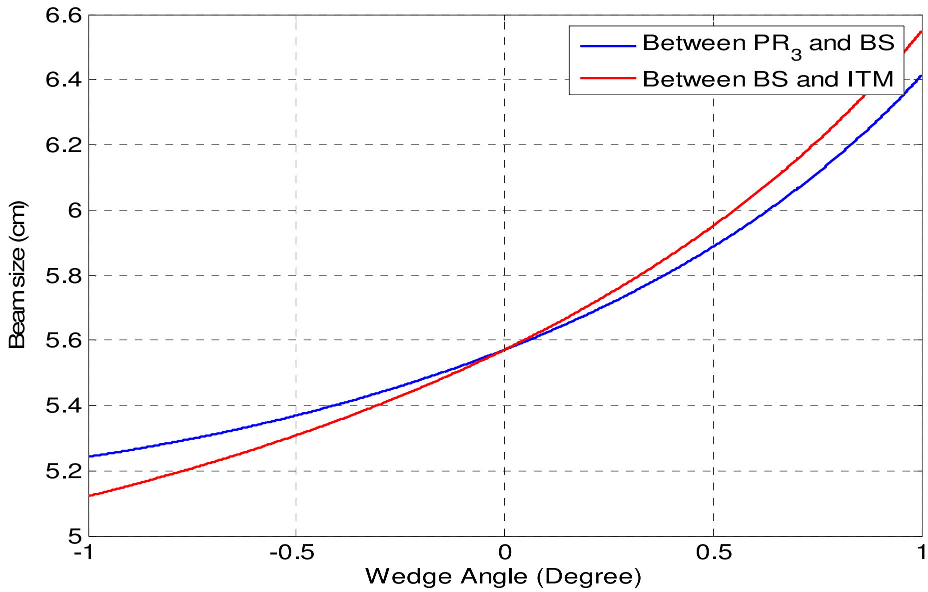


Fig. 2: Beam size as a function of wedge angle. Here the beam size around BS increases for the positive wedge angle at the beam splitter as predicted by Hiro's code.

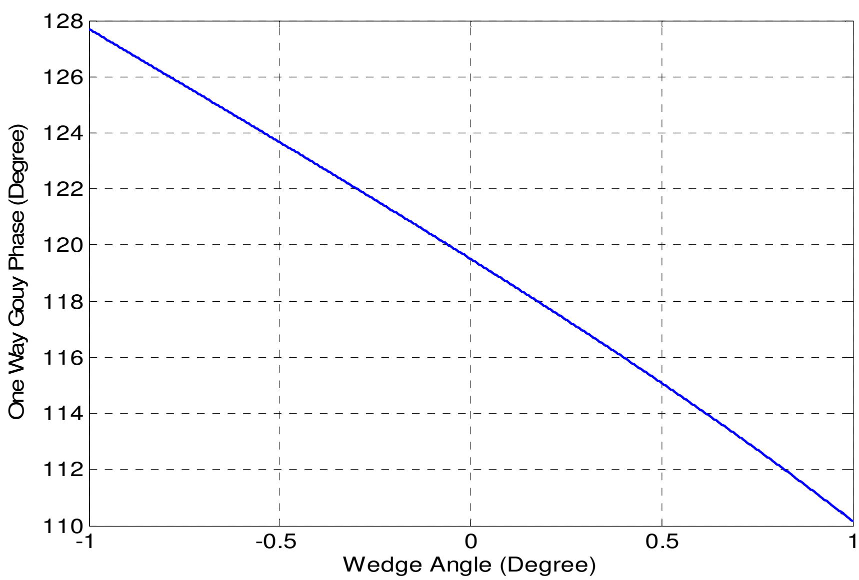


Fig. 3: Gouy phase as a function of wedge angle. The change of Gouy phase is almost linear. However, with higher positive wedge angle, we start to see that the change in Gouy phase starts to increase.

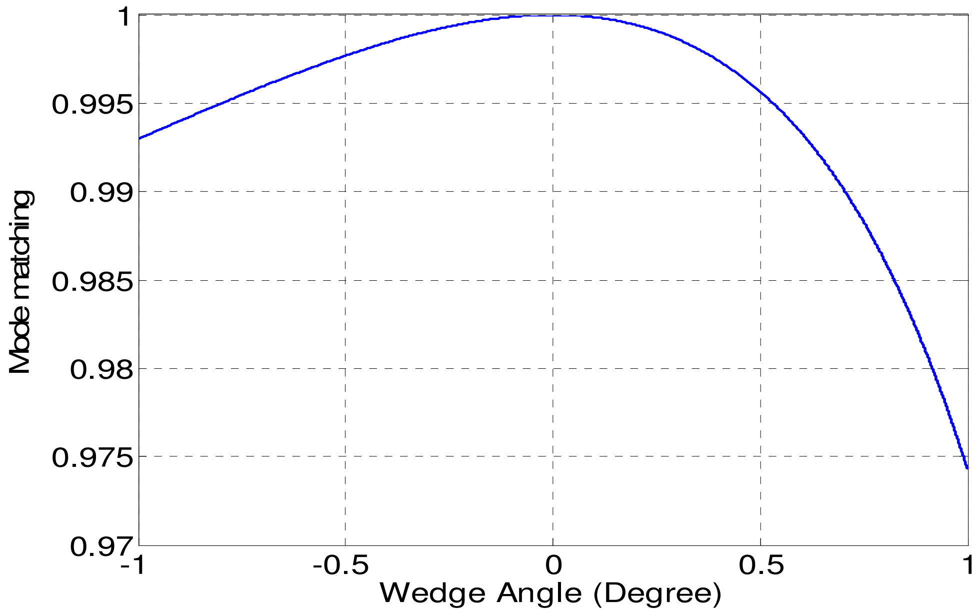


Fig. 4: Mode matching into the nominal mode of the interferometer. Note that here the mode matching calculations assumes that the y-axis mode matching is 100% and the mode matching is power coupling into the nominal mode of the arm cavity.

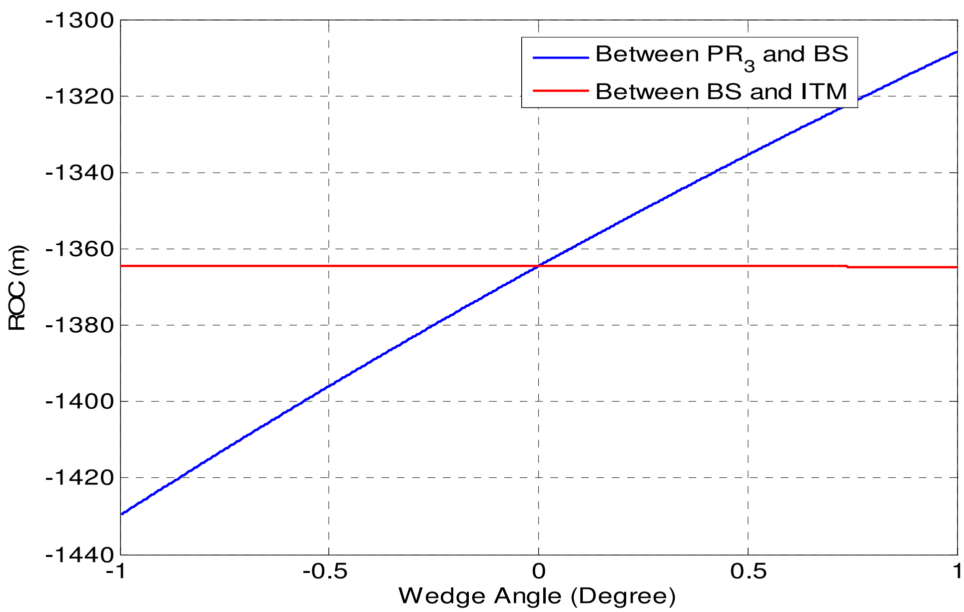


Fig. 5: ROC of the beam around BS.

Note that the beam sizes and the Rocs are measured right at the HR and AR side of the BS. Also the beam sizes and the ROCs are same in both directions. In Hiro's terminology, the in-fit and out-fit values are same. This is I guess due to the fact the coupling of the cavities has been neglected.

## 3.2 Wedge Angle Effect in SRC

For the case of SRC, we model the following path:

$SR_3 \rightarrow BS \rightarrow ITMx \rightarrow BS \rightarrow SR_3 \rightarrow SR_2$

Here the beam coming from SR3, first hits the AR side of the BS and then transmits to the glass side of HR coating, gets reflected, and then propagates towards the ITMx.

```
% LPR based on Mike Smith Document LIGO-T080007-00-D from 01/16/08

lam=1.064e-6;

R_ITM = 1971;
R_ETM = 2191;
L_C = 3994.75;
n=1.44963;
t_ITM_eff = 0.330*n+0.005;  % ITM = 0.2 and CP = 0.13 according to COC documents. Mike has ITM=0.1
t_ITM_m = 0.330/n + 0.005;
%Y-Arm
dIBSy = 4.8764;
t_BSy= 0.0742;
%X-Arm
dIBSx = 4.8218;
t_BSx1 = 0.0729;
t_BSx2 = 0.0742;

dB3 = 19.4350;
R_3 = 34.0;

d32 = 15.6804;
R_2 = -3.261;
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d21 = 15.4214;

R_1 = -15.373;

The results are presented in the following figures.

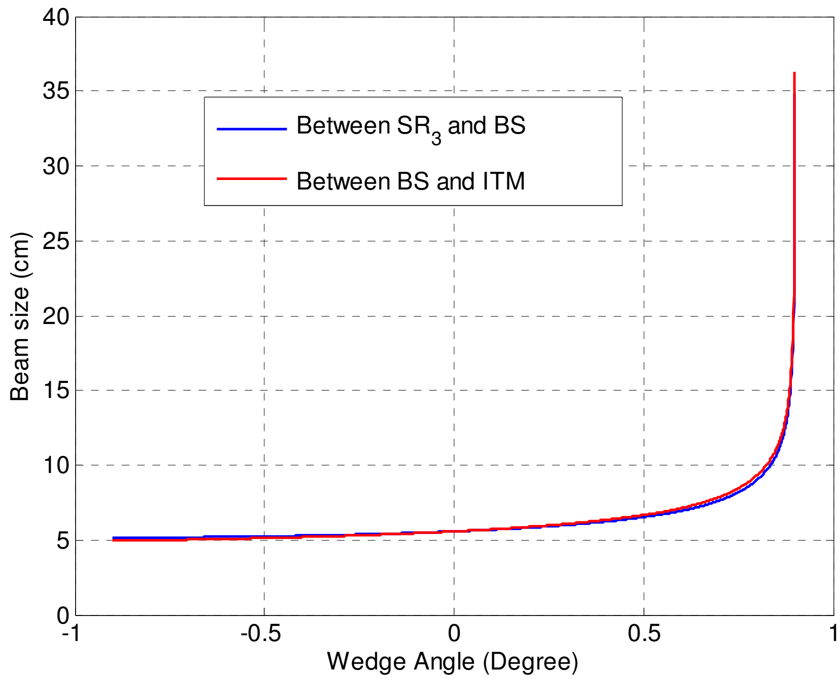


Fig. 6a: Beam size as a function of wedge angle. Here the beam size around BS increases for the positive wedge angle at the beam splitter as predicted by Hiro's code.

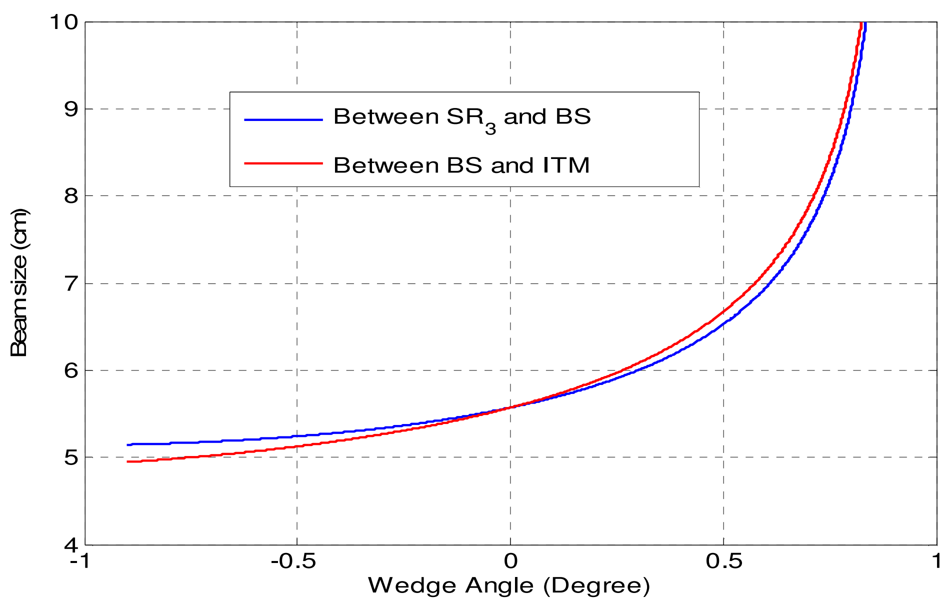


Fig. 6b: Zoomed-in plot of Fig. 6a.

The above plots show that the beam size around 0.8 degree wedge is quite dramatic. It is interesting to note that the Gouy phase around 0.9 degree is 0 radian and the cavity becomes unstable.

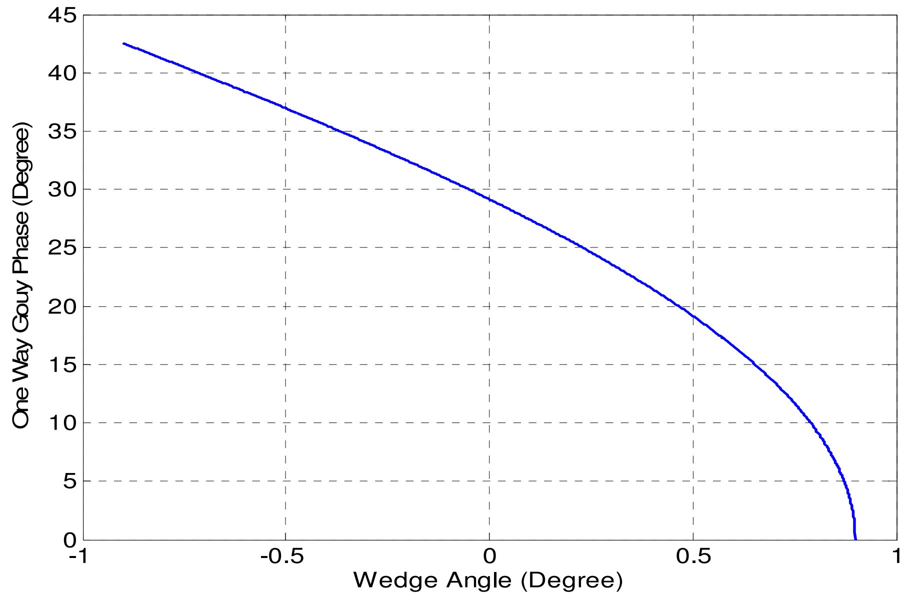


Fig. 7: Gouy phase as a function of wedge angle. The change of Gouy phase is linear in the negative wedge angle region but changes quite fast for wedge angles greater than 0.5 degree.

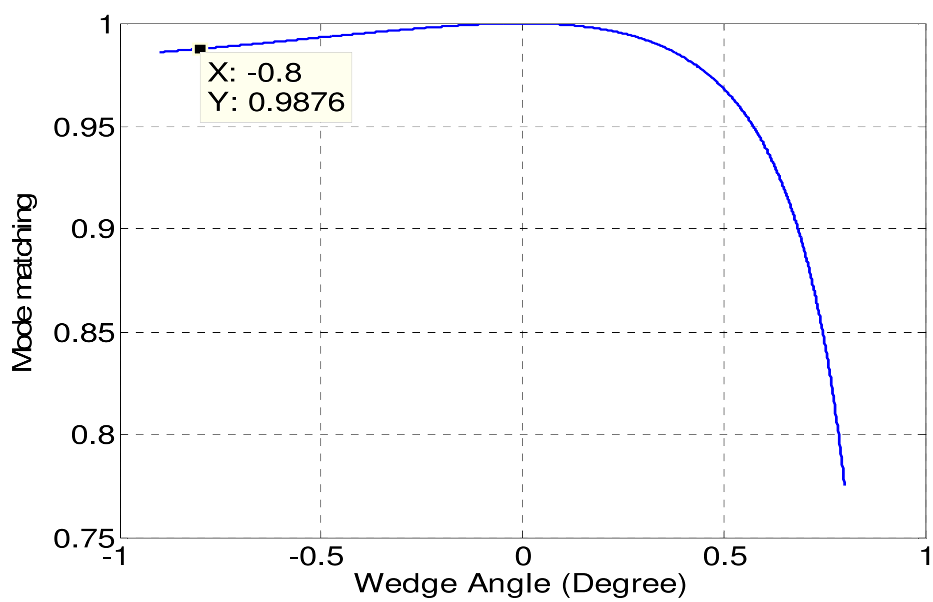


Fig. 8: Mode matching into the nominal mode of the interferometer. Note that here the mode matching calculations assumes that the y-axis mode matching is 100% and the mode matching is power coupling into the nominal mode of the arm cavity.

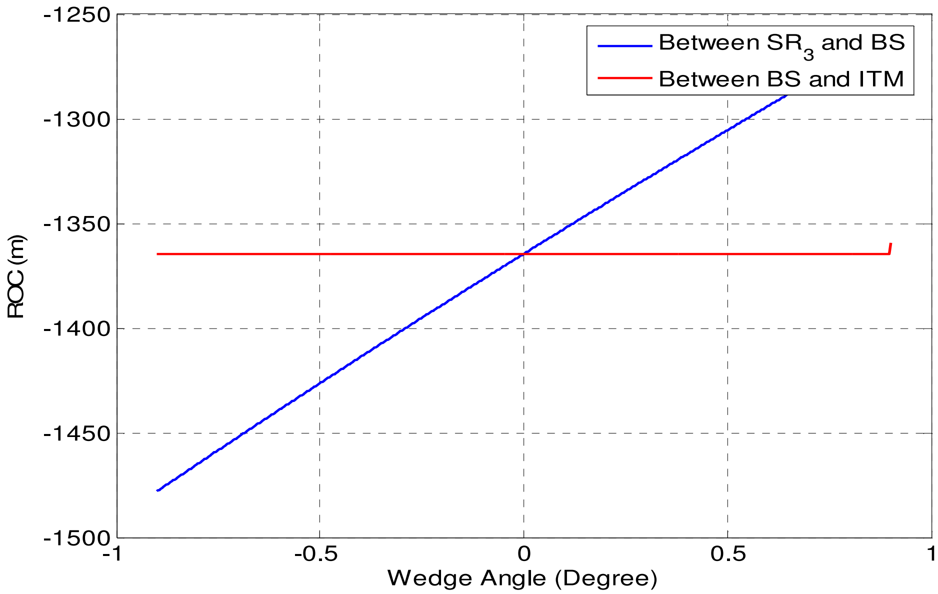


Fig. 9: ROC of the beam around BS.

**Gentler Change for Gouy phase of 0.6 radian in SRC**

We can change the behavior a lot by selecting a different Gouy phase. If we change the ROC of SR2 to -3.825, the Gouy phase of the cavity becomes 0.6 radian or about 30 degree. In this case, the beam size change becomes less steep. Similarly the mode matching drop also improves.

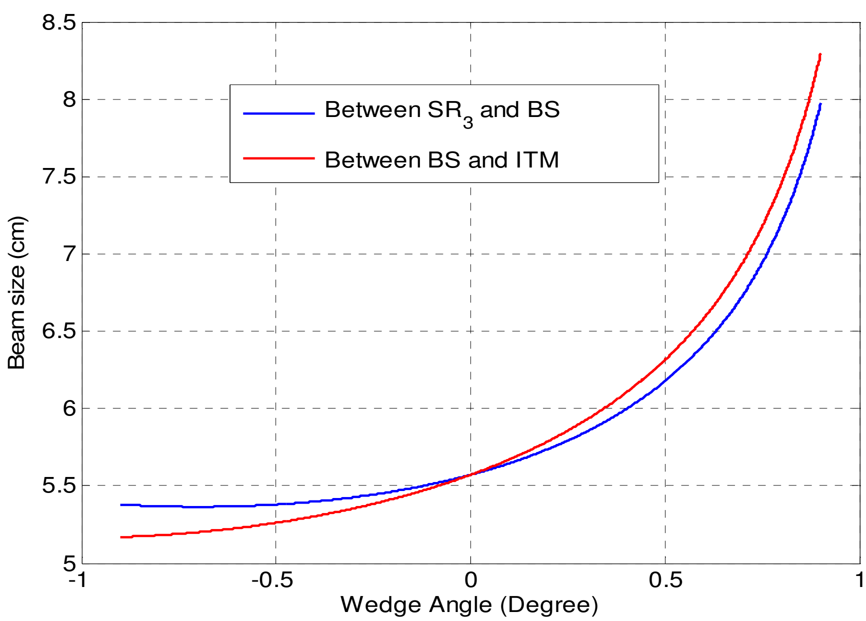


Fig. 10: Beam size change for SRC.

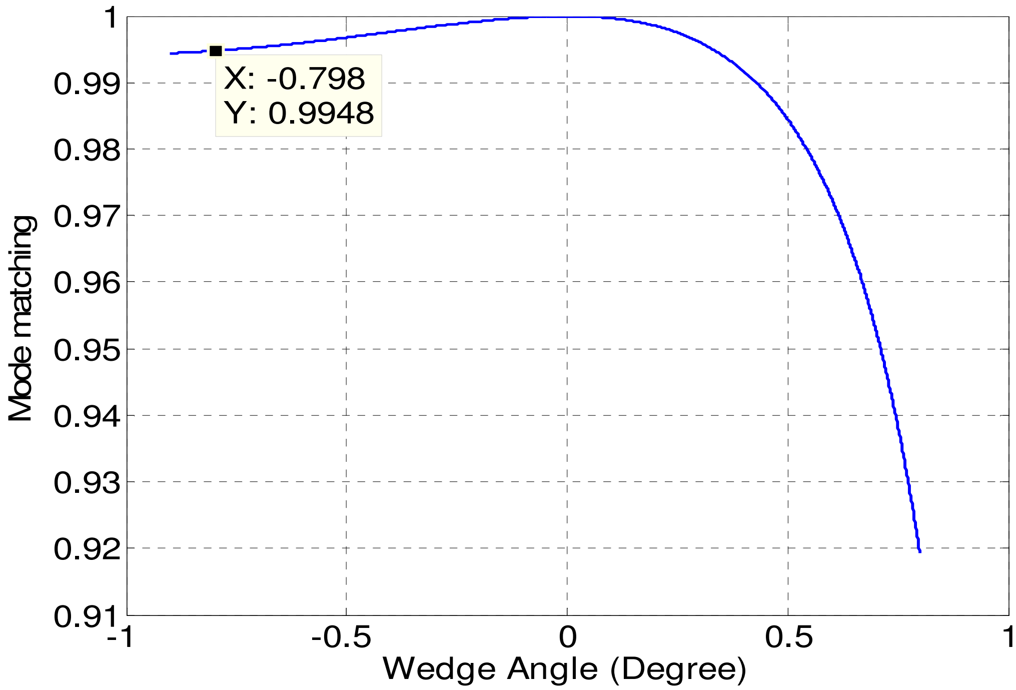


Fig. 11: Mode matching from arm cavity to SRC.

## 4 Summary

As a summary, we can say that the simple modal model also predicts increase in beam size due to BS wedge angle for the current geometry. The situation can be reversed by selecting a negative wedge. Another important point is the sensitivity of the beam size change due to Gouy phase change. We can change the Gouy phase of the RC for BS wedge angle immunity.

The comparison qualitatively shows the same behavior as predicted by Hiro's simulations. However, the SRC shows a little more loss and a little larger beam diameter at the BS. This could be due to the fact that we are neglecting the effect of arm cavity on the RC geometry. As discussed in the meeting, the transmitted beam from the ITM also plays an important part and its omission in the modal model may explain the difference. Another interesting thing to observe is that the difference is dramatic when the cavity becomes unstable. This may be due to the fact that the simple modal model does not represent unstable cavity very well.