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**Polarity of Michelson Length Signal  
Obtained at the Symmetric Port  
In a Fabry-Perot Michelson Interferometer**

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## **ABSTRACT**

We observed during the lock acquisition of the 40m Fabry-Perot Michelson interferometer that the Michelson length control behaves differently between the X-arm locked and Y-arm locked configuration. It was found that this seemingly very peculiar asymmetry can occur with a tiny deviation of the demodulation phase from the ideal one. The mechanism is explained intuitively using the phasor diagram in this report.

## 1. Introduction

We acquire the lock of the 40m Fabry-Perot Michelson interferometer using the following signals for the three degrees of freedom as shown in Table 1 and Fig. 1.

Table 1. Control signals for each DOF.

| DOF                 | Port                | Demodulation Phase |
|---------------------|---------------------|--------------------|
| Michelson length    | Symmetric Port (SP) | Q-phase            |
| X-arm cavity length | POX                 | I-phase            |
| Y-arm cavity length | POY                 | I-phase            |

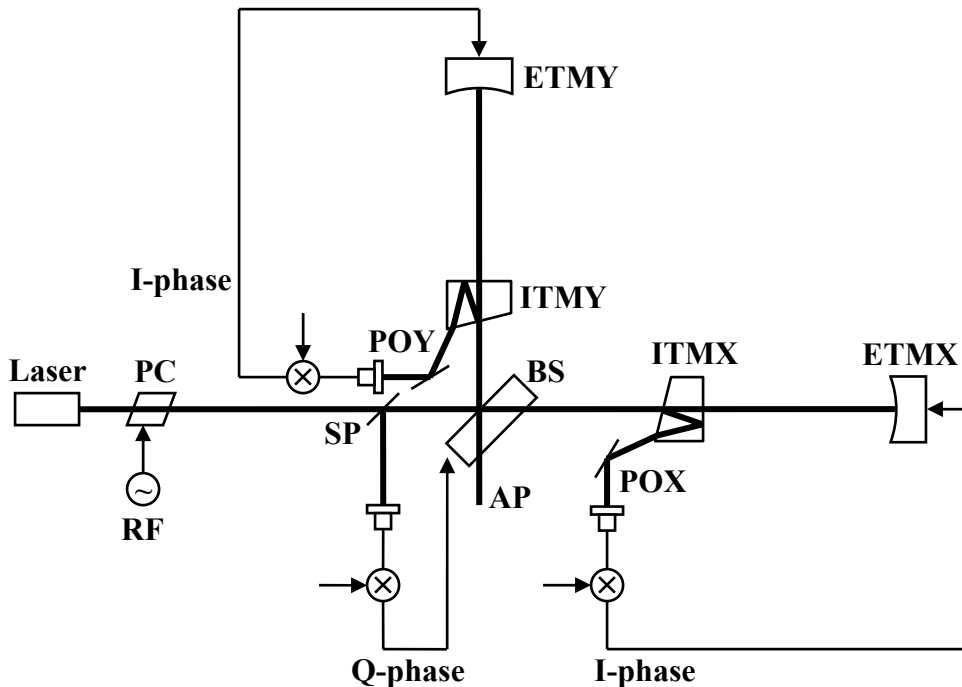


Fig. 1. Schematic diagram of the control system for the lock acquisition of the 40m Fabry-Perot Michelson interferometer.

We found that the fringe at the SP when the Michelson length and one arm is locked is either bright or dark depending on which arm is locked. The mechanism of this seemingly very peculiar asymmetry will be explained in the following chapter using the phasor diagram.

## 2. Phasor diagram

### (1) No arm locked

We start with a simple Michelson interferometer with neither arms locked. As shown in the upper diagram of Fig. 2, the incident light consists of a carrier with an upper and lower sidebands rotating to opposite directions. The phasor diagram in the middle shows that the light from the two arms interferes constructively for the carrier at the SP (The Michelson microscopic length is chosen to give this condition, and will be fixed in the following discussions). When the Michelson length changes slightly to positive (defined as red arrows in the upper diagram) from the ideal operating point, the resultant

upper sideband becomes slightly smaller than the resultant lower sideband as shown in the lower left diagram. Therefore the total power of the light is modulated with the Q-phase signal (compared with  $\sin(\omega t)$ ) and the negative polarity (compared with  $\sin(\omega t + 90^\circ)$ ) as shown in the lower right figure.

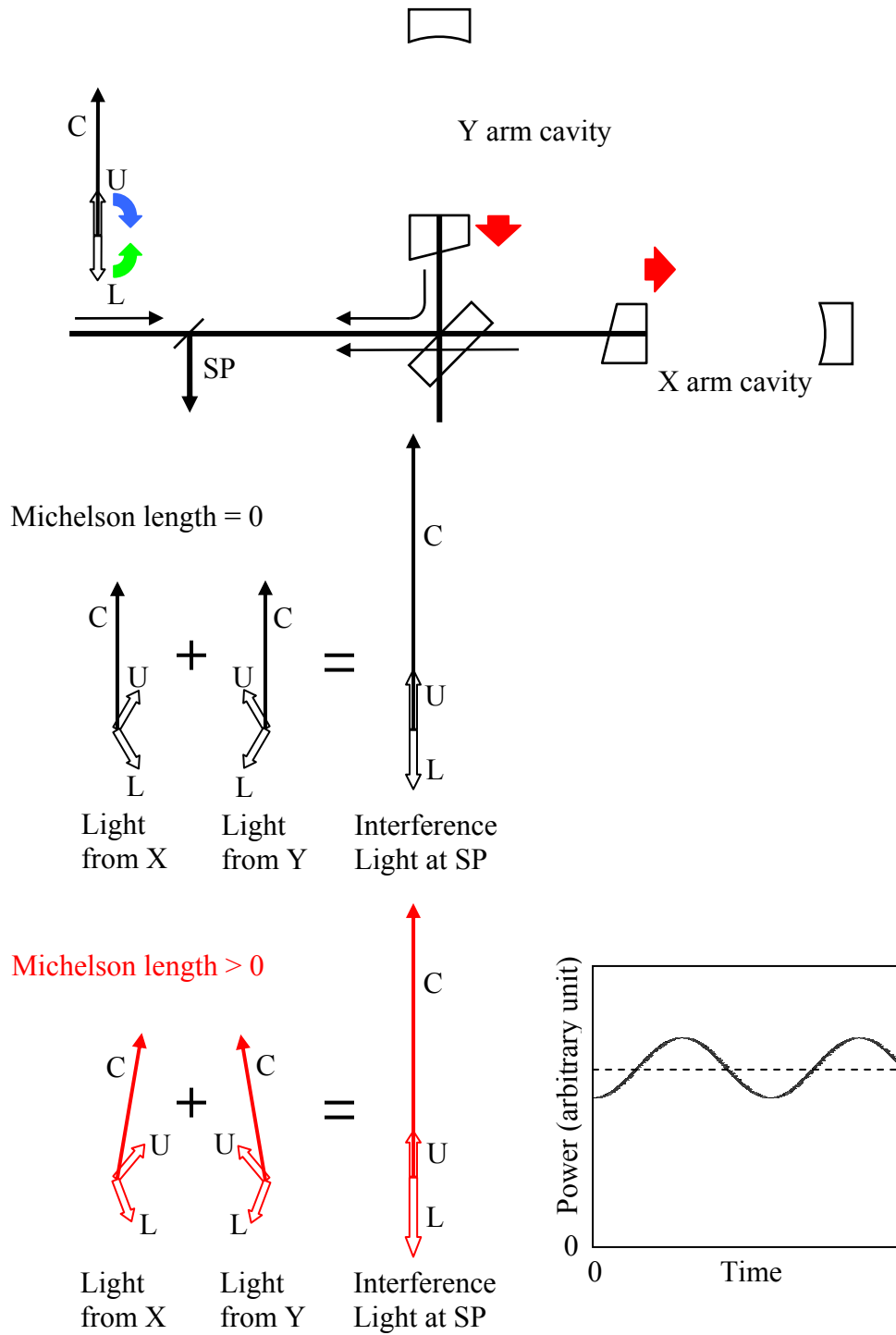


Fig. 2. Phasor diagram for the Michelson signal obtained at the SP in the Michelson interferometer.

(2) X-arm locked

When the X-arm is locked, as shown in Fig. 3, the carrier of the light from the X-arm is flipped with the slightly smaller amplitude. Therefore the interference light has a small carrier left still with the upper direction as shown in the middle diagram. With a small positive deviation of the Michelson length the resultant carrier has two components:  $C_{//}$  and  $C_{\perp}$  as shown in the lower diagram.  $C_{//}$  gives the Q-phase signal with the negative polarity. Note that the amplitude of the signal is much smaller than the no-arm-locked case because of the much smaller carrier left. On the other hand  $C_{\perp}$  gives the I-phase signal with the negative polarity (compared with  $\sin \omega t$ ).

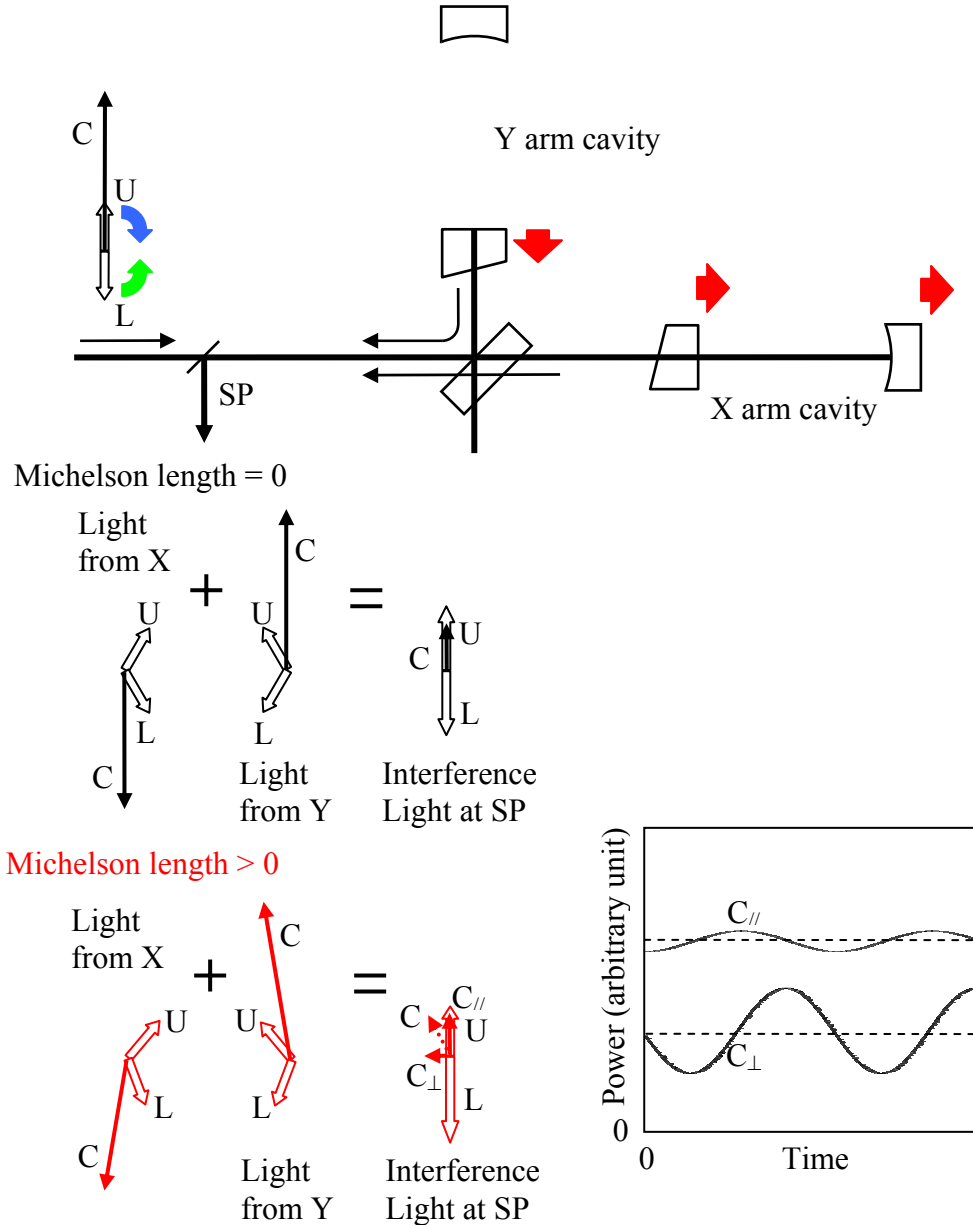


Fig. 3. Phasor diagram for the Michelson signal in the X-arm locked interferometer.

(3) Y-arm locked

When the Y-arm is locked, as shown in Fig. 4, the carrier of the interference light with the Michelson length deviated slightly to positive has two components:  $C_{//}$  and  $C_{\perp}$ , but this time  $C_{\perp}$  with the opposite direction as the X-arm locked case as shown in the lower left diagram.  $C_{//}$  gives the Q-phase signal with the negative polarity in the exactly same manner as in the X-arm locked case. However, since  $C_{\perp}$  has an opposite direction to the X-arm locked case, it gives I-phase signal with the positive polarity (compared with  $\sin \omega t$ ), which is the opposite to the X-arm locked case.

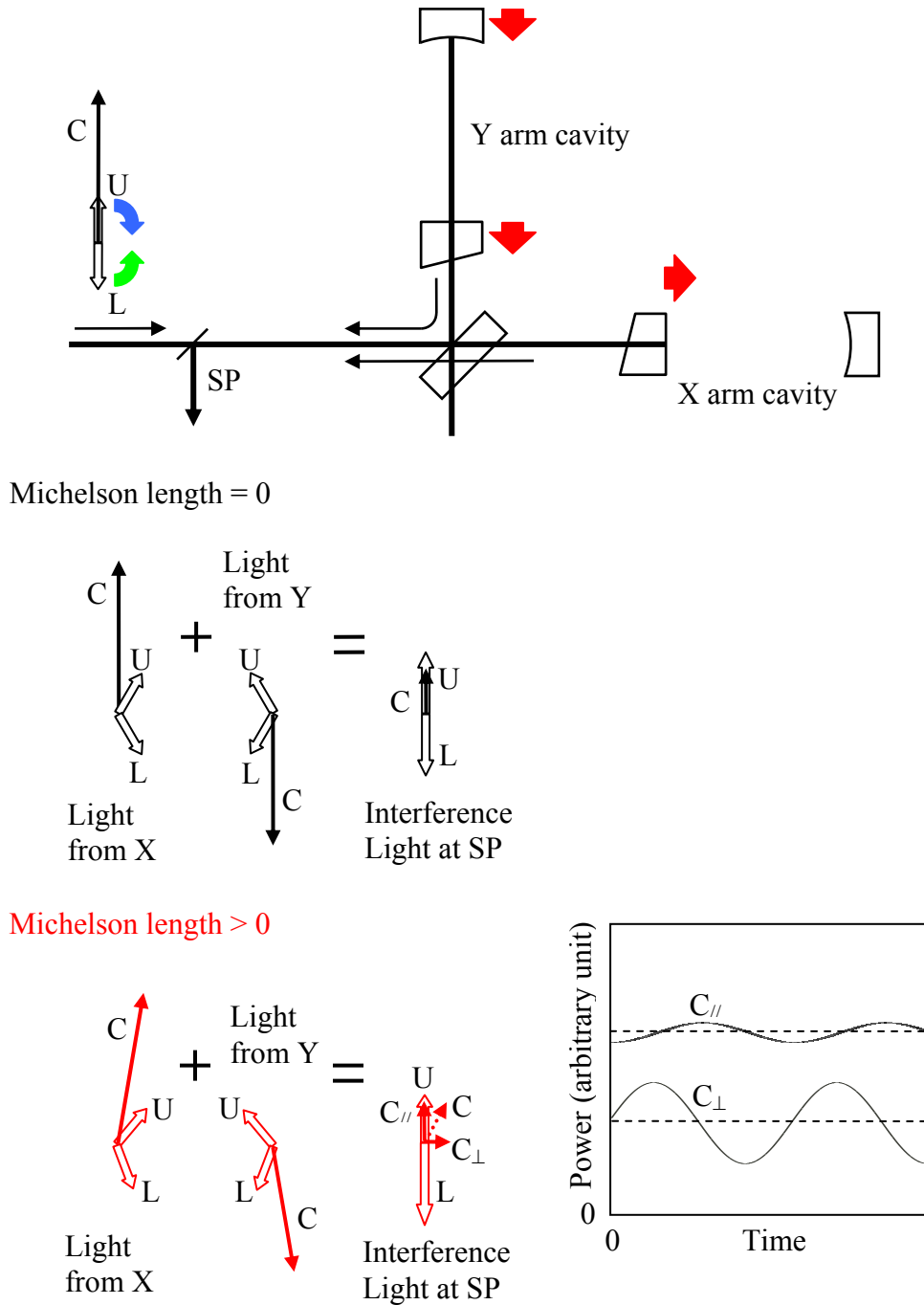


Fig. 4. Phasor diagram for the Michelson signal in the Y-arm locked interferometer.

(4) Both arms locked

When both arms are locked, as shown in Fig. 5, both carriers are flipped whereas sidebands are not affected. Therefore the power of the light at the SP gives the Q-phase signal with the positive polarity. This is the opposite polarity to the no-arm locked case. Note that the signal is slightly smaller than the no-arm locked case because of the slightly smaller flipped carrier of the light.

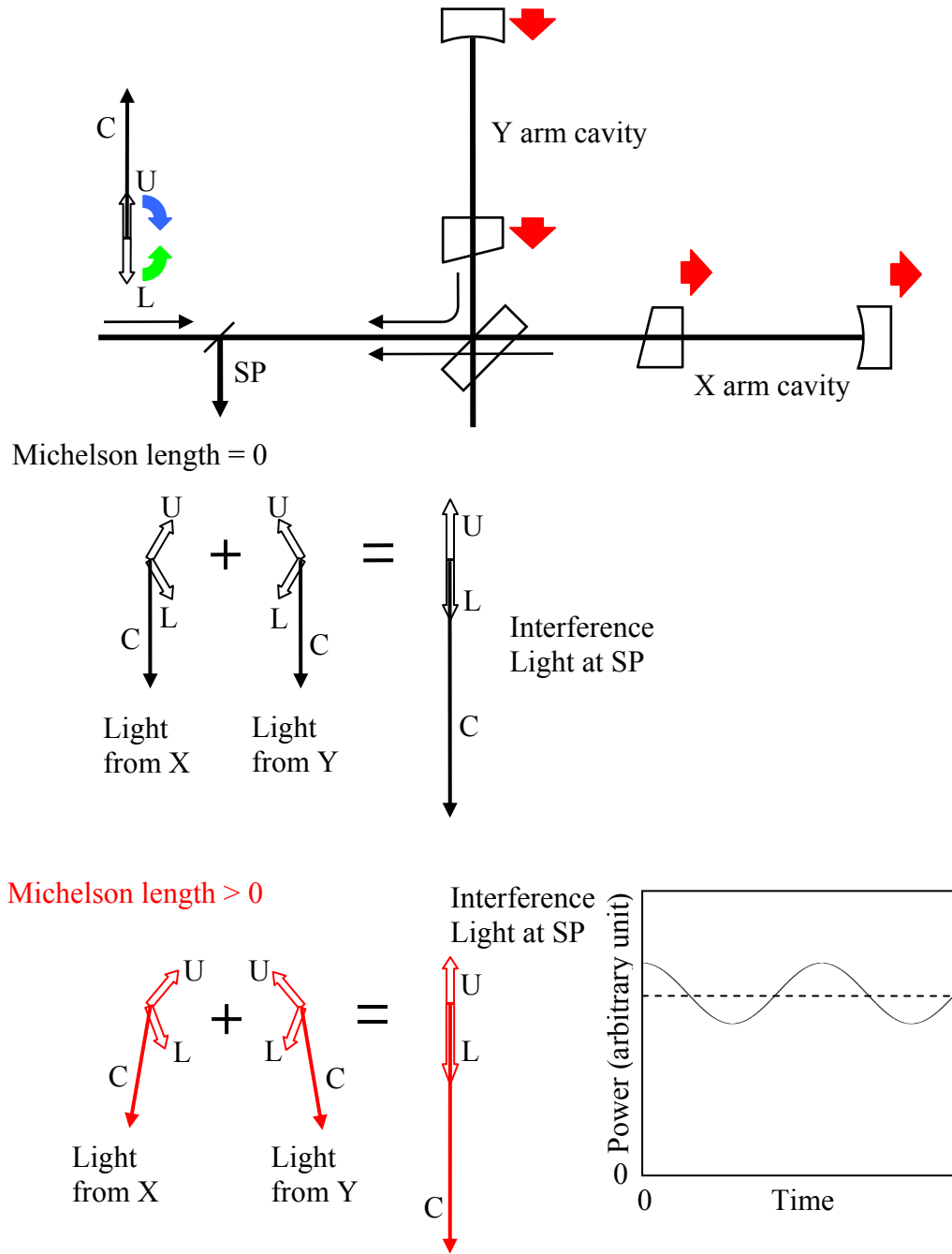


Fig. 5. Phasor diagram for the Michelson signal in the both arm locked interferometer.

Table 2 summarizes the signal type for each configuration **assuming that the Michelson microscopic length is at the ideal operating point (for the final configuration) all the time.**

Table 2. Signal type for each configuration.

| Configuration    | Fringe at SP | Q-phase signal |   | I-phase signal |
|------------------|--------------|----------------|---|----------------|
|                  |              | Polarity       | Amplitude                                     | Polarity       |
| No arm locked    | Bright       | -              |   | Not exist      |
| X-arm locked     | Dark         | -              | Much smaller than the No-arm- locked case     | -              |
| Y-arm locked     | Dark         | -              | Much smaller than the No-arm- locked case     | +              |
| Both arms locked | Bright       | +              | Slightly smaller than the No-arm- locked case | Not exist      |

If the demodulation phase is exactly Q-phase, the polarity of the Michelson length signal for the one-arm locked interferometer should be independent of which arm is locked. The polarity does not change when the configuration changes from the simple Michelson to the one-arm locked configuration, although the signal gain is reduced significantly. If there is an deviation of the demodulation phase from the perfect Q-phase, the I-phase signal is picked up and it gives a different polarity depending on which arm is locked as well as on which side the demodulation phase is deviating.

In the real interferometer the different polarity of the Michelson length signal means the different operating point of the Michelson length if the polarity of the control filter/amplifier is fixed. This should not be confused with the fringe change at the SP, which does not necessarily mean the change of the operating point of the Michelson length.

With this in mind let us explain the evolution of the Michelson locking status (See Fig. 6). We start with the simple Michelson interferometer still with no arm cavities locked. Here we lock the Michelson to the dark fringe at the SP in this configuration. And we do not change the polarity of the control filter/amplifier during the entire process of the lock acquisition. Then we lock the X-arm cavity. Here we assume some deviation of the demodulation phase for the Michelson length signal so that the I-phase signal is dominant over the Q-phase signal, but still with the same polarity of the signal. Therefore the operating point is not shifted, but the fringe at the SP changes into bright because of the flip of the carrier in the light from the X-arm cavity. Then Y-arm cavity is locked; now both cavities are locked. The polarity of the Michelson signal changes as soon as both cavities are locked (See Table 2). Thus the operating point is shifted, which means that the beamsplitter is suddenly moved by the Michelson length servo system. At the same time the carrier of the light from the Y-arm cavity is flipped. As a result the fringe at the SP stays at bright.

The situation is different when the Y-arm cavity is locked first. In this case the polarity of the Michelson signal changes because of the dominant I-phase signal which gives an opposite polarity to the X-arm locked case (See Table 2). It means that the operating point is shifted. With the flipped carrier from the Y arm, the fringe at the SP stays at dark. Then X arm is locked. The polarity of the signal no longer changes and because of the flip of the carrier from the X arm, the fringe at the SP changes into bright.

The evolution will be opposite between the X and Y arm if the deviation of the demodulation phase is opposite.



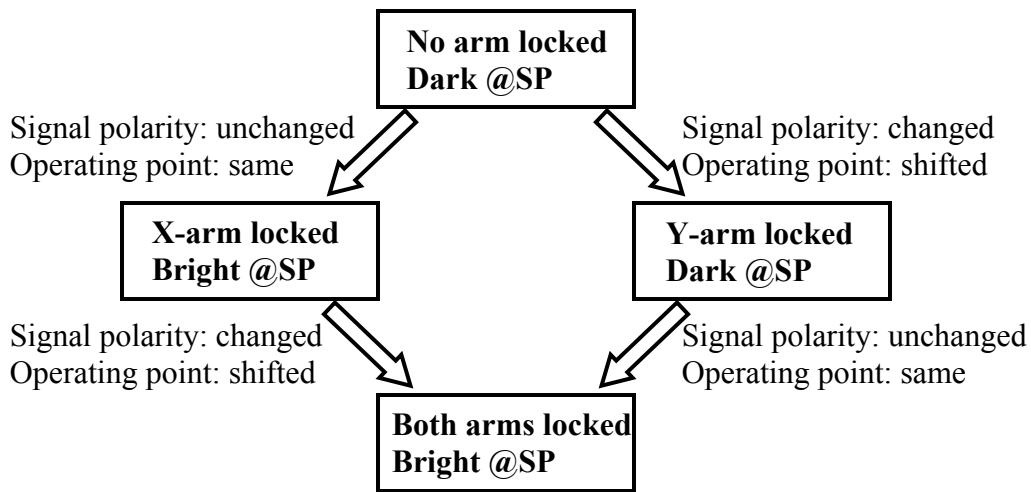


Fig. 6. Evolution of the Michelson locking status.

### 3. FINESSE

The Michelson signal in each configuration was analyzed using “FINESSE”. The result was consistent with the intuitive description given above. With the optical parameters for the 40m interferometer put into “FINESSE”, it was found that the demodulation phase deviation of only  $\sim 1^\circ$  will make the I-phase signal dominant over the Q-phase signal.

### 4. Observation

To verify the theory we investigated the Michelson length locking status of the one-arm locked FP Michelson interferometer with various demodulation phases. The results are shown in Table 3.

Table 3. Fringe at the symmetric port for the Michelson length locking in the one-arm locked FP Michelson interferometer with various demodulation phase.

| Demodulation phase ( $^\circ$ )* | X-arm locked   | Y-arm locked   |
|----------------------------------|----------------|----------------|
| +13                              | Dark           | Bright         |
| +8                               | Dark           | Bright         |
| +7                               | Could not lock | Not tried      |
| +6                               | Could not lock | Not tried      |
| +5                               | Could not lock | Not tried      |
| +4                               | Bright         | Not tried      |
| +3                               | Bright         | Could not lock |
| -2                               | Bright         | Dark           |
| -7                               | Bright         | Dark           |
| -12                              | Bright         | Dark           |

\* A demodulation phase of  $0^\circ$  is chosen to be the one which gives no response to the common-mode arm length signal.

The fringe at the SP for the Michelson length locking in the one-arm locked FP Michelson interferometer depends on the demodulation phase as well as which arm cavity is locked, which is consistent with the theory.