SHORT-TIME FOURIER TRANSFORM (SFT) SPECIFICATION

Gregory Mendell

LIGO Hanford Observatory
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Introduction

The method used to generate Demodulated Fourier Transforms (DeFTs) is discussed in Schutz and Papa [1], Williams and Schutz [2], and S. Berukoff and M. A. Papa, [3]. The generation of DeFT is one part of the strategy used by LIGO to search for gravitational waves from pulsars. The DeFTs, in turn, are generated by summing over Short-time Fourier Transforms (SFTs).

The purpose of this document is to give specifications for the SFTs. To put the SFTs into context, a brief summary of the DeFT algorithm is given here. Following this is the SFT specification.

Given a discrete time series x_a and model of the phase Φ_{ab} one can define the phase demodulate Fourier transform (DeFT) by

$$X_b = \sum_{a=0}^{NM-1} x_a e^{-2\pi i \Phi_{ab}} , \qquad (1)$$

where a is the discrete time index, and b is a discrete frequency index. The number of input data points has been chosen as NM, so as to facilitate breaking the sum into M short time segments containing N samples each

$$X_b = \sum_{\alpha=0}^{M-1} \sum_{j=0}^{N-1} x_{\alpha j} e^{-2\pi i \Phi_{\alpha j b}} , \qquad (2)$$

where α is the segment index, and j is the short time index. We now replace $x_{\alpha j}$ with

$$x_{\alpha j} = \frac{1}{N} \sum_{k=0}^{N-1} X_{\alpha k}^{SFT} e^{2\pi i jk/N} , \qquad (3)$$

where X^{SFT} is the discrete Fourier transform of the short time segments known as an SFT. The time segments are chosen short enough so that modulation effects due to the Earth's motion and intrinsic source evolution are small, and the source frequency will appear monochromatic in the SFT. Substituting Eq. (3) into (2) and rearranging

$$X_b = \sum_{\alpha=0}^{M-1} Q_{\alpha b} \sum_{k=0}^{N-1} X_{\alpha k}^{SFT} P_{\alpha k b} . \qquad (4)$$

The expressions for P and Q simplify if we expand the phase about the midpoint of each SFT:

$$\Phi_{\alpha j b} = \Phi_{\alpha,1/2,b} + f_{\alpha,1/2,b}(t_{\alpha j} - t_{\alpha,1/2}) . \qquad (5)$$

In this case, P and Q are given by

$$
\begin{aligned}
P_{\alpha k b} &= \frac{\sin u_{\alpha k b}}{u_{\alpha k b}} - i\frac{1-\cos u_{\alpha k b}}{u_{\alpha k b}} \\
Q_{\alpha b} &= e^{i v_{\alpha b}} \\
u_{\alpha k b} &= 2\pi\left(\frac{T}{M} f_{\alpha,1/2,b} - k\right) \\
v_{\alpha b} &= -2\pi\left(\Phi_{\alpha,1/2,b} - \frac{T}{2M} f_{\alpha,1/2,b}\right)
\end{aligned}
\qquad (6)
$$

In Eq. (6) T is the total duration of the input time series used to produce a DeFT. Since P is peaked near k* = (T/M)$f_{\alpha 1/2b}$, it has been shown that the sum over k can be limited to approximately k* - 8 to k* + 8. This method is coded in the LALDemod function, written by Berukoff and Papa [3]. In their case, amplitude demodulation will also be accounted for. Their function is also used by the knownpulsardemod DSO search code.

SFT Specification

This specification is for SFTs generated by the knownpulardemod DSO run under the LIGO Data Analysis Systems (LDAS).

I. Input Time Series Specifications

1. The Sample Rate

Theoretical pulsar spin rates are in the 0 to 2000 Hz band. Observed pulsar spin rates are in the 0 to 642 Hz band. Thus, theoretically, gravitational radiation is expected in the 0 to 2000 Hz band, plus higher harmonics. Even to cover all the known pulsars requires searching the 0 to 1284 Hz band. For this reason it is reasonable to choose:

Sample Rate = fSample = 4096 Hz (Time Step Size = deltaT = 1/4096 Hz = 2.44140625e-4 sec.)

2. The Search band

A sample rate of 4096 gives a Nyquist frequency of 2048 Hz. The search band will be:

Search Band = B = 0 to 2048 Hz.

3. Data Conditioning

(i) The gravity wave channel will be resampled to 4096 Hz.
(ii) Resampling is done using the LDAS dataconAPI resample action. This action includes anti-alias filtering. The knownpulardemod DSO code accounts for the time delays introduces by this filtering, by inputing a data segment with some small overlap

with the previous and next segment, and then finding the slice of the input segment after resampling that corresponds to the desired GPS time and duration (tSFT).

No other data conditioning steps are currently used. Calibration will be done down stream. The need for line removal will be studied at a future data.

4. Duration = tSFT

This time must be choosen so that the frequency modulation during tSFT is less than one frequency bin. This is computed by tSFT <= 5.5 X 10^3*sqrt(300 Hz/2048 Hz) s = 2105 s (Schutz and Papa [1]). Choosing the largest power of two less than this upper bound:

Duration = tSFT = 2048 seconds 34.1333... minutes. (Frequency Step Size = deltaF = 1/2048 s = 4.8828125e-4 Hz.)

5. Number of Input data points per SFT = inputN

inputN = (4096 Hz)*(2048 s) = 8388608 = 2^{23} data points.

6. Input memory requirements per SFT

A. The input data is REAL4 data = 4 bytes per datum.

B. Before resampling the sample rate is 16384 Hz = 4 * 4096 Hz.

C. Before resampling:

(16384 Hz) * 2048 s per SFT * 4 bytes per datum * 1 MB / 2^{20} bytes = 128 MB per SFT.

D. After resampling:

(4096 Hz) * 2048 s per SFT * 4 bytes per datum * 1 MB / 2^{20} bytes = 32 MB per SFT.

7. The Channel. SFT are computed from the “gravity-wave” channel, e.g., H1:LSC-AS_Q, H2:LSC-AS_Q, and L1:LSC-AS_Q.

II. Output SFT frequency series specifications

1. Start Frequency = fMin = 0 Hz.

2. Stop Frequency = Nyquist frequency = ½ sample rate = 2048 Hz.

3. Frequency Step Size = Frequency resolution = deltaF = 1/2048 s = 4.8828125e-4 Hz.

4. Output number of data points = nDeltaF:

SFTs are computing using LALForwardRealFFT which,for N input data points, outputs N/2 + 1 frequency bins, which corresponds to frequencies from 0 up to the Nyquist frequency. Thus:

nDeltaF = N/2 + 1 = (4096 Hz)*(2048 s)/2 + 1 = 2^{21} + 1 = 4194305.

5. Computation Method

A. SFTs are computing LALForwardRealFFT, which outputs COMPLEX8 data.

B. It should not make a difference whether an approximate or exact realForwardFFTPlan is used.

6. Normalization

SFTs are normalized by multiplying by deltaT/sqrt(tSFT). Thus SFT units are counts per root Hz.

7. Disk Storage Type

SFTs stored in the proc structure of frame files as COMPLEX8 data. They need to be recast as COMPLEX16 data for use in LALDemod.

8. Disk storage Requirements

A. For COMPLEX8 data is nDeltaF*8 bytes:

B. Storage per SFT: (4194305)*8 bytes per SFT * 1 MB/ 2^{20} bytes = 32.0 MB + overhead per SFT.

C. Storage per Day: (24 hr/day) * (3600 sec/hr) * (1 SFT /2048 Sec) * 32.0 MB/ SFT = 1350.0 MB/day.

D. Seventeen Day E7 Run: 1350.0 MB/day * 17 day * 1 GB/ 2^{10} MB = 22.4 GB + overhead, per interferometer.

9. Total Memory requirements per SFT:

A. Since SFTs are cast into COMPLEX 16 when used by LALDemod, one must reserve 64 MB per SFT in memory.

B. input 32 MB time series data per SFT + output 64 MB per SFT = 96 MB per SFT + overhead.

C. Maximum number of SFTs in 512 MB memory <= 512 MB/64 MB = 5 SFTs.

D. Total duration of data in 512 MB memory = 5 SFTs * 2048 sec per SFT = 10240 sec = 2.8444... hours.

E. To coherently analyze longer stretches of data slices of SFTs in small bands will be read in. The later is possible under LDAS, but testing has only begun recently.

III. Handling Data Quality and Data Drop Out

1. Clean locked stretches of data are identified and used to generate a quality channel. Input time series data of poor quality is “padded” (replaced) with the mean value of the data with good quality before the SFT is generated.

2. -allowgaps option will be used in the LDAS dataPipeline command, and dataconAPI fillgaps action will be used to fill gaps with mean value of input data.

3. The percent of the duration for each SFT that was padded will be stored in the history output structure (see Section IV below).

4. Issues of windowing to reduce leakage due to padding or filling of gaps should be discussed further.

VI. SFT frame format

1. Naming Convention:

$S-P_$LDASJOBNUMBER_SFT_$C-$GPS-$T.gwf

where

A. $S is the site name (e.g., H, L, …).

B. $LDASJOBNUMBER is the number of the LDAS job that produces the SFT.

C. $C is a nickname for the channel the SFT was computed from (e.g., H1, H2, L1).

D. $GPS is the GPS start time of the input data that generated the SFT.

E. $T is the total duration of input data that generated the SFT.

Example: H-P_NORMAL1001-SFT_H2_693644217-2048.gwf

This file contains an SFT computed from 2048 seconds of H2:LSC-AS_Q data that started at GPS time 693644217.

2. Frame Structures

An FrDump of an example SFT file reveals:

[gmendell@vulcan SearchCodeOut]$ FrDump -i H2-SFT_TEST1_4-600000000-1.gwf -d 5 | less
...
History records:
Processed Data:
Proc: sft_output sampleRate=-1 fShift=0.0000e+00 comment:multiDimData to ProcData
Vector:data ndata=9 nDim=1 unitY= count hz^-1/2 unitX=hz startX=0.00e+00 dx=1.28e+02
Data(CD) (7.161711e-03,0.000000e+00) (7.879787e-03,1.053996e-03) ...

A. sft_output = data set name

B. sampleRate and fShift: IGNORE THESE; WILL BE SET TO -1 OR 0.

C. ndata = nDeltaF = 9 = number of frequency bins.

D. unitY will be counts per root hz.

E. startX = Start Frequency = fMin = 0.0

F. dx = deltaF = 128 Hz for this example. (Unit is stored as unitX)

G. Data is stored at COMPLEX8 (A flat vector of Real and Imaginary pairs stored at REAL4 each.)

SFT Meta Data is stored in the History Structure. Example ilwd output:

```
<ilwd name='LDAS_History' size='9'>
    <ilwd name='percent_pad'>
        <real_4 units='percent'>0.0000000e+00</real_4>
    </ilwd>
    <ilwd name='max_power'>
        <real_4 units='power'>7.8124991e-03</real_4>
    </ilwd>
    <ilwd name='freq_pwmax'>
        <real_4 units='hz'>3.8400000e+02</real_4>
    </ilwd>
    <ilwd name='power_mean'>
        <real_4 units='power'>8.6805545e-04</real_4>
    </ilwd>
    <ilwd name='power_stddev'>
        <real_4 units='power'>2.4552315e-03</real_4>
    </ilwd>
    <ilwd name='startSec'>
        <int_4s units='s'>600000000</int_4s>
    </ilwd>
```

```
<ilwd name='startNan'>
    <int_4s units='ns'>0</int_4s>
</ilwd>
<ilwd name='sample_rate'>
    <real_4 units='hz'>2.0480000e+03</real_4>
</ilwd>
<ilwd name='ndata_input'>
    <int_4s units=' '>16</int_4s>
</ilwd> </ilwd>
```


The meta data is:

A. Percent Padding.

B. Maximum power is this SFT.

C. The frequency corresponding to this maximum power.

D. The mean power is this SFT.

E. The standard deviation of the power.

F. The start time from which this SFT was generated (StartSec & StartNan).

G. The sample rate of the input data this SFT was generated from.

H. The number of input data points this SFT was generated from.

References

[1] B. F. Schutz and M. A. Papa, gr-qc/9905018.
[2] P. R. Williams and B. Schutz, gr-qc/9912029.
[3] S. Berukoff and M. A. Papa, LAL Pulsar Package Documentation, Chapter 4.