LIGO-T000102-00-D

Amplitude Modulation Response of Gravitational Wave Interferometers

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Abstract

We describe an algorithm for computing AM response of interferometers. This algorithm is based on previous work by Livas (Liv) and [1] Schutz and Tinto (ST) [2]. Comparisons with work by ST, and Anderson, Brady, and Creighton (ABC) [3] is made.

Goals

- compute the Amplitude Modulation response of any given interferometer to polarized sources of gravitational radiation
- implement for the LAL

Strategy

- follow Livas [1] and Schutz & Tinto [2].
- express the metric perturbation tensor, $h_{\mu\nu}$, in the basis of the Detector frame, and compute the sensitivities
- ignore orbital parallax, nutation of the Earth's rotation, and most other irregularities
- *don't* ignore the tilt of the plane of the IFO, which will be equivalent to an effective location different from the location of the IFO vertex
- $\therefore$ the coordinate transformations reduce to a series of (Euler) rotations.

- sensitivities are given by Schutz & Tinto [2]:

$$
\begin{aligned}
F_+ &= -\sin 2\beta[A_X^x A_X^y - A_Y^x A_Y^y] \\
F_\times &= \sin 2\beta[A_X^x A_Y^y + A_Y^x A_X^y] \\
\delta l/l_0 &= F_+ h_+ + \exp(i\delta) F_\times h_\times
\end{aligned}
$$

where

A = the transformation matrix

β = half the angle between the arms

δ = phase difference between the $+$ and the $\times$ polarizations

δl = difference between the values of $\delta l_n, n = 1, 2$ for the two arms

Coordinate Conventions

- define conventions for the coordinate axes of the reference frames of the source, and the interferometer

- Source reference frame has Z-axis pointing from the source *to* the observer; orientations measured relative to meridians (lines of constant RA), *i.e.* w.r.t. equatorial coordinates

- view from the IFO location to the source would look like:

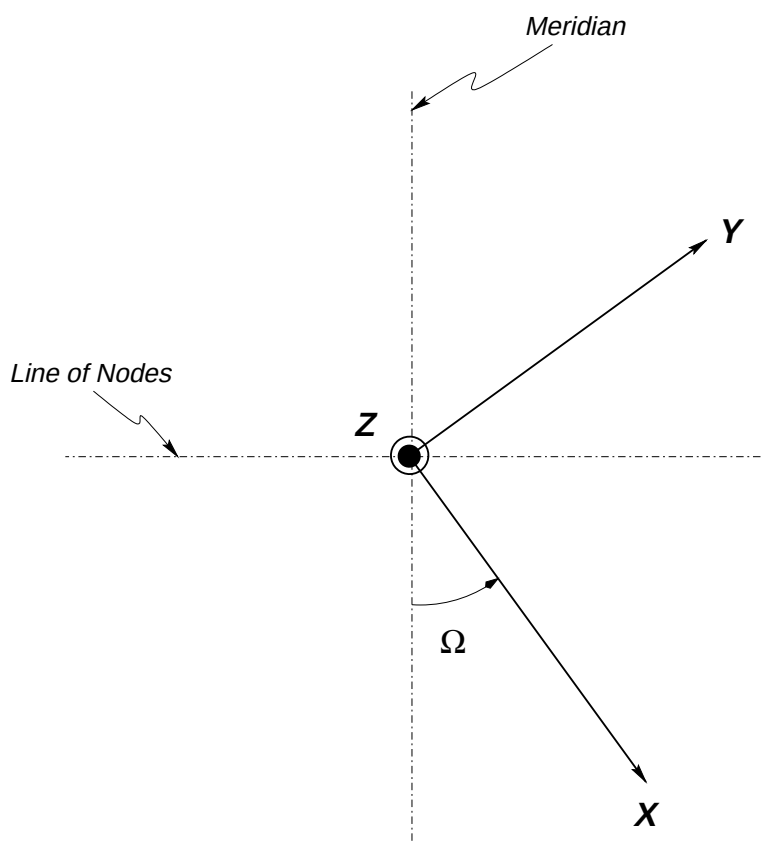


Figure 1: Source Frame

- Source position is given in RA-dec, orientation is measured w.r.t. meridian

- Detector reference frame has Z-axis pointing to its zenith, *i.e.* perpendicular to the plane of the detector

- the plan view of the Detector frame would be:

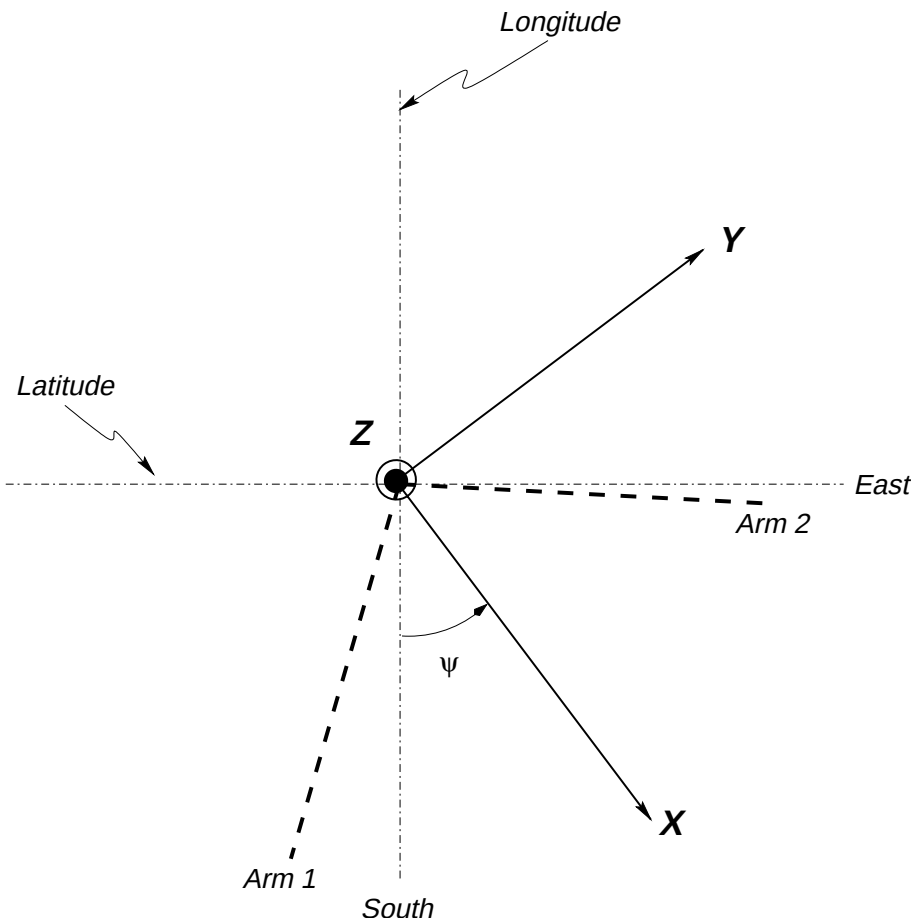


Figure 2: Detector Frame

- NB: orientation angle is measured East of South (counter-clockwise) to the bisector of the arms, as per convention in Schutz & Tinto [2].

- tilt of detector is taken into account by making two additional rotations using LIGO survey data [4]

Coordinate Conventions: Schutz & Tinto

- in Section 2 of Schutz and Tinto's paper [2], they compute the single-detector response function
- their convention for the Source orientation is made relative to the vertical circle of the IFO, *i.e.* w.r.t. horizon coordinates
- vertical circle is the circle that passes through the IFO's zenith and the Source: it is not necessarily a meridian
- position of Source is given as altitude-azimuth, and orientation is measured w.r.t. vertical circle
- this is already implicitly in the Detector frame

Coord. Conventions: Anderson, et al.

- Source frame has Z-axis pointing from Source to Earth.
- Source position is given as RA-dec; orientation is measured w.r.t. Line of Nodes (see Fig. 1)
- IFO frame and orientation are derived by differential geometry using Earth model ellipsoid WGS-84, and LIGO survey data [4]

Euler Rotations

- make Euler rotations about the Z- and Y-axes.
- form of these rotation matrices:

$$R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

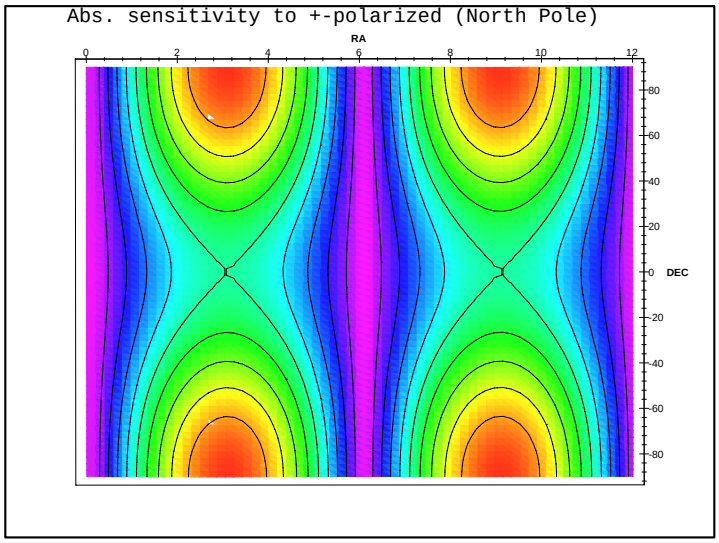
- total transformation is a product of rotations:

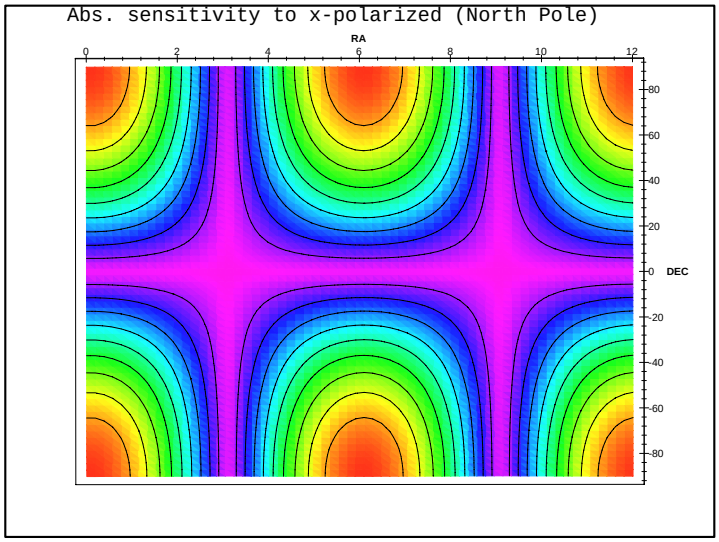
$$
\begin{aligned}
R &= R_z(\psi) R_y(\frac{\pi}{2} - lat) R_z(LMST) \\
& \quad R_z(\pi - RA) R_y(-(\frac{\pi}{2} + dec)) R_z(-\Omega)
\end{aligned}
$$

where

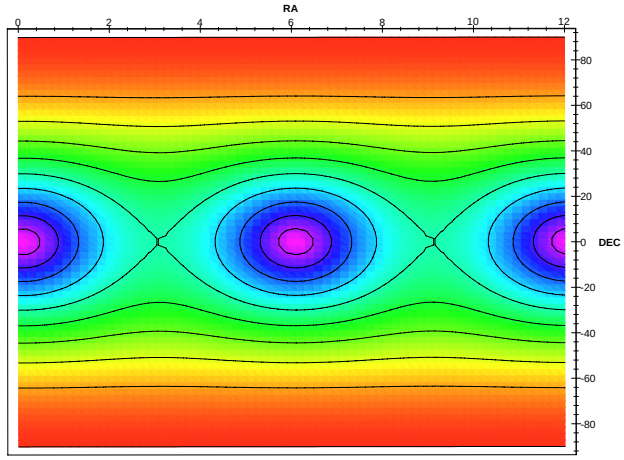
$$
\begin{aligned}
\psi &= \text{orientation angle of IFO} \\
lat &= \text{latitude of IFO} \\
LMST &= \text{Local Mean Sidereal Time} \\
dec &= \text{declination of Source} \\
\Omega &= \text{orientation of Source}
\end{aligned}
$$

Plots of Response (Dec. vs. RA; Detector @ North Pole)

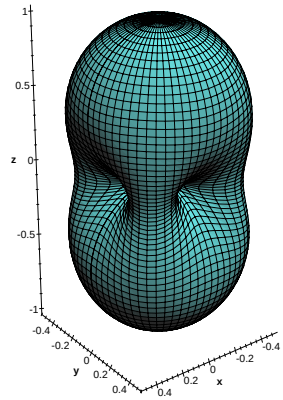




Abs. sensitivity to circular-polarized (North Pole)



Abs. sensitivity to circular-polarized (North Pole)



Comparison Tests: Single Point

- compare results from the three different algorithms for special source location and orientation, and sidereal time

Comparison Tests: TestIFO, Source @ zenith, expect (1,0)

	Plus	Cross	Circular
Livas:	9.9999999852530e-01	-1.6653345369377e-16	9.9999999852530e-01
S&T:	9.9999999852530e-01	-1.6653345369377e-16	9.9999999852530e-01
ABC:	9.9999999852530e-01	-1.8369701960121e-16	9.9999999852530e-01
(L-ST)/ST:	--	--	0.0000000000000e+00
(L-ABC)/ABC:	2.2204460525248e-16	9.3434101133993e-02	2.2204460525248e-16

Comparison Tests: TestIFO, Source @ W hor., expect (0.5,0)

	Plus	Cross	Circular
Livas:	5.0000000005132e-01	-2.6668524318568e-10	5.0000000005132e-01
S&T:	5.0000000005132e-01	4.4447543416675e-10	5.0000000005132e-01
ABC:	5.0000000005132e-01	2.6668534047758e-10	5.0000000005132e-01
(L-ST)/ST:	--	--	2.2204460490224e-16
(L-ABC)/ABC:	2.2204460490224e-16	3.6481907184434e-07	2.2204460490224e-16

Comparison Tests: TestIFO, Source @ N hor., expect (-.5,0)

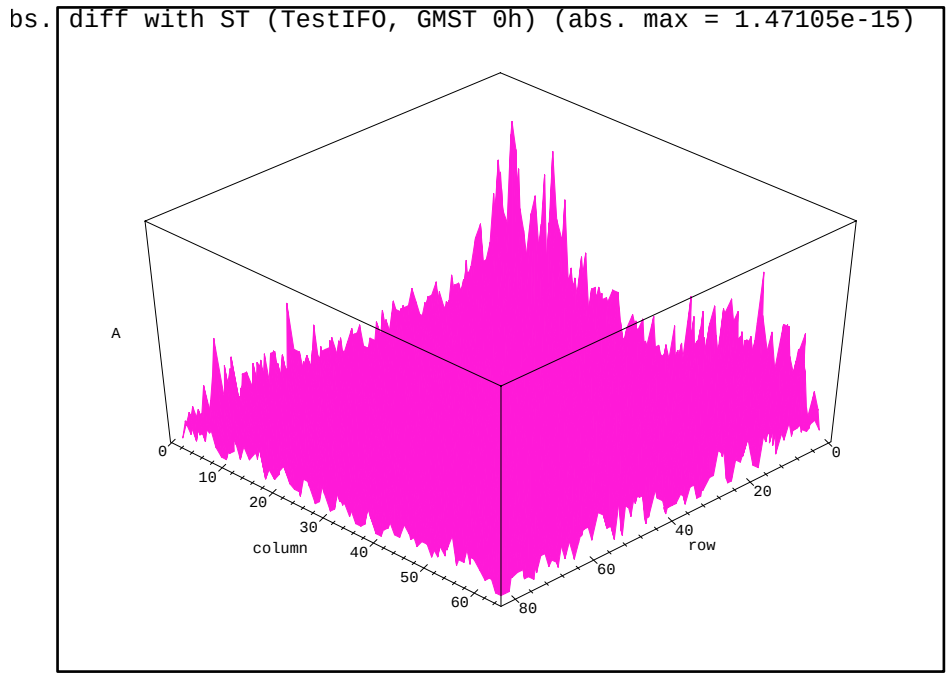
	Plus	Cross	Circular
Livas:	-4.9999999958941e-01	-1.7548263092748e-05	4.9999999989735e-01
S&T:	-4.9999999958941e-01	1.7548260595732e-05	4.9999999989735e-01
ABC:	-4.9999999958941e-01	1.7548263092146e-05	4.9999999989735e-01
(L-ST)/ST:	--	--	1.1102230248531e-16
(L-ABC)/ABC:	1.3322676306442e-15	3.4299010485039e-11	1.3322676298237e-15

Comparison Tests: LHO, Source @ arb. loc.

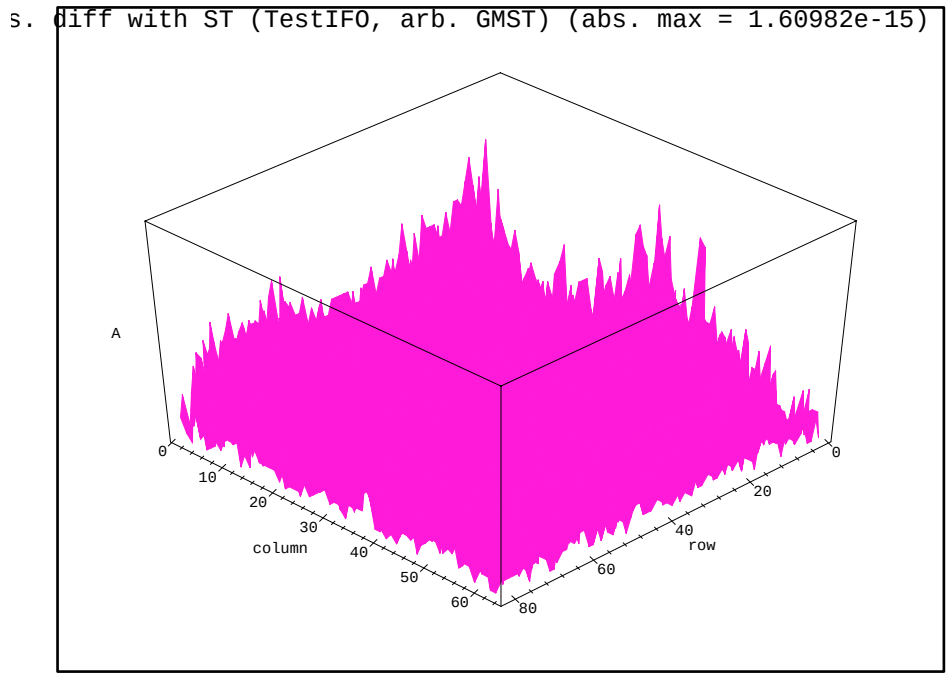
	Plus	Cross	Circular
Livas:	3.8614832967608e-01	3.2538863475748e-01	5.0496365823787e-01
S&T:	2.9667660078718e-01	-4.0862120684851e-01	5.0496365823787e-01
ABC:	-3.8641299959412e-01	-3.2551740739826e-01	5.0524903639158e-01
(L-ST)/ST:	--	--	6.5958589683429e-16
(L-ABC)/ABC:	6.8494051265867e-04	3.9559371589898e-04	5.6514592496918e-04

Circ. Pol. Abs. Diff. Plots: Liv vs. ST [Dec vs. RA]

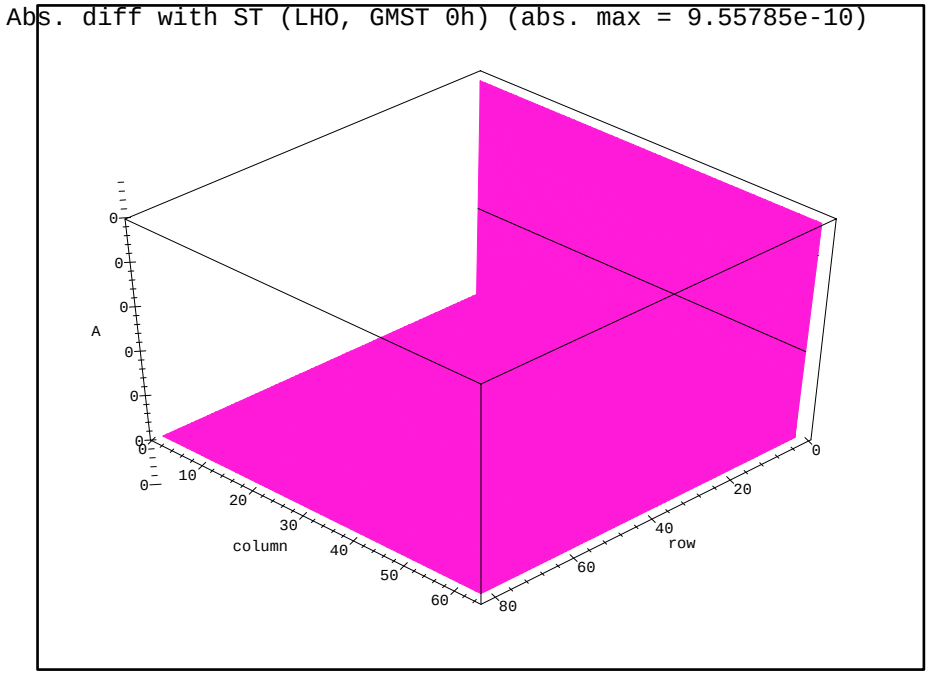
- grid: 82 x 64 = 5248 points
- rows = Right Ascension; cols = Declination
- plot abs. difference between abs. values



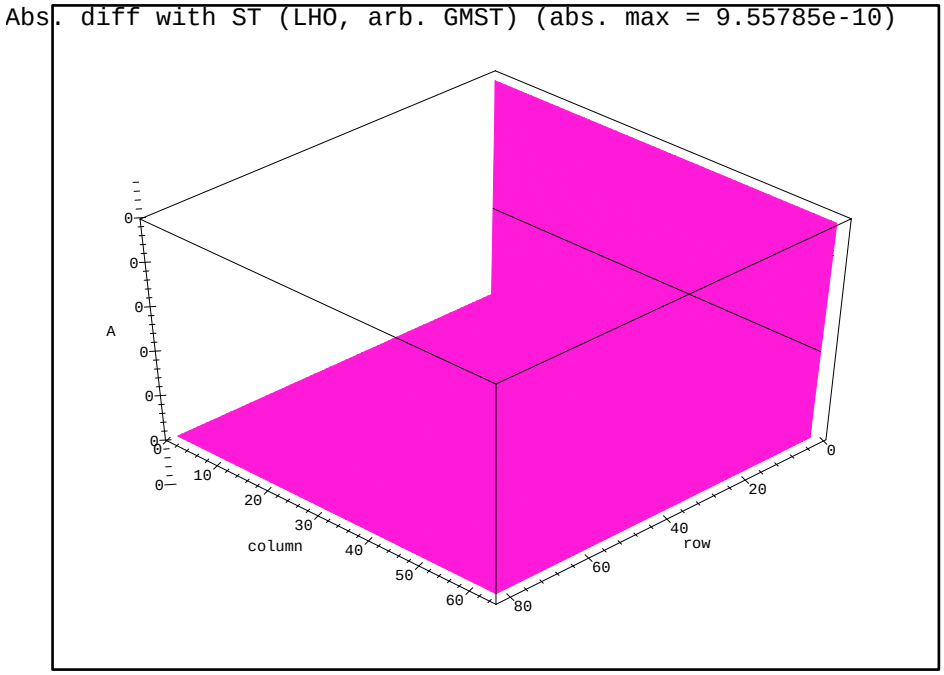

```
IFO:  Test IFO
Date: 1994-05-17 08:20:47 UTC Sun
GMST:     0.255940 secs

RMS(Liv - ST)  =  2.43446984541640e-16
Max|Liv - ST|  =  1.47104550762833e-15
```




```
IFO:  Test IFO
Date: 1994-05-12 05:20:47 UTC Sun
GMST: 74387.909680 secs

RMS(Liv - ST)  =  3.68038732374855e-16
Max|Liv - ST|  =  1.60982338570648e-15
```

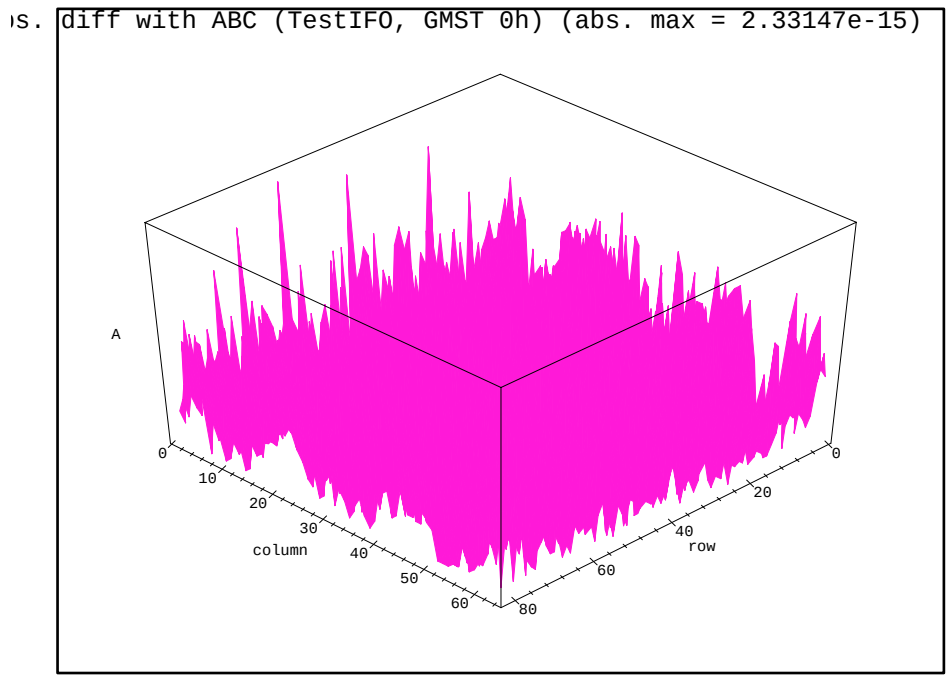



```
IFO:  LHO
Date: 1994-05-17 08:20:47 UTC Sun
GMST:     0.255940 secs

RMS(Liv - ST)  =  1.05548758578373e-10
Max|Liv - ST|  =  9.55784895673162e-10
```

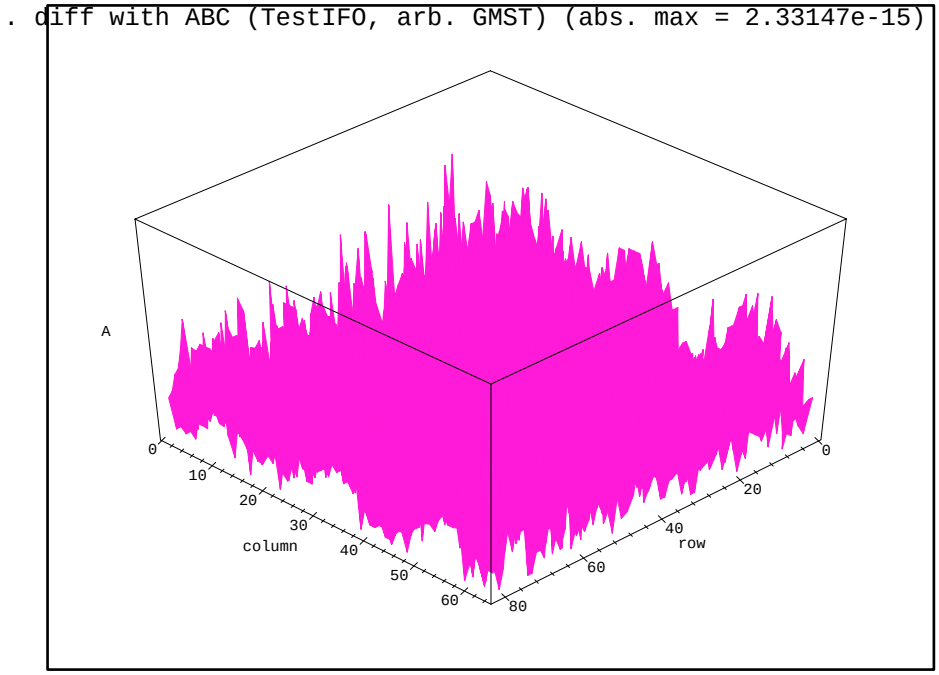



```
IFO:  LHO
Date: 1990-08-01 13:57:09 UTC
GMST: 38444.883737 secs

RMS(Liv - ST)  =  1.05548762808303e-10
Max|Liv - ST|  =  9.55784895673162e-10
```

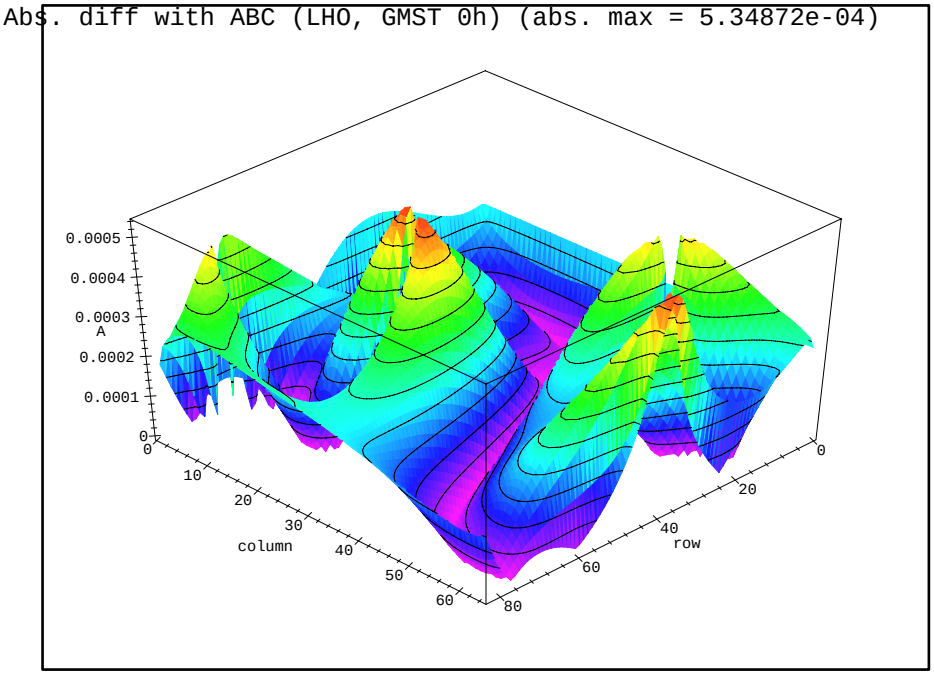
Circ. Pol. Abs. Diff. Plots: Liv vs. ABC [Dec. vs RA]




```
IFO:  Test IFO
Date: 1994-05-17 08:20:47 UTC Sun
GMST:     0.255940 secs

RMS(Liv - ABC)  =  5.83166475061072e-16
Max|Liv - ABC|  =  2.33146835171283e-15
```

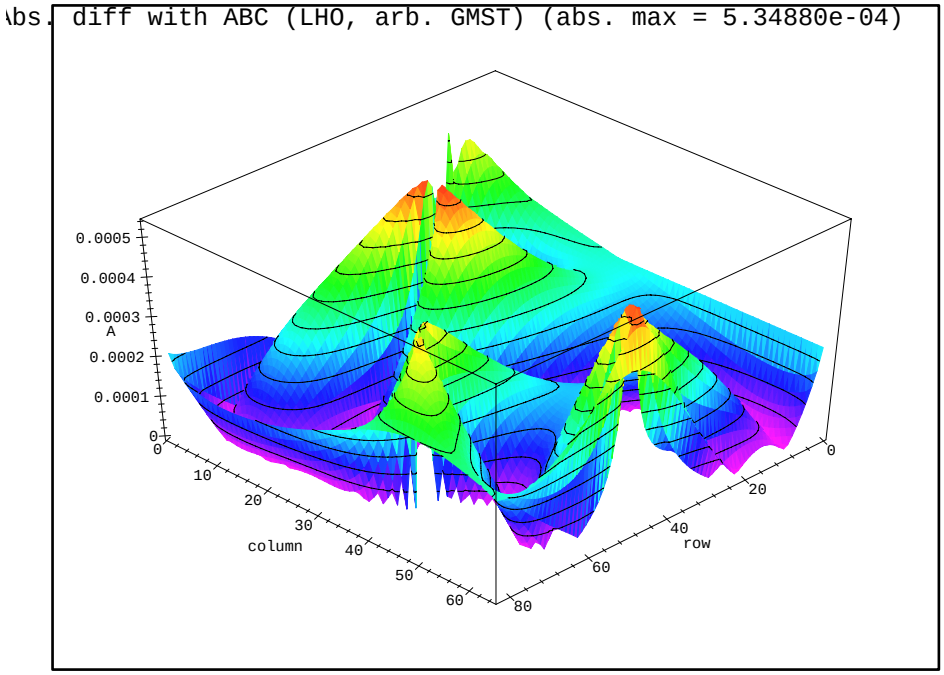



```
IFO:  Test IFO
Date: 1994-05-12 05:20:47 UTC Sun
GMST: 74387.909680 secs

RMS(Liv - ABC)  =  6.02907949384043e-16
Max|Liv - ABC|  =  2.33146835171283e-15
```



IFO: LHO
Date: 1994-05-17 08:20:47 UTC Sun
GMST: 0.255940 secs

RMS(Liv - ABC) = 2.22117795226517e-04
Max|Liv - ABC| = 5.34871669267287e-04




```
IFO:  LHO
Date: 1990-08-01 13:57:09 UTC
GMST: 38444.883737 secs

RMS(Liv - ABC)  =  2.22870725247795e-04
Max|Liv - ABC|  =  5.34880396050097e-04
```

Related Packages: Date and Time

- date and time utility routines have been coded into LAL package: part of LAL beta-0.4 release
- converts time to and from seconds from GPS epoch and seconds from Unix epoch
- converts time in seconds from some epoch to time structure with day, date, residual nanoseconds, etc.

Future Refinements

- vector routine: given initial time, size of time step, and number of time steps, returns vector containing response at each of those time steps
- standardized IFO description parameter data structures, coordinated with J. Romano (UT-Brownsville)
- detailed documentation (LIGO tech note) in preparation

Acknowledgements

We thank W. Anderson, P. Brady, and J. Creighton for sharing their Maple worksheet

Bibliography

References

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[2] B.F. Schutz and M. Tinto, *Mon. Not. R. Astr. Soc.* (1987) **224**, 131-154

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[4] W. Althouse, L. Jones, and A. Lazzarini, Determination of Global and Local Coordinate Axes for the LIGO Sites, LIGO Tech. Note LIGO-T980044-08-E, 1999

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