LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -

LIGO Laboratory/ LIGO Science Collaboration

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Noise from the Fizeau Effect

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The Fizeau effect is a change in the velocity of light when it is traveling through a moving medium. If the motion of the medium is random, due, for example, to seismic motion, this noise can be transmitted to the phase of the transmitted light. This could potentially be a source of limiting noise in interferometers, either at the LIGO sites or the prototypes at the campuses.

To calculate the size of this effect, start with the definition of phase accumulated while traversing an optic;

$$\phi = 2 \pi \nu t \qquad (1)$$

where ν is the frequency of light, and t is the time for the light to pass through the optic. This time can be found from

$$t = d / v1 \qquad (2)$$

where d is the thickness of the optic and v1 is the velocity of light in the transmitting medium. This velocity v1 can be found from the Fizeau formula

$$v1 = c / n + v2 (1 - 1/n^2), \qquad (3)$$

where c is the speed of light in vacuum, n is the index of refraction of the transmitting medium, and v2 is the velocity of the transmitting optic. This formula can be derived from the relativistic addition of velocities formula;

$$v1 = (v2 + c / n) / (1 + (v2 c/n) /c^2) \qquad (4)$$

$$v1 = (v2 + c / n) / (1 + v2 / (c n)). \qquad (5)$$

If we assume that v2, the velocity of the optic relative to the lab frame, is small compared to c, then we can write

$$v1 = (v2 + c / n) (1 - v2 / (c n) + O(v2^2/c^2) \qquad (6)$$

$$v1 = c / n + v2 - v2 / n^2 + O(v2^2/c^2). \qquad (7)$$

For more on the Fizeau effect, see [1].

If v2, the velocity of the optic in the lab frame, is driven by a random process, e.g. seismic noise, then the velocity of the light in the material will be noisy. This will lead to phase noise in the light once it leaves the optic. The differential relationship between v1 and v2 can be written

$$\delta v1 = \delta v2 \ (1 - 1/ n^2), \qquad (8)$$

and using Eq. (1), the differential phase can be written

$$\delta\phi = 2 \pi \nu \ d \ n^2 / c^2 \ (1 - 1 / n^2) \ \delta v2. \qquad (9)$$

Writing the velocity v2 in terms of seismic motion δx2,

$$\delta v2 = 2 \pi f \ \delta x2 \qquad (10)$$

where f is the frequency of the seismic motion, allow the phase noise to be written in terms of seismic noise:

$$(S_\phi(f))^{1/2} = (2 \pi f) (2 \pi \nu) (n / c)^2 d (1 - 1 / n^2) \ (S_x(f))^{1/2}, \qquad (11)$$

or written as frequency noise

$$(S_f(f))^{1/2} = (2 \pi f)^2 \ \nu \ (n / c)^2 d (1 - 1 / n^2) \ (S_x(f))^{1/2}. \qquad (12)$$

Substituting in values for 1.064 μm laser passing through a 5 cm Faraday isolator sitting directly on an advanced LIGO HAM seismic isolaton stack;

$$f = 10 \text{ Hz} \qquad (13)$$

$$\nu = 3 \ 10^{14} \text{ Hz} \qquad (14)$$

$$n = 2 \qquad (15)$$

$$d = 5 \text{ cm} \qquad (16)$$

$$(S_x(f))^{1/2} = 2 \ 10^{-13} \text{ m/Hz}^{1/2} , \qquad (17)$$

gives a value for frequency noise at 10 Hz,

$$(S_f(f))^{1/2} = 2.8 \ 10^{-12} \text{ Hz/Hz}^{1/2}. \qquad (18)$$

This is compared to the advanced LIGO requirement for frequency noise at 10 Hz for RF readout [2];

$$(S_f(f))^{1/2} = 10^{-6}\ \mathrm{Hz/Hz}^{1/2}. \tag{18}$$

Based on this, it is unlikely that noise from the Fizeau effect will be a noticeable source of noise in advanced LIGO or in a prototype.

References:

[1] A. Sommerfeld, *Optics*, (Academic Press, New York, 1964), pg 69.
[2] P. Fritschel, Advanced LIGO Systems Design, LIGO-T010075-00-D.