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Calculation of cavity lengths and RF frequencies for RSE at the 40m

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Abstract

Signal recycling and dual recycling optical configurations require control of all relevant length degrees of freedom, including the position of the signal mirror. This can be accomplished by adding more RF sidebands to standard LIGO-I configuration, for use in Pound-Drever locking of the signal extraction cavity. These, in turn, put constraints on the lengths of the various optical cavities, and the RF frequencies required to control them. Using a simple frontal modulation scheme developed by Mason and Willems, we calculate the RF sideband frequencies and required cavity lengths, for LIGO and for the 40m upgrade.

1 Introduction

A simple realization of RSE or dual recycling, with minimal modification to the LIGO-I optical configuration, is shown in Fig. 1. We have two nearly-identical Fabry-Perot (FP) arms, a power recycling cavity (PRC), and, with the addition of one more mirror at the dark port, a signal recycling cavity (SRC).
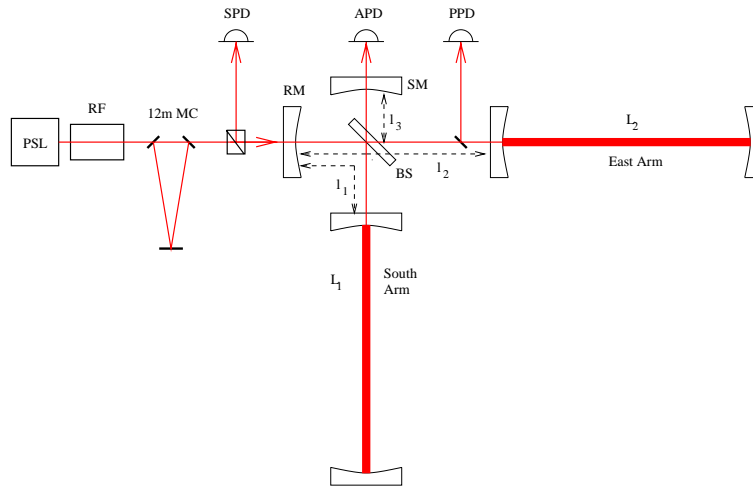

Figure 1: Configuration for a power-recycled Michaelson IFO with Fabry-Perot arms, with a signal recycling mirror (SM) for resonant sideband extraction (RSE).

1.1 Coupled cavities

For the purposes of analyzing the PRC or SRC, we can imagine that the beam splitter (BS) plus arms can be folded together, thus represented by a single FP arm (this simplified representation of course requires modification when considering the Schnupp asymmetry). The addition of the PRC or the SRC can then be analyzed as a coupled cavity, as in Fig. 2. Light exiting from the orthogonal direction (*e.g.*, when analyzing the PRC plus arms, light exiting the dark port) can be thought of as a loss.
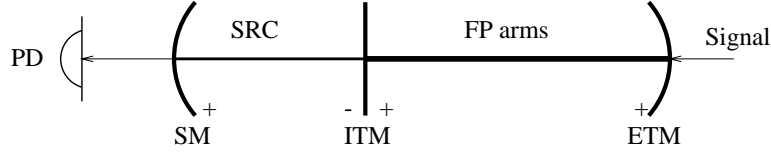

Figure 2: Simple model of signal-recycling IFO as a coupled cavity. Note the sign conventions for the reflected light for each mirror.

Note the sign convention for the reflected light in Fig. 2.

1.2 RSE

The signal recycling mirror (SM) at the dark port sees no carrier light (if the contrast is perfect), unless there is a gravity wave signal. We can think of the latter as signal sidebands on the carrier, at acoustic frequencies. The signal sidebands in the arms exit to the dark port through the ITM plus SM, seen as a compound mirror, with reflectivity

$$r_{cm}(\phi) = r_{ITM} - \frac{t_{ITM}^2 r_{SM} e^{-i\phi}}{1 - r_{ITM} r_{SM} e^{-i\phi}},$$

where $\phi = 2\pi\nu$ is the phase advance of the carrier plus GW signal, round trip in the SRC.

If the mirrors are held at resonance ($\phi = 0$), the transmission through the output cavity can be smaller than the ITM transmission by itself, so that the signal sidebands leak out the arms faster (resonant sideband extraction, RSE [1]).

Conversely, if the mirrors are held at anti-resonance, the transmission through the output cavity can be larger than the ITM transmission by itself, so that the signal sidebands are stored in the arms longer (signal recycling).

Thus, one can independently change the finesse (and therefore the storage time and IFO bandwidth) of the arms, for the gravity wave signal only, while leaving the finesse for the unmodulated carrier unchanged.

These considerations are illustrated in Fig. 3 and 4.

1.3 Controlling the cavity lengths

The carrier must be kept at resonance in the arms and the PRC, by controlling the longitudinal positions of the mirrors with the Pound-Drever locking scheme: an (RF) sideband is placed on the light, so that the carrier is resonant in the FP arms and the sideband is not resonant; beats between the reflected carrier and sidebands, demodulated at the RF frequencies, are sensitive to the phase shift of the carrier in the arms and thus to the longitudinal positions of the mirrors.
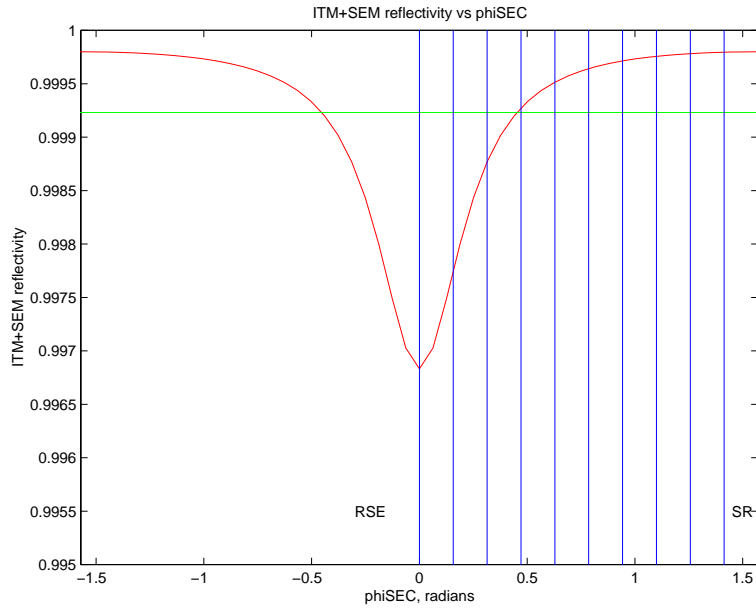
Figure 3: Amplitude reflectivity of the compound mirror formed by the ITM plus SM, for different detunings. The horizontal line is the ITM reflectivity. The vertical lines indicate detunings $\phi_{cs} = 2\pi\nu_{cs}$ in radians which correspond to the shot noise curves in Fig. 4.
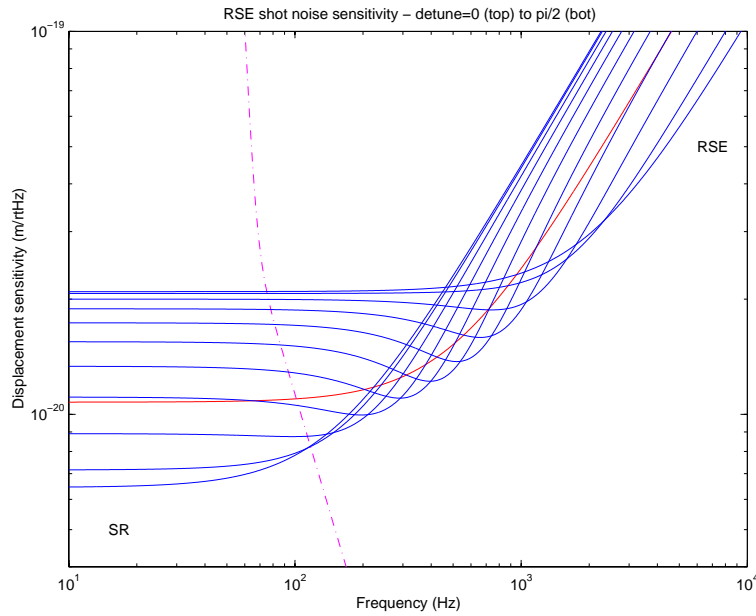
Figure 4: Shot noise displacement sensitivity versus GW frequency, for different SRC tunings. The middle, red curve, with no dip, corresponds to the absence of a SM; the other blue curves are in the presence of a SM, with tunes corresponding to the vertical lines in Fig. 3. They range from the narrow-band, $\phi_s = \pi/2$, "signal recycling" limit (bottom-most curve on the left), to the widest-band, $\phi_s = 0$, "resonant sideband extraction" limit.

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In the LIGO-I configuration, a single RF sideband is placed on the beam before it is injected into the IFO (frontal modulation). The carrier is resonant in both the arms and the PRC; the RF sideband is resonant only in the PRC. Longitudinal degrees of freedom are controlled as follows:

- The average arm length $L_+ \equiv (L_1 + L_2)/2$, and the average PRC length $l_+ \equiv (l_+ + l_-)/2$, are controlled using signals from the in-phase beats of carrier and RF sideband, from the bright port (symmetric photodiode, SPD) and the PRC pickoff port (pickoff photodiode, PPD).

- The PRC arm length difference $l_- \equiv l_1 - l_2$ is controlled using signals from the quad-phase beats of carrier and RF sideband, from the bright port (symmetric photodiode, SPD) *or* PRC pickoff (PPD).

- The arm length difference $L_- \equiv (L_1 - L_2)$, *i.e.*, the gravity wave signal, is controlled using the signal from the quad-phase beats of carrier and RF sideband, from the dark port (asymmetric photodiode, APD).

This exhausts the information that can be extracted using one pair of phase-modulated RF sidebands.

1.4 Controlling the SRC

The addition of a signal recycling mirror (SM) adds a new degree of freedom to be controlled, the Signal Recycling Cavity (SRC) length l_s. The control of the SM requires additional RF sideband(s), at frequencies different from the one used for controlling the LIGO-I degrees of freedom.

Call the "LIGO-I" sidebands SB1, and the additional ones required for SRC control (and perhaps more), SB2.

1.5 Mach-Zender IFO for sideband application

These additional sidebands can also be placed on the beam before injection into the IFO (frontal modulation). It may not, however, be desirable to add these in series, *i.e.*, with the use of two phase-modulating Pockels cells placed one after the other, since the second will place sidebands on not only the carrier but also on the first set of sidebands. These secondary sidebands may confuse the control plant.

One way to eliminate this problem is to split the beam in two, to apply the two sets of sidebands in two separate paths, and then recombine the beams. Such a Mach-Zender interferometer is shown schematically in Fig. 5.

Ideally, the sidebands should be placed on the carrier prior to introduction into the mode cleaner, so that spatial and frequency noise introduced by the phase-modulating Pockels cells and associated optical elements are filtered out. However, this would require a mode cleaner that is at least twice as long as the PRC, or 4.58 meters. This may not be practical at the 40m. Detailed studies of the noise introduced by the sideband application system has not been done, but it has been suggested (Whitcomb, Raab) that it is not likely to be a serious problem at the 40m. We therefore plan on sticking with a fixed-spacer 1 meter mode cleaner, and applying the sidebands in between the mode cleaner and power recycling mirror, in the input vacuum chamber.
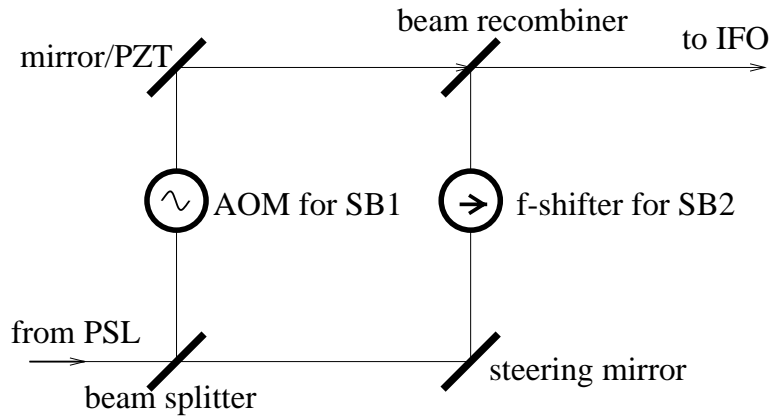
Figure 5: Schematic of a Mach-Zender interferometer used to apply two sets of sidebands onto the beam from the PSL, before injection into the IFO (and, ideally, before injection into the mode cleaner).

1.6 Frequency-shifted subcarrier

The acousto-optical modulators (AOMs) employed at LIGO apply phase modulation to generate a pair of sidebands at $f_c \pm f_1$. Both sidebands are useful for extracting control signals.

The same can be said for the sidebands SB2 applied to control the SRC, *if* the carrier is made to resonate in the SRC. However, as discussed below, it may be advantageous to operate the SRC in a detuned configuration. In that case, only one sideband of the pair is useful and the other contributes only noise.

One can instead apply only one SB2 sideband onto the incoming light, using a frequency-shifted subcarrier technique (on one arm of the M-Z interferometer mentioned above). This is the method chosen by Mason and Willems.

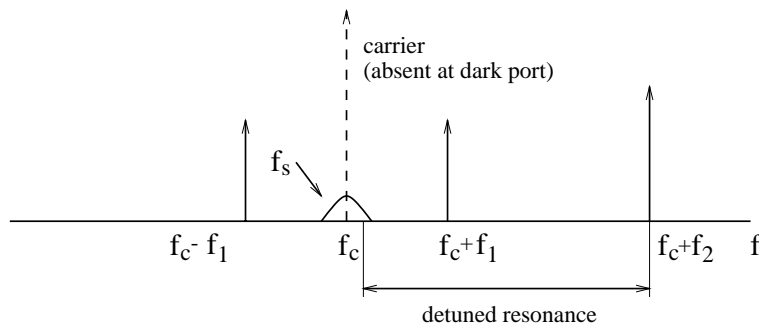The frequency spectrum in this case is summarized in Fig. 6.

Figure 6: Frequency spectrum of carrier and applied sidebands for the control of a dual recycling IFO. Note that in a detuned configuration, the SRC length is fixed to resonate at $f_2 - f_s$.

1.7 GW signal

In this scheme, SB1 (beating against the carrier) is used for the control of the l_+, l_-, and L_+ degrees of freedom, as in LIGO-I.

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Beats between SB1 and SB2 (at $f_2 - f_1$) are used to control the SRC length l_s.

Beats between SB2 and the carrier (at f_2) are used to control L_- and thus constitute the GW signal.

1.8 Notation: Resonance tune

In order to efficiently control the cavity lengths, the RF sidebands should be resonating in the appropriate cavities. Since the arm cavities have rather high finesse, if a frequency component is *not* resonant in the arm cavity, it is very nearly anti-resonant.

The resonance tune of a component of frequency f in a cavity of length L is defined as $\nu = 2Lf/c = 2L/\lambda$, corresponding to a round-trip phase advance $\phi = 2\pi\nu$. Depending on the signs of the reflectivities of the mirrors in the cavity, resonance is achieved when the tune is an integer or a half-integer.

We will refer to the tunes of different frequency components of the laser light, in different cavities, using subscripts. The first subscript denotes the frequency component: c for carrier, 1 for SB1, and 2 for SB2. The second subscript denotes the cavity: a for the arms, p for the PRC, and s for the SRC. For exmaple, $\nu_{cs} = 2l_s f_c/c$ is the round-trip tune of the carrier in the SRC.

For the sidebands, the frequency of the light is displaced from the carrier frequency f_c by an amount $\pm f_1$ or $\pm f_2$ (for phase-modulated sidebands), where f_1 and f_2 are RF frequencies (in the MHz to GHz band). The total tune for the sidebands in a cavity is given by the carrier tune ν_c (whose integral part is very large and, in this note, ignored), plus the tune shift $\nu_{1,2} = 2Lf_{1,2}/c$ (whose integral part we will keep for pedagogy).

1.9 Resonance conditions

- The carrier should be resonant in the arms and the PRC.

- The carrier will be resonant in the SRC for resonant sideband extraction, anti-resonant for signal recycling, and at an *optimized tune ν_s* in the general case (see below).

- The first set of sidebands, SB1, should be nearly anti-resonant in the arms (but not perfectly so, or its first harmonic will be resonant); and resonant in the PRC.

- The second set of sidebands, SB2, should also be nearly anti-resonant in the arms and resonant in the PRC.

- One of the sidebands should be resonant in the SRC, and the other nearly anti-resonant, so that beats between them can be used in the Pound-Drever locking.

This is summarized in Table 1.

We see from the table that in the PRC, carrier resonance is achieved with an integral tune, because the arm is overcoupled for the carrier, so that the reflectance flips sign. Not so for the two sidebands.

1.10 SRC tune ν_{cs}

By changing the length of the SRC, l_s, over a 1.064 μm distance, one can change the tune of the carrier (c) in the SRC (s), ν_{cs}, arbitrarily. This, in turn, changes the transmission of the "compound mirror" formed by the ITM plus SM, making it larger or smaller, as in Fig. 3.

Table 1: Resonance conditions. R means resonant, A means nearly-antiresonant. Referring to the sign conventions in Fig. 2, a "+" subscript means that the tune $\nu = 2Lf/c$ is an integer, and a "−" subscript means that the tune $\nu = 2Lf/c$ is a half integer.

Cavity	arms	PRC	SRC
carrier	R_+	R_+	ν_s
SB1	A	R_-	A
SB2	A	R_-	R_-

1.11   Optimal tune

In the presence of overwhelming noise sources at low frequencies (seismic and thermal), it is clear that the shot noise curve can be optimized. Given a sufficiently reliable estimate of the low-frequency noise, the tune ν_{cs} can be chosen by optimizing the SNR of a binary inspiral event [2].

This procedure has been done for the expected LIGO-II parameters [3], and a tune of around $\phi_{cs} = 0.45$ radians may be optimal.

For the 40m prototype RSE experiment, which is not expected to observe gravity waves and which has a very different (and much larger) low-frequency noise contribution, it is not sensible to optimize the tune, but rather, establish operation at a tune appropriate for LIGO-II, *i.e.*, around 0.45 radians (does this make sense?).

2   Analysis, for 40m prototype IFO

After some consideration, the following procedure seems to be the most efficient way to establish the cavity lengths and sideband frequencies appropriate for an RSE experiment at the 40m. Presumably, it is not a unique procedure!

2.1   Constraints on cavity lengths

The PRC average length $l_+ \equiv (l_+ + l_-)/2$ and Schnupp asymmetry $l_- \equiv l_1 - l_2$ was carefully measured, including effects of refraction in optics, by Logan and Rakhmanov [4] at the beginning of the recycling experiment in 1996. They obtained $l_+ = 229.4$ cm and $l_- = 54.2$ cm.

As can be seen from the optical layout [5] in Fig. 7, most of the PRC lies on the "arm-side" of the beam splitter (BS). Only 25cm of the PRC lies in the region between the recycling mirror (RM) and the BS; the rest of the average length, and all of the asymmetry, is in the arms. This is a consequence of placing the ITMs in separate chambers from the BS chamber. There is some room for adjustment of the relative positions of the BS and ITMs, at the level of 25 cm or so.

If we take this as a constraint, then we see that the signal recycling cavity length l_s cannot be significantly shorter than the average PRC length l_+, and most likely it will need to be bigger.

2.2   FP Arm length

We require the carrier ($f_c = 2.82 \times 10^{14}$ Hz) to be resonant in the arms with integral tune, *i.e.*, the arm length must be an integral multiple of 1.064 μm. For the 40m, the nominal arm length is
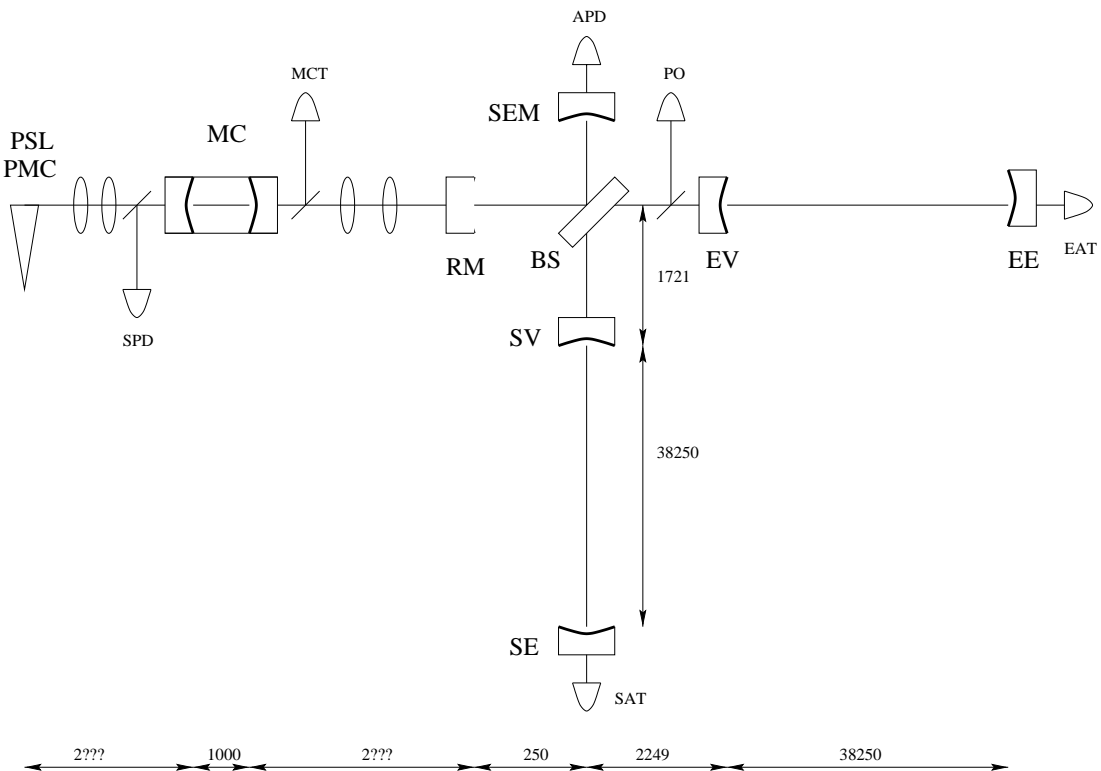
38.25 m, and the arm length at resonance is very close to this, fixed by the Pound-Drever locking control system.

Figure 7: Optical layout of 40 m IFO with signal mirror.

2.3 PRC length

We choose a PRC length close to the nominal $l_+ = 229.4$ cm. Again, the carrier should be resonant in the PRC, with integral tune ν_{cp}, so the length must be an integral multiple of 1.064 μm.

We want SB1 to be resonant in the PRC, so its tune shift ν_{1p} should be a half integer. The smallest half-integer, 1/2, then gives $f_1 = \nu_{1p}c/(2l_+) = 32.7$ MHz, already rather high (for current RF and photodiode technology), so let's not consider the next tune up, $\nu_{1p} = 3/2$ (but see later!).

For reference, in LIGO, $\nu_{1p} = 5/2$, since the PRC is much longer than at the 40m.

Now we must check that SB1 is not resonant in the arms. Since the integral part of $\nu_{ca} = 0$ there, we have the tune shift $\nu_{1a} = 8.352$ (where we keep the integral part for your edification).

The fractional part is far from 0 and 1/2, clearly far from resonance.

2.4   SB2 frequency

Now we choose a SB2 frequency that is also resonant in the PRC, but is *not* the same as SB1, since they must have different resonant properties in the SRC.

In the case of the 40m, the only way to go is up. Since $\nu_{1p} = 1/2$, we must choose $\nu_{2p} = 3/2$ or some even larger half-integer. Let's stop here, since that gives $f_2 = \nu_{2p}c/(2l_+) = 98.2$ MHz, which is pushing the RF technology.

Now we must check that SB2 is not resonant in the arms. Since the integral part of $\nu_{ca} = 0$ there, we have the tune shift $\nu_{2a} = 25.055$. The fractional part is far from 0 and 1/2, at least for the very high finesse arm cavity, and thus, far from resonance.

2.5   Tune of the carrier and of SB2 in the SRC

As discussed above, the tune of the carrier in the SRC is subject to optimization. We want to consider both RSE ($\nu_{cs} = 0$) and slightly detuned (*e.g.*, $\nu_{cs} = 0.1$ or $\phi_{cs} = 0.63$ radians).

We must make SB2 resonant in the SRC and SB1 non-resonant (or vice versa; but since we chose $f_2 = 3f_1$ to be a multiple of f_1, is SB1 is resonant, SB2 will be, as well. So in this case, we must make SB2 resonant and SB1 non-resonant in the SRC).

The *total* tune of SB2 is given by the tune of the carrier, ν_{cs}, plus the tune shift ν_{2s}. A *microscopic* change of the length l_s can change ν_{cs}; if the total tune is fixed, then ν_{2s} must change, leading to a *macroscopic* change of the length l_s. Thus, it will be rather difficult to change the tune of the carrier in the SRC, without making big changes to the position of the SM with respect to the BS.

2.6   Choosing the length of the SRC

Once we have chosen a carrier tune in the SRC, we can choose a corresponding *total* tune of SB2 and thus the tune shift ν_{2s}.

For RSE, we choose $\nu_{cs} = 0$ (at least, the fractional part thereof). the total tune for SB2 is thus equal to the tune shift ν_{2s}; this must be half-integral to make SB2 resonant. The tune shift of SB1, ν_{1s}, will be 1/3 of this, since we chose $f_2 = 3f_1$ for the 40m; this must not be half-integral.

If we choose ν_{2s} to be less than 5/2, then we obtain $l_s = \nu_{2s}c/(2f_2)$ to be less than the PRC length l_+. As argued above, having $l_s < l_+$ cannot be easily arranged. So we choose $\nu_{2s} = 5/2$, giving $l_s = 381.7$ cm. Any larger half-integral value of ν_{2s} results in impractically large l_s.

We then must check that SB1 is not resonant in the SRC. Since $\nu_{1s} = \nu_{2s}/3 = 5/6$, we're safe.

Now let's detune, by choosing $\nu_{cs} = 0.1$. Then the SB2 tune shift in the SRC must be $\nu_{2s} = 5/2 - 0.1$, leading to $l_s = \nu_{2s}c/(2f_2) = 366.4$ cm, *i.e.*, 15.3 cm from the RSE position. Then, the fractional part of $(\nu_{cs} + \nu_{1s})$ is 0.90, still pretty far from resonance.

The signal recycling extreme, with $\nu_{cs} = 0.25$, gives $l_s = 343.5$ cm. Unfortunately, in this case (which is not being considered for LIGO-II), SB1 is also resonant in the SRC; one has to go to a total tune of SB2 in the SRC of 7/2 to avoid this.

2.7   Is there room?

To accomodate a SM a meter or more from the BS, an output chamber must be constructed and placed between the beam splitter chamber and the wall. Actually, an output chamber already

exists, with the same dimensions as the input chamber; it needs only a seismic stack and optical table.

From the above, we see that SRC's with lengths varying from 350 cm to 382 cm are workable. This works out to a SM/BS distance of (350-229+25) = 146 cm to 178 cm.

Is that much space available, given the current 40m layout and the dimensions of the output chamber?

A rough sketch of the situation is shown in Fig. 8. We see that 146 cm is a tight but doable squeeze, while 178 cm will require some clever redesign of the layout.
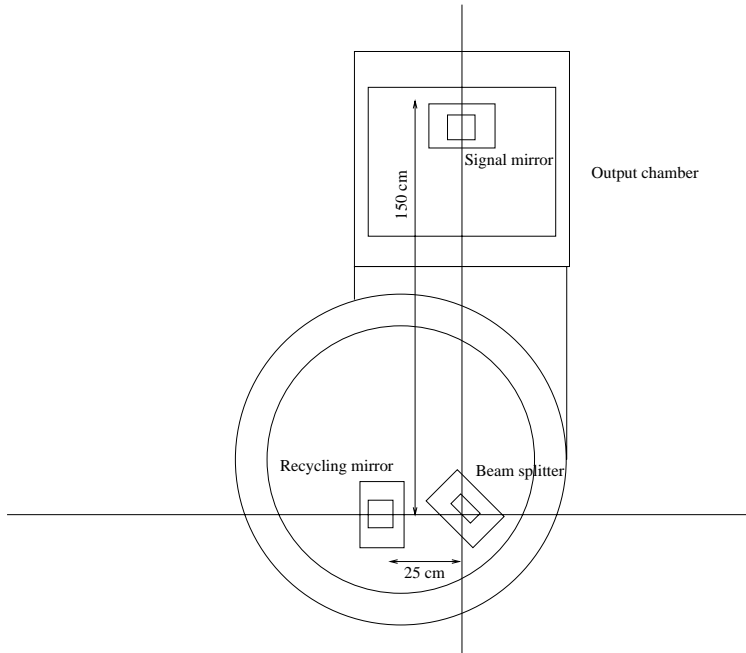

Figure 8: Sketch of the 40m beam splitter chamber with a new output chamber added (to the north), showing the space available for a signal recycling cavity.

2.8   Asymmetry

In this scheme, SB1 is not resonant in the SRC and is not used to control the GW signal L_-. Rather, it is SB2 which must make it out the dark port via a Schnupp asymmetry in the PRC. This asymmetry results in a phase difference between the SB2 light in the two arms of $\delta\phi_{2p} = 2\pi f_2 l_-/c$.

The power in SB2 exiting the BS towards the dark port is given by

$$P_{2s} = P_{laser} \left(\frac{t_{RM} r_{ITM} \sin(\delta\phi_{2p})}{1 - r_{RM} r_{ITM} \cos(\delta\phi_{2p})} \right)^2$$

and the value of $\delta\phi_{2p}$ which maximizes this is

$$\delta\phi_{2p}^{opt} = r_{RM} r_{ITM}.$$

This corresponds to $l_-^{opt} = (c/2\pi f_2) \cos^{-1}(r_{RM} r_{ITM})$.

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For the particular 40m optical configuration summarized below, the fact that f_2 is three times larger than f_1, and the RM reflectivity is higher, means that the optimal asymmetry is smaller than at the 40m recycling experiment. This gives $l_- = 20$ cm, which is, I believe, easily obtainable in the 40m vacuum envelope.

2.9 Transfer functions

An analysis of the transfer functions between mirror motions and demodulated signals from the output ports, using `twiddle`, is in progress.

3 Summary

A potentially workable configuration for a tuned RSE demonstration at the 40m is summarized below, and in Table 2.

- SRC carrier tune: $\nu_{cs} = 0.1$

- SB1 frequency: $f_1 = 32.729$ MHz

- PRC cavity length: $l_+ = 229.0$ cm

- SB2 frequency: $f_2 = 3f_1 = 98.188$ MHz

- SRC cavity length: $l_s = 366.4$ cm

- tunes and tune shifts are summarized in Table 2.

Table 2: Tunes (fractional part, for carrier) and tune shifts (for sidebands) in the three cavities.

Cavity	arms	PRC	SRC
carrier	$\nu_{ca} = 0.00$	$\nu_{cp} = 0.00$	$\nu_{cs} = 0.10$
SB1	$\nu_{1a} = 8.35$	$\nu_{1p} = 0.50$	$\nu_{1s} = 0.80$
SB2	$\nu_{2a} = 25.05$	$\nu_{2p} = 1.50$	$\nu_{2s} = 2.40$

3.1 Optical parameters

The choice of optical parameters for an RSE experiment at the 40m is summarized in Table 3.

The (naively) predicted shot noise sensitivity for such an optical configuration is shown in Fig. 9. Warning! The thermal and suspension noise components shown in this figure are based on damping parameters that I have made up; they're bound to be wrong, and I welcome advice on what numbers to put in!

4 Acknowledgements

Many thanks to Jim Mason and Phil Willems, whom I hope will find all the conceptual errors in this draft.

Table 3: Mirror parameters and other parameters for one proposed optical configuration for an RSE experiment at the 40m.

mirror	Loss (ppm)	$T = t^2$	R_{curv} (m)	ω_{beam} (cm)
ETMs	20	15 ppm	90.5	0.40
ITMs	20	1547 ppm	90.5	0.40
BS	750	0.500	∞	0.42
RM	20	0.161	60.3	0.42
SM	20	0.630	60.0	0.42

Arm cavity finesse = 3919
Arm cavity Gain = 2409
PRC Gain = 7.4
SRC tune $\phi_{cs} = 0.45$
$h_{shot}(DC) = 2.2 \times 10^{-22}$
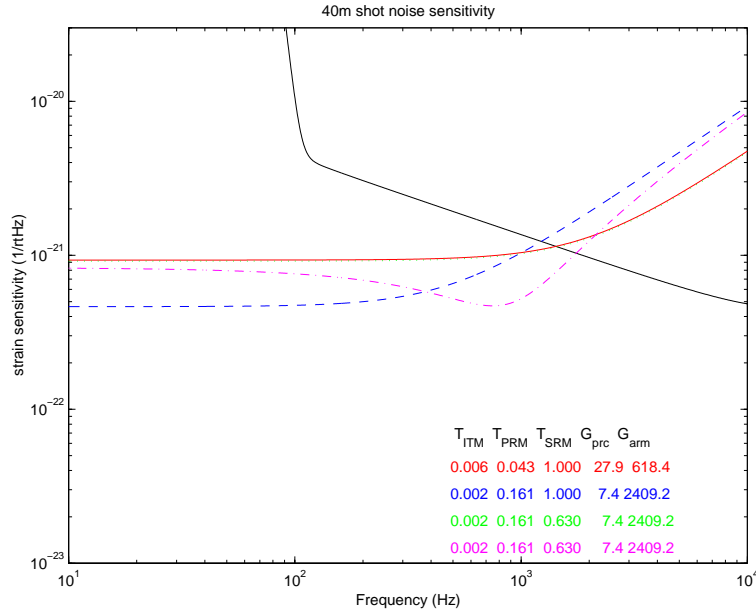$h_{shot}(750Hz) = 1.2 \times 10^{-22}$

Figure 9: The (naively) predicted shot noise sensitivity for one proposed optical configuration for an RSE experiment at the 40m.
Black curve: very crudely estimated strain noise due to seismic, thermal, and suspension noise.
Red solid curve with $h(DC) = 2.4 \times 10^{-22}$: 40m with no RSE, high PRC gain, $f_{pol} = 2000$ Hz;
Blue dashed curve with $h(DC) = 1.2 \times 10^{-22}$: 40m with no RSE, low PRC gain, $f_{pol} = 500$ Hz;
Green dotted curve with $h(DC) = 2.4 \times 10^{-22}$ (indistinguishible from red curve): 40m with RSE, low PRC gain, $f_{pol} = 2000$ Hz, $\phi_{cs} = 0$;
Magenta dash-dot curve with $h(DC) = 2.1 \times 10^{-22}$: 40m with RSE, low PRC gain, $f_{pol} = 2000$ Hz, $\phi_{cs} = 0.45$ rad.

References

[1] Jun Mizuno, MPQ-203, LIGO-P950029-00-D, July 1995.

[2] S. Finn, "LIGO Science Benchmarks", LIGO T970167-00 (1997); and the `cbi.m` program.

[3] LSC white paper on Detector R&D, `http://www.stanford.edu/group/ligo/WhitePaper30Sept98.pdf` (1998).

[4] J. Logan and M. Rakhmanov, LIGO Note T960013-02 (1996).

[5] `http://www.ligo.caltech.edu/LIGO_web/dcc/docs/D961304-06.pdf`.