LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
– LIGO –

*LIGO Laboratory / LIGO Scientific Collaboration*

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# Auxiliary Suspended Optics Displacement Noise Requirements

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This is an internal working note
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# 1 Introduction & Purpose

This note lays out the $f > 10$ Hz displacement noise requirements for the auxiliary (i.e., non-Test Mass) suspended optics. The noise requirements are determined by calculating the coupling of the auxiliary optics to the relevant signals, as described below. For optics that couple directly to the gravitational-wave signal, the noise requirement is set so that their influence is 10× below the test mass displacement noise given in Section 2. Since displacement noise is typically overridden by sensing noise above ~100 Hz, the noise requirements are given only in the interval $f = 10 - 100\ \mathrm{Hz}$. The noise requirements given refer to the displacement of the reflecting surface of the optic, in the presence of vertical-horizontal coupling as given. The displacement noise input to the suspension subsystem is the seismic (SEI) platform motion requirement given in LIGO-E990303-03-D.

# 2 Test Mass Displacement Noise Limit

By operating an Advanced LIGO signal-recycled interferometer at low power and/or with zero signal-detuning, it is possible to reduce the low-frequency quantum noise to essentially the level of suspension and internal thermal noise. Since this is a possible mode of operation, auxiliary-optic noise requirements are determined against the test mass suspension and internal thermal noise level. This noise level is shown in Figure 1, for the following test mass parameters:

- 40 kg sapphire test mass, 31.4 cmϕ × 13 cm
- 6.0 cm gaussian beam radius on the test mass
- Ribbon fibers: 1.15 mm × 0.115 mm (stress = 750 MPa)
- Penultimate mass: 80 kg (to keep vertical mode below 10 Hz)

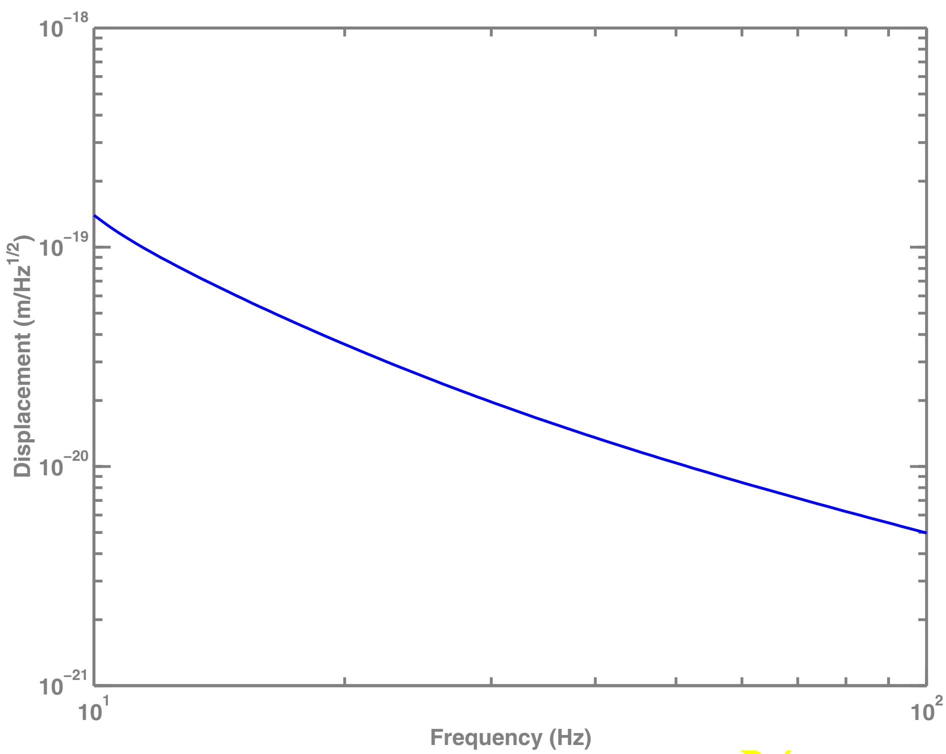


*Figure 1.* Test mass displacement noise due to root-square-sum of suspension and internal thermal noise.

# 3 AUXILIARY OPTICS

Displacement noise upper limits for the auxiliary optics are shown below in Figure 2. The subsections below describe how the levels were determined. Since the local suspension damping can compromise the vibration isolation and potentially add noise, the status of the local damping also becomes part of the requirement. When the interferometer is locked there are global sensing signals that can provide damping in lieu of the suspension local damping–specifically there are five such global longitudinal signals. Four of these are reserved for damping the test masses. The fifth could be assigned to one of the auxiliary optics; this could be a useful option for the BS suspension, as its displacement noise limit is lower than that of the RMs. However for greater flexibility, it is preferrable that the displacement noise requirement for all auxiliary optics be met with the local damping active. (There is no single optic which would obviously benefit from lifting the local damping criterion–the BS's extra isolation stage probably makes up for its lower noise requirement; one of the MC mirrors could have local damping disengaged, but since the other two still require local damping it wouldn't impact the suspension design.)

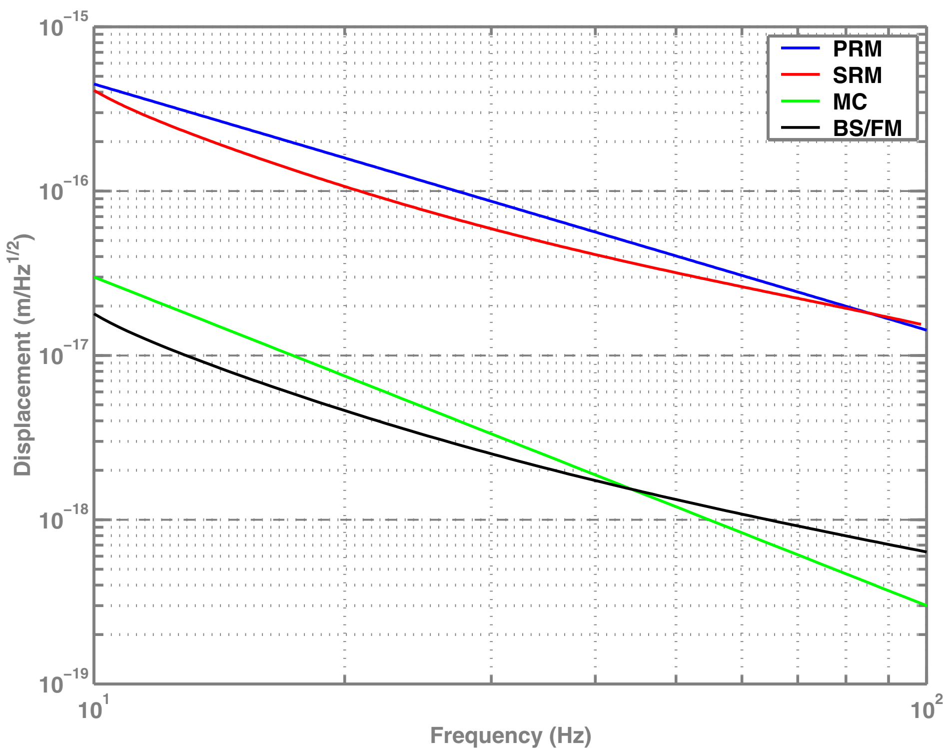


*Figure 2.* Displacement noise upper limits for various auxiliary optics. PRM: power recycling mirror; SRM: signal recycling mirror; MC: mode cleaner mirror; BS: beam splitter; FM: fold mirror

## 3.1. Power recycling mirror (PRM)

Motion of the PRM primarily affects the fidelity of the error signal used to stabilize the laser frequency to the interferometer; in the baseline sensing configuration this is the reflection port signal, demodulated at 9 MHz. This error signal is mostly sensitive to the common arm length, but is also somewhat sensitive to PRM motion. The latter must be small enough that it does not compromise the required level of frequency stability.

The required frequency stability is calculated using `'rsenoiseDC3.m'`[1] with the following stipulations:

- relative matching of arm input mirror transmission: 1%
- round trip loss difference in the arms: 20 ppm
- rf readout used to set requirement (more stringent than dc readout by ~order of magnitude)
- frequency noise limit set to be 10× below the strain noise determined by the test mass motion given in Figure 1

The resulting frequency noise requirement can be approximated, in the 10-100 Hz band, by:

$$\delta\nu \leq 6\times10^{-7}(10\mathrm{Hz}/f)^{1/2}\ \mathrm{Hz}/\sqrt{\mathrm{Hz}} \qquad (1)$$

This is converted into an equivalent stability of the arm common mode by: $\delta L_{+} = L(\delta\nu/\nu_l)$, with $L = 4$ km. The relevant strengths of the arm common mode and the PRM degrees-of-freedom as sensed with the reflection port 9 MHz signal are calculated using 'Finesse 0.64'.[2] The displacement noise limit of the PRM is then set so that its level on the reflection port signal corresponds to the frequency noise requirement given in Eq. (1).

The displacement requirement holds in the presence of a vertical-horizontal coupling of $1.8\times10^{-3}$ (see LIGO-T010076-01).

## 3.2. Signal recycling mirror (SRM)

SRM motion can couple directly to the gravitational-wave signal, by impressing signal sidebands on any carrier light leaking out the antisymmetric port of the beamsplitter. 'Finesse' simulations show the coupling to be 10-100× stronger for the rf readout (180 MHz demodulation), compared to dc readout (presumably from the rf sideband interaction with the arms–still under investigation). 'Finesse' is used to calculate the relative signal strengths of differential arm and SRM motion, using the rf readout to be conservative. This signal ratio is scaled by the arm differential noise level determined by Figure 1 to determine the SRM noise upper limit (including standard factor of 10 margin).

The displacement requirement holds in the presence of a vertical-horizontal coupling of $5.9\times10^{-3}$ (see LIGO-T010076-01).

## 3.3. Beamsplitter (BS) & Fold mirrors (FM)

Motions of the BS & FMs directly produce a phase difference between the arms. A longitudinal motion $x$ of any of these optics produces a phase difference at the beamsplitter of $(4\sqrt{2}\pi x/\lambda)$. The limit on $x$ is set by taking the root-square-sum of the three optics (over-conservative for the non-folded interferometers), and comparing to the phase difference produced by the test mass

1. matlab code by J Mason: `http://www.phys.ufl.edu/LIGO/LIGO/STAIC.html`
2. Frequency domain simulation code by A Freise: `http://www.phys.ufl.edu/LIGO/LIGO/STAIC.html`

motion, $(8F/\lambda)(2x_{tm})$, where $F$ (=1238) is the cavity finesse and $x_{tm}$ is the displacement given in Figure 1.

The displacement requirement holds in the presence of a vertical-horizontal coupling of $9\times10^{-3}$ for the BS (TBD for the FMs; see LIGO-T010076-01).

## 3.4. Mode cleaner mirrors (MC)

Displacement noise of the mode cleaner mirrors is set to be consistent with the MC frequency stability requirement given in T010075-00-D, *Advanced LIGO Systems Design*: $3\times10^{-3}(10\text{Hz}/f)\ \text{Hz}/\sqrt{\text{Hz}}$ from 10-1000 Hz. The noise level given in Figure 1 for a MC mirror corresponds to a frequency stability of $1.5\times10^{-3}(10\text{Hz}/f)^2\ \text{Hz}/\sqrt{\text{Hz}}$. Radiation pressure fluctuations will limit the frequency stability at a similar level.

The displacement requirement holds in the presence of a vertical-horizontal coupling as limited by the suspension design (the mode cleaner optic axis can be leveled to $10^{-4}$ or better).

## 3.5. Input Telescope Mirrors

Motion of the input mode matching telescope mirrors creates frequency fluctuations of the input light via doppler shifts. The motion must be small enough that the frequency fluctuations do not compromise the frequency stability demanded of the mode cleaner. Demanding that the root-square-sum of 5 telescope optics (2 steering mirrors and 3 figured mirrors) produce frequency fluctuations no greater than 1/3 of the MC frequency stability requirement gives a displacement noise limit of:

$$x \le 7\times10^{-12}(10\text{Hz}/f)^2\ \text{m}/\sqrt{\text{Hz}}$$

Note that this is already larger than the motion of the seismic (SEI) platform.