Search for Gravitational Radiation from Known Periodic Sources in October 1999 40-Meter Data

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As interferometric detectors become increasingly sensitive over the next few years, it is likely that they may provide direct evidence of the periodic gravitational radiation believed to be emitted by asymmetric spinning neutron stars. Such radiation is a likely candidate for early detection for many reasons, including the relative simplicity of its form and the fact that it does not originate from a single, isolated cosmic event. Additionally, the parameter space associated with directed searches for periodic radiation from known pulsars is considerably smaller than that for more general searches, making the former more accessible given present computing power.

Despite the belief that detections are not yet possible, it is nonetheless desirable at the present time to use available data to set appropriate upper limits. The present paper endeavors to accomplish this by performing an analysis of data obtained from the 40 meter LIGO prototype interferometer at the California Institute of Technology and subsequently performing a Monte Carlo study to determine its effectiveness. The analysis presented here consists of two primary stages, a preprocessing stage in which environmental correlations are estimated and removed in order to increase the signal-to-noise ratio, and a search stage consisting of an algorithmic procedure for detecting evidence of periodic gravitational wave signals due to fixed pulsars within the Differential Mode Readout (DMRO) channel. The details of each stage are discussed below in sections 1 and 2. It is hoped that, within the next few years, this analysis may be applied to data taken from the larger LIGO interferometers, at which point the likelihood of actual signal detection will be greater. For this reason, the software written to perform the analysis presented here has been collected into a cohesive package and made publicly available [3].

1 Increasing the Signal-to-Noise Ratio

Due to the high sensitivity required to detect gravitational wave signals, it is critical to make every effort to reduce the signal-to-noise ratio within the DMRO channel. One method of doing so, due to Allen, *et al* [1] attempts to use data contained within the environmental monitor channels to estimate and remove environmental contaminations. A description of this method is not repeated here, but the assumptions required for the method's validity are stated for clarity. There are two such assumptions, namely that all environmental contaminations are linear in nature and that the linear transfer function connecting the environmental channels to the DMRO channel are slowly varying over frequency bands of lengths on the order of 1

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Hz[2]. While these assumptions are not true in general, the first two are valid for smoothly varying linear contaminations. By contrast, spurious correlations in narrow frequency bands and nonlinear contaminations are not addressed.

With these assumptions, the frequency-space representation of the linear transfer functions connecting environmental monitor channels and the DMRO channel are taken to be constant on small frequency bands. The values of these transfer functions may then be estimated as those values that minimize the power remaining in the DMRO channel following contamination removal. In practice, this technique must be altered slightly in order to prevent the removal of so-called "false correlations." A thorough analysis of methods by which this may be accomplished is contained within the paper of Allen, *et al* [1], while the precise method utilized in the present analysis is described in the user's manual that accompanies the above-mentioned software package [3].

Though the validity of the above procedure seems plausible, the degree to which the signal-to-noise ratio is decreased, as well as the impact of the procedure upon the phase of an actual signal, must be determined through a Monte Carlo analysis. Such an analysis, which studies the effectiveness of this procedure coupled with the search algorithm described below, is described in section 3.

2 An Algorithm for Periodic Source Searches

Despite the relatively simple form of a periodic gravitational wave in the source's frame of reference, it is well-known that the response of an interferometer to such a wave is complicated by the relative motion of the earth and source. In particular, one must cope not only with the familiar Doppler modulation of frequency, but also with modulation of the detector's sensitivity to the wave, and hence apparent amplitude of the detected signal, as its orientation with respect to the source changes in time. One method of analysis, which takes both of these forms of modulation into account, is presented here[3].

2.1 Accounting for Frequency Modulation

The most natural setting for an algorithm designed to search for periodic signals is Fourier space, and hence the Discrete Fourier Transform, or DFT, plays a critical role in the following analysis. If it is suitable to assume that both forms of modulation discussed above are sufficiently slow that there exist time intervals over which the amplitude and frequency of a detected signal are nearly constant, a time-frequency analysis is permitted. In particular, the period of time from which data has been taken may be broken into intervals of the appropriate size and individually DFT'ed. From this sequence of DFT's, a time series may be constructed by taking a single coefficient, corresponding to the bin in which the signal frequency is expected to be found (this is hereafter referred to as the "signal bin"), from each DFT. Such a time series, referred to as the "signal series," would essentially demonstrate the time evolution of the signal and any spurious noise contributions of similar frequencies. Similar time series', termed "neighboring series'," may also be constructed by taking DFT coefficients whose bin indices differ from those of the signal bins by a fixed amount. At this point, one method of approach would be to compare the mean power of the signal series to the rms of the mean powers of neighboring series'. While this would succeed in detecting a relatively strong frequency-modulated signal, this method fails to account for amplitude modulation and, additionally, discards the phase information encoded within the DFT. Thus, a few modifications need to be made.

[2] Actually, the length of the frequency bands over which this assumption is made is a parameter of the method. In the current paper, however, the assumption was made for bands of length ~ 1 Hz.

[3] In order to account for frequency and amplitude modulation, the algorithm of the present paper requires a determination of their forms given a target pulsar in the sky. The Tempo timing code [4] was used to compute the frequency modulation, while the method of Anderson, *et al* [2] was used to compute amplitude modulation.

The first modification considered is one which incorporates the DFT coefficient phases into the previously-described method. Naively, an appropriate approach appears to be a computation of the normalized vector sum of the DFT coefficients within the signal and neighboring series', rather than a computation of the mean power. In this manner, non-alignment of the coefficients gives rise to cancellation that decreases the noise level. In order to utilize such a method, though, the DFT coefficients due to the signal must align exactly. Unfortunately, this does not occur automatically, as coefficients from successive DFT's are phase shifted relative to one another due to the fact that they are computed with respect to different reference, or "starting", times. These phase shifts may be computed, though, and compensated in order to assure signal coefficient alignment. To do this, the DFT of an artifically constructed pure signal of unit amplitude displaying the expected frequency modulation[4] is computed. [5]. Within a given time interval, the magnitude, R, of the articifical signal DFT and the phase shift, Ψ, relative to the first interval are determined. Once this is accomplished, the entire DMRO channel DFT within the current interval is rescaled through multiplication by $e^{-i\Psi}/R$. The exponential factor compensates the shift in phase of the signal bin coefficient so that the signal bin coefficients align perfectly in the absence of noise. The $1/R$ factor is a rescaling of the DFT which accounts for leakage from the signal bin into neighboring bins caused by the discrete nature of the DFT. Inclusion of this factor ensures that, in the absence of noise, the signal bin magnitude reflects the magnitude of the input signal. Since it is applied globally, it has no effect on the signal-to-noise ratio within a given DFT.

Following this rescaling of the DFT, the vector sum procedure described above may be applied. Components of DFT coefficients due to an astrophysical signal from a specified target will add constructively while, in general, noise contributions will undergo some cancellations. This will result in an overall increase of the signal-to-noise ratio, enabling the detection of increasingly feeble signals.

Before closing this section, it is necessary to briefly discuss a subtle issue regarding the above procedure. While the artifically generated pure signal used to determine phase shifts resembles a real signal in many ways, the overall phases of the two may not agree. This may cause a problem since the phase of a DFT does not depend linearly on the phase of the original signal. To see this explicitly, consider a signal of the form $x_n = \cos(2\pi fn + \phi)$ and break it up into "left-moving" and "right-moving" components:

$$x_n = \frac{1}{2}\left(e^{i(2\pi fn+\phi)} + e^{-i(2\pi fn+\phi)}\right)$$

The DFT, $\tilde{x}_L$, of a left-moving complex wave of the form $e^{2\pi ifn}$ is given by:

$$\left(\tilde{x}_L\right)_m = \sum_{n=0}^{N-1} e^{i(2\pi n(m+f))}e^{i\phi}$$

Though this sum is easily computed, at present we simply write it in the form $A_m e^{i(\alpha_m+\phi)}$, where A and α are both real.

Similarly, the DFT, $\tilde{x}_R$, of a right-moving complex wave of the form $e^{-2\pi ifn}$ is given by:

$$\left(\tilde{x}_R\right)_m = \sum_{n=0}^{N-1} e^{i(2\pi n(m-f))}e^{-i\phi}$$

[4]The expected frequency modulation is determined with the TEMPO timing code [4].

[5]One might think that since the apparent phase shift caused by a shift in starting times is easy to compute (being simply ωT), proceeding with a more complicated procedure utilizing the DFT of an artifically constructed pure signal is not necessary. However, if the pure signal is not used, it is possible that accumulated phase stemming from the frequency modulation within a given interval becomes sufficiently large to destroy the effectiveness of the subsequent analysis.

We write this sum as $B_m e^{i(\beta_m - \phi)}$ with B and β real. With this notation, the DFT of the signal becomes:

$$\tilde{x}_m = e^{i\phi} A_m e^{i\alpha_m} + e^{-i\phi} B_m e^{i\beta_m}$$

From this expression, it is obvious that a shift in ϕ by an amount δ does not, in general, shift the complex phase of the DFT by δ[6]. Rather, the shift in the DFT phase may be determined to be:

$$\Delta\phi = \delta + \arctan\left(\frac{-B^2\sin(2\delta) + AB\left(\sin(2\phi+\alpha-\beta) - \sin(2\phi+\alpha-\beta+2\delta)\right)}{A^2 + B^2\cos(2\delta) + AB\left(\cos(2\phi+\alpha-\beta) + \cos(2\phi+\alpha-\beta+2\delta)\right)}\right)$$

In frequency bins such that $m \sim f$, A will be much larger than B in general. If we expand the above result in B/A, we find that the leading behavior of the additional term is B/A, which is negligible in our current analysis.

2.2 Accounting for Amplitude Modulation

If the interferometer response to an incident gravitational wave signal displayed no amplitude modulation, then the above procedure could be applied without difficulty. The phase shifts would ensure that the signal bin DFT coefficients corresponding to the target pulsar add constructively while destructive interference decreases the noise in neighboring bins. With the rescaling, the average of the signal bin would yield a good estimate of the gravitational wave strength, and the rms in the neighboring bins a good estimate of the associated uncertainty. Unfortunately, the situation is not this simple, so the above method must be modified even further. The first step, though, namely the multiplication by $e^{-i\Psi}/R$, remains unchanged. It is merely necessary to replace the process of adding DFT coefficients from successive intervals with something more sophisticated that takes amplitude modulation into account.

Consider the vector $\vec{x}$, whose ith component is the DFT coefficient of the signal bin within the ith time interval. Now, let $\vec{F}^{+}$ and $\vec{F}^{\times}$ denote the N-dimensional vectors whose ith components contain the true sensitivities of the interferometer to a unit plus-polarized gravitational wave and a unit cross-polarized gravitational wave, respectively, at the median time of the ith time interval[7]. Since the sensitivity is assumed to be relatively constant within any given time interval, a gravitational wave of the form $h_{+}\hat{e}^{(+)} + h_{\times}e^{i\phi}\hat{e}^{(\times)}$, where h_{+} and $h_{\times}$ are real and ϕ denotes the phase difference between the plus and cross polarization components, gives rise to the DFT coefficient $x_i = h_{+}F_i^{+} + h_{\times}F_i^{\times}e^{i\phi}$, within the ith time interval, following application of the above procedure[8]. The reason for this is that the rescaling maintains the amplitudes h_{+} and $h_{\times}$, while the phase shifting assures that there is no phase difference between DFT coefficients from different time intervals, x_i and x_j. Equivalently, this rescaling and phase shifting assures that the plus- and cross-polarized components of x_i have the same amplitude and phase as the respective components of x_j. This only holds in the ideal case, though, in the absence of noise. To treat the actual case, let $\vec{g}$ denote an N-dimensional complex vector whose ith component, written as $g_j \exp(i\psi_j)$ with g_j and ψ_j both real, represents the DFT of the data channel less that of a pure signal. It is assumed that the noise represented by $\vec{g}$, and hence g_j and ψ_j, is random in nature. Including the effects of this noise, the vector $\vec{x}$ properly takes the following form:

[6] The frequency almost never sits at a bin boundary so A and B are nonvanishing.

[7] The sensitivities are computed via the method of Anderson, *et al* [2]

[8] Actually, this expression is correct only up to an arbitrary phase, determined by the starting time of the first interval, which is fixed for all subsequent intervals. Since this constant phase simply multiplies the entire response vector, it may be ignored.

$$x_j = h_+ F_j^+ + h_\times F_j^\times e^{i\phi} + g_j e^{i\psi_j}$$

To estimate h_+ given $\vec{x}$, $\vec{F}^+$, and $\vec{F}^\times$, first look at the weighted sum, $x_j \vec{F}_j^+$, where values of x_j originating in time intervals of relatively high sensitivity to plus polarized gravitational waves are given greater emphasis. The result of this weighted sum is the following, where summation over repeated indices is implied:

$$x_j F_j^+ = h_+ \left(F_j^+ F_j^+\right) + h_\times \left(F_j^\times F_j^+\right) e^{i\phi} + g_j e^{i\psi_j} F_j^+$$

The norm of $\vec{F}^{(+)}$ is much greater than its projection onto $\vec{F}^{(\times)}$ so, provided $h_+ F^+$ is larger than g_j, which is required for a signal detection, x_j is dominated by the $h_+ \left(F_j^+ F_j^+\right)$ term. For this reason, the estimated value of h_+, termed $h_+^?$, is taken to be the magnitude of the complex number $x_j \vec{F}^+ / |\vec{F}^+|^2$. In terms of the actual values, this implies that the estimate, $\left(h_+^?\right)^2$, is given by:

$$\begin{aligned} \left(h_+^?\right)^2 &= h_+^2 + h_\times^2 \frac{\left(F_j^\times F_j^+\right)^2}{\left(F_k^+ F_k^+\right)^2} + \frac{\left(g_j F_j^+\right)^2}{\left(F_k^+ F_k^+\right)^2} + 2\frac{\left(F_j^\times F_j^+\right)}{\left(F_k^+ F_k^+\right)} h_+ h_\times \cos\phi \\ &+2\frac{F_j^+ g_j \cos\psi_j}{\left(F_k^+ F_k^+\right)} h_+ + 2\frac{\left(F_j^\times F_j^+\right)}{\left(F_k^+ F_k^+\right)^2} F_r^+ g_r \cos\left(\phi - \psi_r\right) h_\times \end{aligned}$$

Since ψ_j is a random phase and F_j^+ is a smoothly varying function of time, one might expect the fluctuations of $\cos\psi_j$ and $\cos\left(\phi - \psi_j\right)$ to cause $F_j^+ g_j \cos\psi_j$ and $F_r^+ g_r \cos\left(\phi - \psi_r\right)$ to become negligibly small if sufficiently many intervals are considered. One must be careful, though, because these terms are only first order in the noise factors, g_r, while the only other noise-dependent term is of second order in the g_r. For this reason, all terms are kept to yield a conservative estimate of the error.

The above expression may be rewritten as follows, where $(,)$ is the standard Euclidean inner product:

$$\begin{aligned} \left(h_+^?\right)^2 &= h_+^2 + \frac{\left(\vec{g}, \vec{F}^+\right)^2}{\left(\vec{F}^+, \vec{F}^+\right)^2} + h_\times^2 \frac{\left(\vec{F}^\times, \vec{F}^+\right)^2}{\left(\vec{F}^+, \vec{F}^+\right)^2} + 2\frac{\left(\vec{F}^\times, \vec{F}^+\right)}{\left(\vec{F}^+, \vec{F}^+\right)} h_+ h_\times \cos\phi \\ &+2\frac{\left(\vec{F}^+, \vec{g}\right)}{\left(\vec{F}^+, \vec{F}^+\right)} h_+ \cos\psi_j + 2\frac{\left(\vec{F}^\times, \vec{F}^+\right)\left(\vec{F}^+, \vec{g}\right)}{\left(\vec{F}^+, \vec{F}^+\right)^2} h_\times \cos\left(\phi - \psi_j\right) \end{aligned}$$

Thus, in the limit in which many time intervals are considered, the estimated value of $\left(h_+^?\right)^2$ differs from the actual h_+^2 by an amount $\delta\left[\left(h_+^?\right)^2\right]$, given by the following:

$$\begin{aligned} \delta\left[\left(h_+^?\right)^2\right] &= \frac{\left(\vec{g}, \vec{F}^+\right)^2}{\left(\vec{F}^+, \vec{F}^+\right)^2} + h_\times^2 \frac{\left(\vec{F}^\times, \vec{F}^+\right)^2}{\left(\vec{F}^+, \vec{F}^+\right)^2} + 2\frac{\left(\vec{F}^\times, \vec{F}^+\right)}{\left(\vec{F}^+, \vec{F}^+\right)} h_+ h_\times \cos\phi \\ &+2\frac{\left(\vec{F}^+, \vec{g}\right)}{\left(\vec{F}^+, \vec{F}^+\right)} h_+ \cos\psi_j + 2\frac{\left(\vec{F}^\times, \vec{F}^+\right)\left(\vec{F}^+, \vec{g}\right)}{\left(\vec{F}^+, \vec{F}^+\right)^2} h_\times \cos\left(\phi - \psi_j\right) \end{aligned}$$

The terms involving the overlap of $\vec{F}^{+}$ and $\vec{F}^{-}$ would be zero if the sensitivity functions due to the two independent polarizations were orthogonal, but this is not the case in general. This is a manifestation of the fact that while the two independent polarization states are orthogonal to one another, the components which are detected by the interferometer, which acts as a polarization filter, generally are not.

It is possible to determine $\vec{F}^{+}$ and $\vec{F}^{\times}$ with the algorithm of Anderson, *et al* [2], while an estimation of the noise is obtained by performing the same procedure as that applied to the signal bin, namely phase shifting, rescaling, projecting onto $\vec{F}^{+}$, and dividing by the norm squared of $\vec{F}^{+}$, on the neighboring frequency bins. Very near the signal bin, the neighboring bins will contain signal leakage, but if a sufficiently large neighborhood is used, then the rms value of $\left(\vec{x},\vec{F}^{+}\right)^{2} / \left(\vec{F}^{+},\vec{F}^{+}\right)^{2}$ within that neighborhood will provide a good estimate for the g_r appearing in $\delta\left[\left(h_{+}^{?}\right)^{2}\right]$. Unfortunately, the true values of h_{+} and $h_{\times}$, as well as the phase difference, ϕ, between the plus and cross polarized components, are generally unknown. Thus, the following procedure is used for determining the error in $\left(h_{+}^{?}\right)^{2}$. Presumably the h_{+}^{2} estimate will be largely due to the existence of a plus polarized gravitational wave, but in addition it will be artifically increased by the presence of a cross polarized wave. Similarly, the $h_{\times}^{2}$ estimate will be inflated if a plus polarized component exists. Because of this, the estimated values of h_{+}^{2} and $h_{\times}^{2}$ are used in the expression for $\delta\left[\left(h_{+}^{?}\right)^{2}\right]$ since, being larger than the actual values, they will produce an overestimate, rather than an underestimate, of the error. In addition, the trigonometric factors are all set to one to obtain a conservative estimate. Once the expression for $\delta\left[\left(h_{+}^{?}\right)^{2}\right]$ is obtained, standard error propagation techniques are used to estimate $\delta h_{+}^{?}$. The preceding algorithm may also be applied for $h_{\times}$. The resulting expressions for both h_{+} and $h_{\times}$ are summarized below for clarity:

$$\left(h_{+}^{?}\right)^{2} = \frac{\left(\vec{x},\vec{F}^{+}\right)}{\left(\vec{F}^{+},\vec{F}^{+}\right)^{2}} \qquad \left(h_{\times}^{?}\right)^{2} = \frac{\left(\vec{x},\vec{F}^{\times}\right)}{\left(\vec{F}^{\times},\vec{F}^{\times}+\right)^{2}}$$

$$\begin{aligned}\delta\left[\left(h_{+}^{?}\right)^{2}\right] &= \frac{\left(\vec{g},\vec{F}^{+}\right)^{2}}{\left(\vec{F}^{+},\vec{F}^{+}\right)^{2}} + h_{\times}^{2}\frac{\left(\vec{F}^{\times},\vec{F}^{+}\right)^{2}}{\left(\vec{F}^{+},\vec{F}^{+}\right)^{2}} + 2\frac{\left(\vec{F}^{\times},\vec{F}^{+}\right)}{\left(\vec{F}^{+},\vec{F}^{+}\right)} h_{+}h_{\times}\cos\phi \\ &\quad +2\frac{\left(\vec{F}^{+},\vec{g}\right)}{\left(\vec{F}^{+},\vec{F}^{+}\right)} h_{+}\cos\psi_j + 2\frac{\left(\vec{F}^{\times},\vec{F}^{+}\right)\left(\vec{F}^{+},\vec{g}\right)}{\left(\vec{F}^{+},\vec{F}^{+}\right)^{2}} h_{\times}\cos\left(\phi-\psi_j\right)\end{aligned}$$

$$\begin{aligned}\delta\left[\left(h_{\times}^{?}\right)^{2}\right] &= \frac{\left(\vec{g},\vec{F}^{\times}\right)^{2}}{\left(\vec{F}^{\times},\vec{F}^{\times}\right)^{2}} + h_{+}^{2}\frac{\left(\vec{F}^{+},\vec{F}^{\times}\right)^{2}}{\left(\vec{F}^{\times},\vec{F}^{\times}\right)^{2}} + 2\frac{\left(\vec{F}^{+},\vec{F}^{\times}\right)}{\left(\vec{F}^{\times},\vec{F}^{\times}\right)} h_{\times}h_{+}\cos\phi \\ &\quad +2\frac{\left(\vec{F}^{\times},\vec{g}\right)}{\left(\vec{F}^{\times},\vec{F}^{\times}\right)} h_{\times}\cos\psi_j + 2\frac{\left(\vec{F}^{+},\vec{F}^{\times}\right)\left(\vec{F}^{\times},\vec{g}\right)}{\left(\vec{F}^{\times},\vec{F}^{\times}\right)^{2}} h_{+}\cos\left(\phi-\psi_j\right)\end{aligned}$$

3 Monte Carlo Analysis

References

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