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Sideband Requirements in Advanced LIGO
Part I

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Intensity and phase noise on each sideband can mimic a gravitational wave signal. The subject of this note is to calculate the requirements for the phase and intensity stability in the input beam. These requirements depend strongly on the sensing scheme and the bandwidths of the various feedback loops.

1 Intensity Noise in Sidebands

Changes in the modulation index will change the intensity of the sidebands and of the carrier. The requirements for the relative intensity noise for the two different sensing schemes are given in [1]. Below 100 Hz the requirements are driven by balancing of the power in the two arm cavities and the subsequent sensitivity to technical radiation pressure noise. With a requirement on the balancing of the average arm power of 1%, the relative intensity noise has to be below $2 \cdot 10^{-9}/\sqrt{\mathrm{Hz}}$ at 10 Hz and $2 \cdot 10^{-8}/\sqrt{\mathrm{Hz}}$ at 100 Hz. Above 100 Hz the requierements are different for the two sensing schemes. For RF-sensing, the requirements above 100 Hz stay between $2-4 \cdot 10^{-8}/\sqrt{\mathrm{Hz}}$ up to 10 kHz. For DC-sensing, the requirements continue to roll off roughly with frequency f. This and the fact that the intensity stabilization feedback loop will measure the intensity of all frequency components and not only of the carrier can be used to establish requirements on the modulation index.

1.1 DC-sensing

The electric field behind the phase modulator can be written as:

$$E = E_o \left(J_0(m) + J_1(m)(e^{i\Omega t} - e^{-i\Omega t}) \right)$$

The total intensity I_o will be stabilized with a feedback loop onto a signal measured behind the mode cleaner. Changes in the modulation index will not alter the total intensity but the intensity in each frequency component. The intensity in the carrier is

$$I_c = I_o J_0^2(m)$$

The intensity in the carrier changes with the modulation index:

$$I_c = I_o J_0^2(m) + I_o \frac{\partial J_0^2(m)}{\partial m} \delta m$$

with

$$\frac{\partial J_0^2(m)}{\partial m} = -2 J_0(m) J_1(m)$$

The relative intensity noise in the carrier is therefor:

$$RIN_c = 2 \frac{J_1(m)}{J_0(m)} \delta m \approx 2m\delta m$$

The requirements on the intensity noise for DC-sensing and below 100Hz for RF-sensing are driven by requirements based on the carrier. The required stability of the modulation index is therefor:

$$\delta m < \frac{10^{-9}}{m\sqrt{\mathrm{Hz}}} \frac{f}{[10\,\mathrm{Hz}]}$$

1.2 RF-sensing

The intensity noise requirements below 100 Hz are independent from the sensing scheme and do not impose different requirements on the stability of the modulation index. Above 100 Hz changes in the intensity of the sidebands dominate the sensitivity of the GW-signal for intensity fluctuations. The intensity of the sidebands is

$$I_{SB} = I_o J_1^2(m)$$

The intensity of the sideband will change with the modulation index:

$$I_{SB} = I_o J_1^2(m) + I_o 2 J_1(m) \frac{\partial J_1(m)}{\partial m} \delta m$$

The derivative of the first order Bessel function is:

$$\frac{\partial J_1(m)}{\partial m} = J_0(m) - \frac{J_1(m)}{m} \approx \frac{1}{2}$$

The relative intensity noise is defined as intensity change over average intensity. Therefor the requirement has to scale with the total intensity:

$$\frac{\delta I_{SB}}{I_o} \approx J_1(m)\delta m$$

This analysis focused only on one sideband, mainly because only one sideband will transmit effectivly through the interferometer, while its counterpart is not really resonant. It will be suppressed, but the suppression factor depends on the final tuning and finesse of the SR-cavity. If we assume no suppression, we end up with:

$$\delta m < \frac{10^{-9}}{m\sqrt{\mathrm{Hz}}} \frac{f}{[10\,\mathrm{Hz}]} \qquad < 100\,\mathrm{Hz}$$

$$\delta m < \frac{10^{-8}}{m\sqrt{\mathrm{Hz}}} \qquad > 100\,\mathrm{Hz}$$

2 Phase Noise in Sidebands

2.1 Theory

The second possible noise source is the remaining phase noise in the modulation frequency:

$$E = E_o \left[J_0(m) + J_1(m) \left(e^{i(\Omega t + \beta(t))} - e^{-i(\Omega t + \beta(t))} \right) \right]$$

$$\beta(t) = \int \tilde{\beta}(f) e^{i2\pi f t} df$$

In a simple sideband picture for one particular noise frequency f we can write this as:

$$E = E_o e^{i\omega_o t} e^{im\cos\left[\Omega t + \frac{\delta\nu}{2\pi f} sin(2\pi f t)\right]}$$

$$E = E_o e^{i\omega_o t} \cdot \left[1 + \frac{im}{2} \left(e^{i\Omega t} + e^{-i\Omega t} \right) \right]$$

$$+ E_o e^{i\omega_o t} \cdot \left[\frac{im\delta\nu}{8\pi f} \left(e^{i(\Omega + 2\pi f)t} - e^{i(\Omega - 2\pi f)t} - e^{-i(\Omega - 2\pi f)t} + e^{-i(\Omega + 2\pi f)t} \right) \right]$$

where $\delta\nu$ is the amplitude of the frequency noise. The noise sidebands will beat with the carrier at the dark port and can mimic a signal. In addition, the phase noise sidebands will also show up at the local oscillator at the electronic mixer (see below).
The electric field at the final photodetector can be written as:

$$E_{pd} = E_o e^{i\omega_o t} \cdot \left[t_c + \frac{im}{2} \left(t_+ e^{i\Omega t} + t_- e^{-i\Omega t} \right) \right]$$

$$+ E_o e^{i\omega_o t} \cdot \left[\frac{im\delta\nu}{8\pi f} \left(t_{++} e^{i(\Omega + 2\pi f)t} - t_{+-} e^{i(\Omega - 2\pi f)t} - t_{-+} e^{-i(\Omega - 2\pi f)t} + t_{--} e^{-i(\Omega + 2\pi f)t} \right) \right]$$

where we used the frequency dependent transfer functions of the interferometer.
This generates a photocurrent that will be demodulated by the oscillator signal with the same phase noise:

$$LO \propto \cos\Omega(t - \tau) + \frac{\delta\nu}{4\pi f} \left[\cos\left(\Omega + 2\pi f\right)(t - \tau) - \cos\left(\Omega - 2\pi f\right)(t - \tau) \right]$$

τ is the time delay between LO and photodiode signal and defines the demodulation phase. The demodulated signal is:

$$S = I_o \frac{\delta\nu}{8\pi f} \left[\left(2\Im\left\{ t_c \left(t_+ + t_- \right) \right\} \cos 2\pi f\tau - \Im\left\{ t_c \left(t_{++} + t_{--} + t_{+-} + t_{-+} \right) \right\} \right) \sin 2\pi f t \sin\Omega\tau \right.$$
$$- \left(2\Im\left\{ t_c \left(t_+ + t_- \right) \right\} \sin 2\pi f\tau + \Re\left\{ t_c^* \left(t_{++} - t_{--} + t_{-+} - t_{+-} \right) \right\} \right) \cos 2\pi f t \sin\Omega\tau$$
$$+ \left(2\Re\left\{ t_c^* \left(t_+ - t_- \right) \right\} \cos 2\pi f\tau + \Re\left\{ t_c^* \left(t_{--} + t_{-+} - t_{++} - t_{+-} \right) \right\} \right) \sin 2\pi f t \cos\Omega\tau$$
$$\left. - \left(2\Re\left\{ t_c^* \left(t_+ - t_- \right) \right\} \sin 2\pi f\tau - \Im\left\{ t_c \left(t_{++} + t_{--} - t_{-+} - t_{+-} \right) \right\} \right) \cos 2\pi f t \cos\Omega\tau \right]$$

The demodulation phase $\Omega\tau$ is not yet defined. Most likely we will demodulate both quadratures and reconstruct the signal anyway. The signal has the form:

$$S(f) \propto [X_1 \sin 2\pi ft + X_2 \cos 2\pi ft] \sin\Omega\tau + [Y_1 \sin 2\pi ft + Y_2 \cos 2\pi ft] \cos\Omega\tau$$

The noise spectral density in each quadrature component can be written as:

$$I(f) = \sqrt{\frac{X_1^2 + X_2^2}{2}} \qquad Q(f) = \sqrt{\frac{Y_1^2 + Y_2^2}{2}}$$

The transfer function from the input to the output is:

$$t(f) = \frac{t_p t_{MI} t_s e^{i(\phi_p+\phi_s)/2}}{1 + r_p r_f e^{i\phi_p} - r_s r_b e^{i\phi_s} - r_p r_s \left(r_f r_b - t_{MI}^2\right) e^{i(\phi_p+\phi_s)}}$$

where

$$t_{MI} = t_{bs} r_{bs} \left(r_{cav1} e^{-i\phi_{MI}/2} - r_{cav2} e^{i\phi_{MI}/2}\right)$$
$$r_f = r_{bs}^2 r_{cav1} e^{-i\phi_{MI}/2} + t_{bs}^2 r_{cav2} e^{i\phi_{MI}/2}$$
$$r_b = t_{bs}^2 r_{cav1} e^{-i\phi_{MI}/2} + r_{bs}^2 r_{cav2} e^{i\phi_{MI}/2}$$

are the transmissivity, the front reflectivity, and the back reflectivity of the cavity enhanced MI.

The cavity reflectivities are:

$$r_{cav} = \frac{r_{ITM} - r_{ETM}\left(r_{ITM}^2 + t_{ITM}^2\right) e^{i\phi_c}}{1 - r_{ITM} r_{ETM} e^{i\phi_c}}$$

The phases are:

$$\phi_{MI}: \quad \text{Michelson differential}$$
$$\phi_p: \quad \text{PR -- Cavity}$$
$$\phi_s: \quad \text{SR -- Cavity}$$
$$\phi_c: \quad \text{Cavity Phase}$$

2.2 Results

The above set of equations was used to calculate the power transfer function for frequency noise in the oscillator:

$$P_{DP}(f) = T(f)\delta\nu(f)$$

The maximum allowed frequency noise was then calculated by comparing this with the maximum allowed technical power fluctuations $P_{max}^{RF}(f)$ in the dark port (see [2]):

$$\delta\nu_{max}(f) = \frac{P_{max}^{RF}(f)}{T(f)}$$

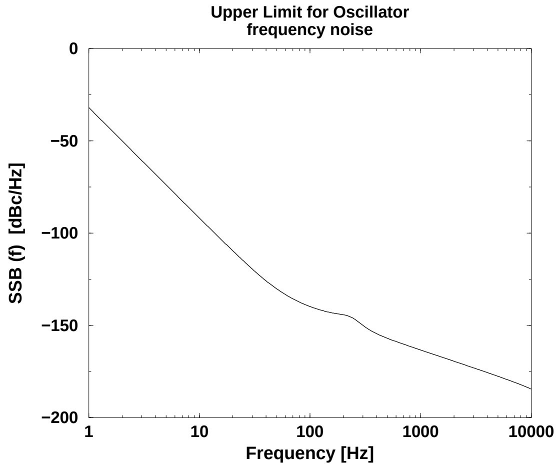


Figure 1: Oscillator Phase Noise requirements for the standard set of parameters for Advanced LIGO.

This was then transfered into the single sideband oscillator noise intensity:

$$SSB_{max}(f) = 20 \cdot Log_{10}\left(\frac{\delta\nu_{max}(f)}{4\pi f}\right)$$

for easier comparison with commercially available oscillators.

The sensitivity of the Advanced LIGO detector to oscillator phase noise depends strongly on the asymmetries. The calculated requirements are therefore only as good as the assumptions that were used to calculate them. In this document I present first the requirements as they evolve for the basic set of parameters that I used for all other requirements presented in the IO-designs. In a second chapter I will try to specify the asymmetries with which oscillator phase noise scales to give a more complete view on the problem.

2.2.1 Advanced LIGO Baseline

The parameters for the optical components can be found in [2]. The single sideband noise requirements are shown in Fig. 1. The requirements at certain frequencies are:

$$I_{SSB}(10\,\mathrm{Hz}) < -92\,\frac{\mathrm{dBc}}{\mathrm{Hz}} \qquad I_{SSB}(100\,\mathrm{Hz}) < -140\,\frac{\mathrm{dBc}}{\mathrm{Hz}} \qquad I_{SSB}(1\,\mathrm{kHz}) < -163\,\frac{\mathrm{dBc}}{\mathrm{Hz}}$$

For comparison: Stanford Research PRS10 10 MHz Rubidium frequency standard has the following specs:

$$I_{SSB}(10\,\mathrm{Hz}) < -130\,\frac{\mathrm{dBc}}{\mathrm{Hz}} \qquad I_{SSB}(100\,\mathrm{Hz}) < -140\,\frac{\mathrm{dBc}}{\mathrm{Hz}}$$

If we simply transfer the 10 MHz to 180 MHz by harmonic generation, we also transfer the noise sidebands from x Hz to 18x Hz. In other words, the requirement of -

145 dBc/Hz at 180 Hz in the modulation frequency transforms into a required -145 dBc/Hz at 10 Hz in the 10 MHz frequency standard. Thats a problem.

2.2.2 Specific Asymmetries

The result derived above can only be used as an example as the parameters are only one example of an infinte set of possible parameters. The sensitivity to oscillator noise sidebands scales with the carrier field at the dark port. The parameters that change the carrier at the dark port are losses and differential detunings. The requirements on these parameters that need to be met that the requirements on oscillator phase noise are valid are:

$$\Phi_- < 10^{-7}\text{rad} \qquad \phi_- < 10^{-4}\text{rad}$$

$$\Delta T_{ITM} < 40\% \Rightarrow T_{ITM1} = (0.5-0.1)\%, T_{ITM2} = (0.5+0.1)\%$$

$$\Delta Losses = 15\,\text{ppm} \qquad \text{in ETMs and ITMs}$$

The critical values in the above example were the difference in the losses in the test masses. The requirements on oscillator phase noise are relatively insensitive to the transmission of the ITMs as long as the losses are equal. The ratio between the differential detunings Φ_- and ϕ_- is caused by the phase gain in the arm cavities.

$$\Phi_- = 10^{-7}\text{rad} \Rightarrow \Delta L_- \approx 10^{-14}\text{m} \Leftrightarrow \delta\nu \approx .6\,\text{mHz}$$

2.3 FSR-Detuning

Over the course of the run, the arm cavities would change their lengths due to tidal effects and thermal drifts. Either we let the cavities drift and let the frequency of the laser and the lengths of the mode cleaner track that drift or we have to use actuators on the suspension systems of the arm cavity optics to take out the drift and lock the common arm cavity lengths to another frequency standard. If we assume that we will change the length of the mode cleaner to track common arm cavity motions we will also change its free spectral range and offset the RF-sidebands from beeing exactly on resonance. This change will increase the transformation of oscillator phase noise into amplitude noise in the sidebands. The limits for amplitude noise are:

$$\delta m(f) < \frac{10^{-9}}{m\sqrt{\text{Hz}}}\frac{f}{[10\,\text{Hz}]} \qquad \text{for DC sensing}$$

$$\delta m(f) < \sqrt{\left(\frac{10^{-9}}{m\sqrt{\text{Hz}}}\frac{f}{[10\,\text{Hz}]}\right)^2 + \left(\frac{10^{-8}}{m\sqrt{\text{Hz}}}\right)^2} \qquad \text{for RF sensing}$$

Note that the amplitudes of the noise sidebands are half of the variation in the modulation index:

$$a_{AM}(f) < \frac{\delta m(f)}{4}$$

The limits on the relative amplitude of the phase noise sidebands are:

$$a_{PM}(f) < \sqrt{\left(\frac{0.05}{f^3}\right)^2 + \left(\frac{10^{-5}}{f}\right)^2}\frac{1}{\sqrt{\mathrm{Hz}}}$$

A detuned mode cleaner changes phase modulation partly into amplitude modulation with a conversion factor given by the detuning divided by the transmissivity T of the MC mirrors:

$$a_{AM\leftarrow PM}(f) = \frac{2\phi_{RT}}{T}a_{PM}(f)$$

where ϕ_{RT} is the round trip phase shift of the RF-sideband in the mode cleaner:

$$\phi_{RT} = 2\pi\frac{\Delta\nu}{FSR} = 2\pi\frac{\Delta\nu}{[9\,\mathrm{MHz}]}$$

Using the condition that the AM sidebands caused by PM in a detuned MC should stay below the allowed AM sidebands based on the intensity noise requirements ($a_{AM\leftarrow PM} < a_{AM}$), the maximal allowed phase shift is then:

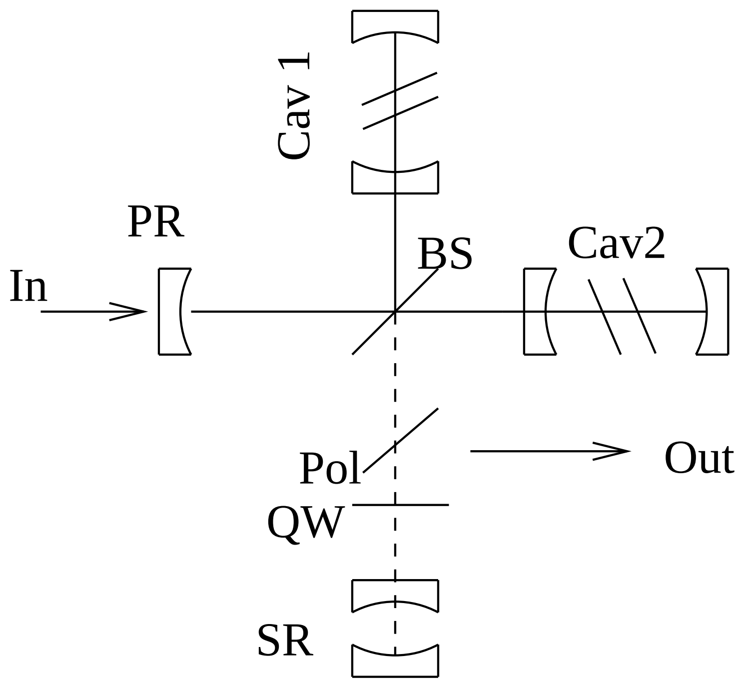
$$\phi_{RT}^{max} < \frac{Ta_{AM}(f)}{2a_{PM}(f)}\left(\frac{f_{GW}}{[10\,\mathrm{Hz}]}\right)^4 < 10^{-5}\mathrm{rad} \qquad \Delta\nu < 14\,\mathrm{Hz}$$

This is the required maximum detuning of the MC if we try to reach the planned sensitivity at a GW-frequency of 10 Hz. Note that this value scales with the GW-frequency up to the forth power.

The tidal effects were calculated in [3]. The common arm cavity length will change over a period of 12 h by a maximum amount of 110 μm. The differential arm cavity length would change by about 30 μm. The later has to be taken out by actuators. If we track the former with the laser frequency and mode cleaner length we would have to change the lengths of the mode cleaner by about 500 nm. This would change the free spectral range of the mode cleaner by 0.3 Hz.

2.4 Possible Solution

The major problem in the actual setup seem to be the sensitivity towards oscillator phase noise. One possible solution might be to use a non transmissive cavity instead of a simple mirror at the dark port.



This extra cavity should be resonant for the carrier only. The reflectivity of its end mirror should be as high as possible (≈ 1). Its finesse has to be optimized, but shouldn't be to high. Its free spectral range should be twice the modulation frequency. The position should be such that we have resonant sideband extraction without the end mirror. This ensures a symmetric interferometer for the sidebands. The necessary phase shift for the carrier to tune the instrument to the correct frequency can be achieved via a small detuning of the SR-cavity alone. First calculations show a reduced sensitivity towards oscillator phase noise. The requirements on oscillator phase noise could be relaxed by two orders of magnitude or 40dBc.

Some other noise sources like modulation index stability and intensity noise if the laser are limited by their impact on the carrier intensity stability in the two arm cavities which in turn has very stringent requirements set by technical radiation pressure noise. Symmetric sidebands won't relax the requirements for these kind of problems. Pointing requirements are set by amount of carrier in the fundamental mode they create. This does not depend on the read out scheme and it can not be expected that symmetric sidebands would reduce the requirements on pointing.

2.5 DC-Sensing

The requirements for DC-Sensing on oscillator stability are yet unknown. There is no direct way oscillator noise couples into the signal because the sidebands are not detected by the main photodetector. However, due to the different feedback loops and the coupling between the different degrees of freedom, oscillator phase noise will affect the signal. Details TBD.

References

[1] Advanced LIGO Systems Design (LIGO-T-010075-00-D, June 27. 2001) LSC ed. Peter Fritschel.

[2] *Signal Sensing in Advanced LIGO, Part I* (LIGO-T-020023-00-D), Guido Mueller

[3] *The Effect of Earth Tides on LIGO interferometer*, F. Raab, M. Fine (LIGO-T970059-01-D, 02/20/1997)