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The Wave-front signals from E2E's 6-optics LIGO system

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Purpose of this Note:

SimLIGO, a model for the LIGO interferometers has been built-up recently using the software tools of the End-to-End (E2E) model [1]. Besides many advanced features, this model includes the full Length and Alignment sensing control systems. The alignment sensing system is based on a clever arrangement of the wavefront sensor sytem and feedback control.

While implementing this, it was important to check whether or not the complex simulation set-up of the 6-optics LIGO system was generating correct signals at various output ports.

This note is prepared to keep record of this validation of the misalignment signals that we obtain from a model of a 6-optics system. We compared our results from E2E models with those presented in Table 2 of the 1998 paper by Fritschel, et al [2].

The 6-optics system considered:

The 6-optics system that we considered is same as that described in Appendix A of that paper. This is a 4-Km interferometer but many of its parameters are different from the as-built LIGO. The following points are worth noting down:

- (i) There is a typographical mistake in the parameter Table presented in Appendix A. The Schnupp asymetry length is quoted there as $(\ell_1 - \ell_2) = 0.21$, but actually it should be $(\ell_1 - \ell_2)/2 = 0.21$. This makes lot of differences especially for the common mode to the reflected or pick-off port.

- (ii) To avoid phase-rotation of the quadrature and in-phase signal at the dark port due to finite distance between the beam-splitter and the recycling mirror, the length between these two mirrors is set to zero.

- (iii) Although Figure 1 shows the pick-off point as ”From Beam-splitter to ITMX”, actual calculation was done with the field that get reflected from the RM.

- (iv) The paper does not give values for the modulation index of sidebands used for generating the Tables. One must note that the signal values presented in Table 2 are scale down-converted signals by $2J_0(\Gamma)J_1(\Gamma)P_{in}$ to make them independent of the modulation depth and the incident light power.

Simulation Models:

We compared results from 3 different models for the same system:

- (i) The Table 2 of the paper. This Table was generated from the MIT modal model code based on Mathematica software package.

- (ii) The MIT FFT model [3]

- (iii) The E2E model.

E2E and FFT:

The FFT and E2E generate essentially the same amplitudes of lights at the outputs, except that the ratio of the imaginary and carrier TEM00 amplitudes at various ports for the E2E run (e.g., for the internal light at the recycling mirror, this is of the order of 0.01) is larger than those for FFT runs (essentially zero). One of the reasons behind this could be the limited number of modes used in E2E but we donot yet know the exact origin of this discrepancy. However, these differences do not get reflected in the actual misalignment signals calculated from these amplitudes. So, FFT and E2E generate the same signals at the output ports.

In the following we discuss only about the comparison between the paper and the E2E signals.

Calculations in E2E:

The procedure followed in E2E is as follows:

The light in the cavities of the 6-optics system is built up and a small angular perturbation (we used 1e-8 rad in each mirror involved) in a particular mode are applied to the appropriate mirrors.

The following 8 amplitudes representing the coefficients of the Hermite-Gaussian modes at three different output ports (reflected, asymmetric and the recycling cavity pick-off) are recorded: The Carrier TEM00, $\overline{\mathrm{CR}}_{00}$, the carrier TEM01 (for pitch modes), $\overline{\mathrm{CR}}_{01}$, the positive sideband TEM00, $\overline{\mathrm{SP}}_{00}$, the positive sideband TEM01, $\overline{\mathrm{SP}}_{01}$, the negative sideband TEM00, $\overline{\mathrm{SM}}_{00}$, the negative sideband TEM01, $\overline{\mathrm{SM}}_{01}$.

The *quadrature* and *in-phase* signals are then calculated as

$$Q = 2\mathrm{Re}\Big[S_1 e^{-i\eta} + S_2 e^{+i\eta} + S_3 e^{-i\eta} + S_4 e^{+i\eta}\Big] \quad (1)$$

$$I = -2\mathrm{Im}\Big[S_1 e^{-i\eta} + S_2 e^{+i\eta} + S_3 e^{-i\eta} + S_4 e^{+i\eta}\Big] \quad (2)$$

where

$$S_1 = \overline{\mathrm{CR}}_{00}\overline{\mathrm{SP}}^*_{01} \quad (3)$$

$$S_2 = \overline{\mathrm{CR}}_{01}\overline{\mathrm{SP}}^*_{00} \quad (4)$$

$$S_3 = \overline{\mathrm{SM}}_{00}\overline{\mathrm{CR}}^*_{01} \quad (5)$$

$$S_4 = \overline{\mathrm{SM}}_{01}\overline{\mathrm{CR}}^*_{00} \quad (6)$$

where * represents the conjugation and η is the Guoy phase between the TEM00 and TEM01 modes at the output ports. If R_i and I_i represent the real and imaginary parts respectively of S_i, then the signals can be expressed as

$$Q = 2[(R_1 + R_2 + R_3 + R_4)^2 + (I_1 - I_2 + I_3 - I_4)^2]^{1/2} \quad (7)$$

$$\eta_Q = tan^{-1}\left[\frac{(I_1 - I_2 + I_3 - I_4)}{(R_1 + R_2 + R_3 + R_4)}\right] \quad (8)$$

$$I = -2[(I_1 + I_2 + I_3 + I_4)^2 + (R_2 - R_1 + R_4 - R_3)^2]^{1/2} \quad (9)$$

$$\eta_I = tan^{-1}\left[\frac{(R_2 - R_1 + R_4 - R_3)}{(I_1 + I_2 + I_3 + I_4)}\right] \quad (10)$$

Comparing E2E results with those in the Paper:

We now need to connect the signals calculated above from E2E outputs with the numbers presented in Table 2 of the paper.

This connection needs to be made by using Eq.(16) of the paper, which describes the RF amplitude-modulated alignment signal at each port as:

$$WFS(\eta, \Theta, \Gamma) = P_{in} f(\Gamma) f_{split} k_{PD}^{10} \sum_{i=1}^{5} A_i \Theta_i \cos(\eta - \eta_i) \cos(\omega_m t - \phi_{Di}) \quad (11)$$

Table 2 of the paper presents the numbers A_i and η_i. While η_i values obtained from above calculations can be directly compared with the paper, we need to convert the E2E's signals by appropriate conversion factors to get the corresponding values for A_i.

We already described how we calculate this expression at each output port for each of the 5 misalignment modes, i . The following considerations need to be kept in mind:

- (i) P_{in}, the total power was taken to be 1 in E2E calculations.

- (ii) We can say that, in E2E calculations, the quantity, f_{split}, the fraction of a particular port's light that is directed to the Wavefront sensor is just 1. However, for the recycling cavity pick-off port, the paper uses a factor of 3×10^{-4} and so that number needs to be multiplied to the corresponding E2E signals at that port.

- (iii) The E2E values should be divided by the factor $f(\Gamma) = 2J_0(\Gamma)J_1(\Gamma)$ for appropriate Γ for signals involving resonant and nonresonant sidebands.

- (iv) For this discussion, we need not include the factor k_{PD} that accounts for the difference between the specific photodiode geometry and the idealized half-plane geometry.

- (v) The E2E values should be divided by the normalized angles, Θ_i, which for Differential ETM and ITM and Common ETM and ITM signals are approximately equal to $\sqrt{2} \times 10^{-3}$. Note that the mirrors were rotated by 0.01 microradians and the divergence angle is approximately 10^{-5} rad [Accurate values: For recycling cavity, the divergence angle = 9.9975e-06, Rayleigh range = 3388; For arm cavities: the divergence angle = 9.6486e-06, Rayleigh range = 3638]

- (vi) The coefficients in Eq.11 are calculated when the intensity is integrated over a half-plane detector, which is one that subtracts and integrates over two mirror-symmetric half-infinite planes located left(up) and right(down) of the y(x) axis. In the expressions for E2E signals presented above, we have dealt with only the Hermite-Gaussian mode coefficients and have not put this integral factor. So, we need to multiply the expressions for the Q and I signals in Eqs.7 and 9 by $2 \times 1/\sqrt{2\pi}$, twice the integral over half-plane.

After these conversions, E2E signals and the angles match well with the numbers presented in Table 2 of the paper.

Concluding remarks:

One should note that in SimLIGO, the actual e2e model for the as-built LIGO interferometer, the fields obtained at the output ports are transmitted through Guoy phase telescopes and then made to fall on quadrant photodetectors-cum-demodulators where the signals are automatically calculated.

In the presentation above of the calculations of signals from the field amplitudes we just checked if signals are correct at the output ports. E2E's *telescope* or *photodetector-cum-demodulator* modules have been validated separately.

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References

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[2] P. Fritschel, N. Mavalvala, D. Shoemaker, D. Sigg, M. Zucker, G. Gonzalez, "Alignment of an interferometric gravitational wave detector", Applied Optics, **37**, 6734-6747 (1998).

[3] Bochner, B., *Modelling the performance of Interferometric Gravitational-wave detectors with realistically imperfect optics*, Ph.D theses (MIT, 1998), LIGO Doc. P980004-00R