LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Blade bend radius

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This is an internal working note
of the Advanced LIGO Project, prepared by members of the UK team.

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1 Background

In a previous note (T030285-01-K) it was observed that the method used for calculating blade bend radius was only one of two possibilities and it would be useful to compare them. In this note I set out to do so.

2 The methods

2.1 Current method (method "A")

The method currently used is to calculate the deflection from standard beam bending theory and then to work out a radius from that:

$$\lambda = \alpha \frac{4Pl^3}{Eah^3}$$

and

$$\lambda = R\left(1 - \cos(l/R)\right)$$

Where the symbols have their standard meanings (see for example T030285-01-K).

2.2 Alternate method (method "B")

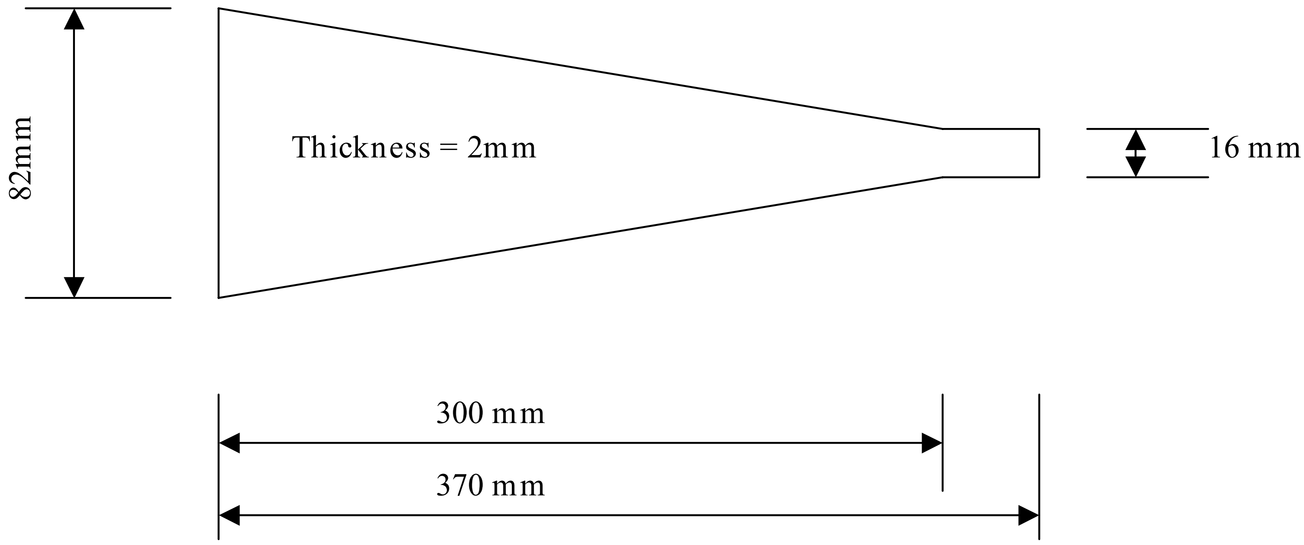
Derive the bend radius directly from

$$R = \frac{EI}{M}$$

This is part of the standard beam bending formulae. In the case of a triangular blade, I varies in the same way as M along the blade and so the bend radius will be constant. In the case of a trapezoidal blade that will not hold true – right at the tip the blade will be flat as the bending moment vanishes but the second moment of area does not.

3 Example

Taking the blade which was used for internal mode tests because it is basically a triangle and will therefore have a constant bend radius:



For a stress of 600 MPa at the root, the load is

$$\sigma = \frac{My}{I}$$

$$600 = \frac{P \times 370 \times 1}{82 \times 2^3 / 12}$$

$$P = 88.6\text{N}$$

Method A gives

$$\lambda = \alpha \frac{4Pl^3}{Eah^3}$$

$$\lambda = 1.5 \frac{4 \times 88.6 \times 370^3}{176000 \times 82 \times 2^3}$$

$$\lambda = 233\text{mm}$$

And the resulting radius is **240mm.**

Method B gives

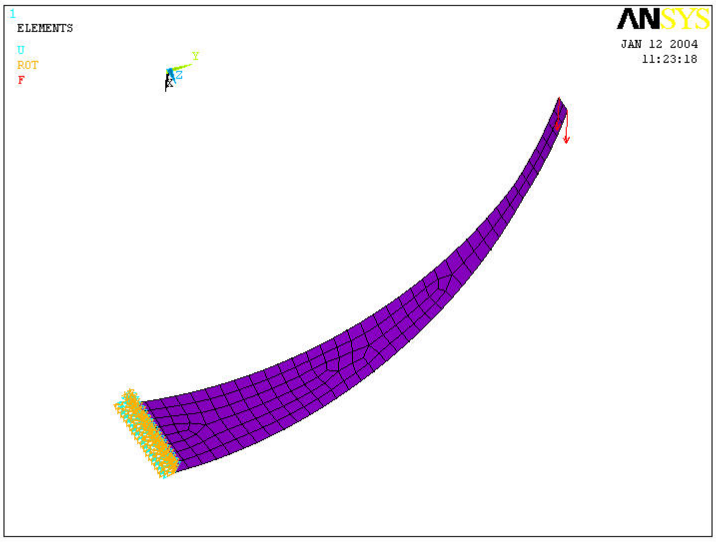
$$R = \frac{EI}{M}$$

$$R = \frac{176000 \times 82 \times 2^3}{12 \times 370 \times 88.6}$$

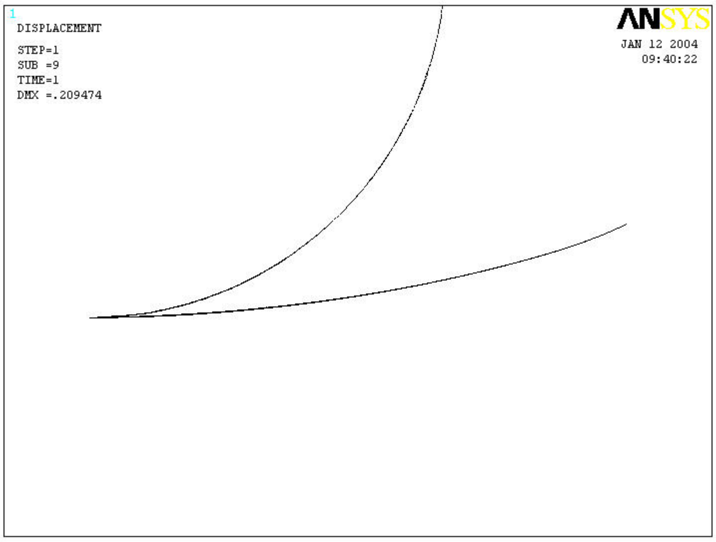
$$\mathbf{R = 293.5mm}$$

3.1 Test by nonlinear FEA

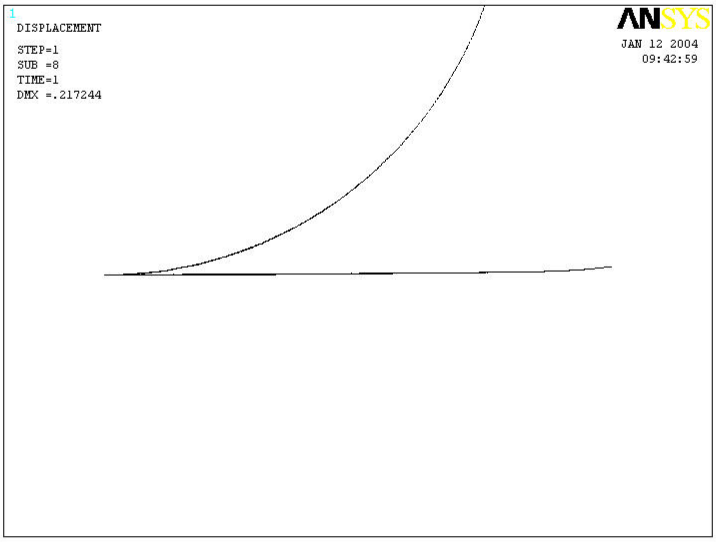
Obviously the best way to test this would be with a real model. In the absence of that, try FEA. A macro is given in appendix 1. Here is the model:



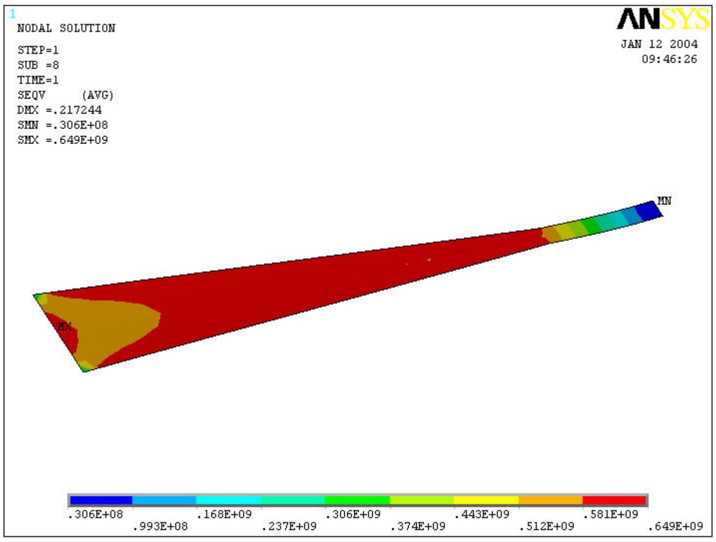
The result for a radius of 240mm is below. The undeflected and the deflected shape are shown.



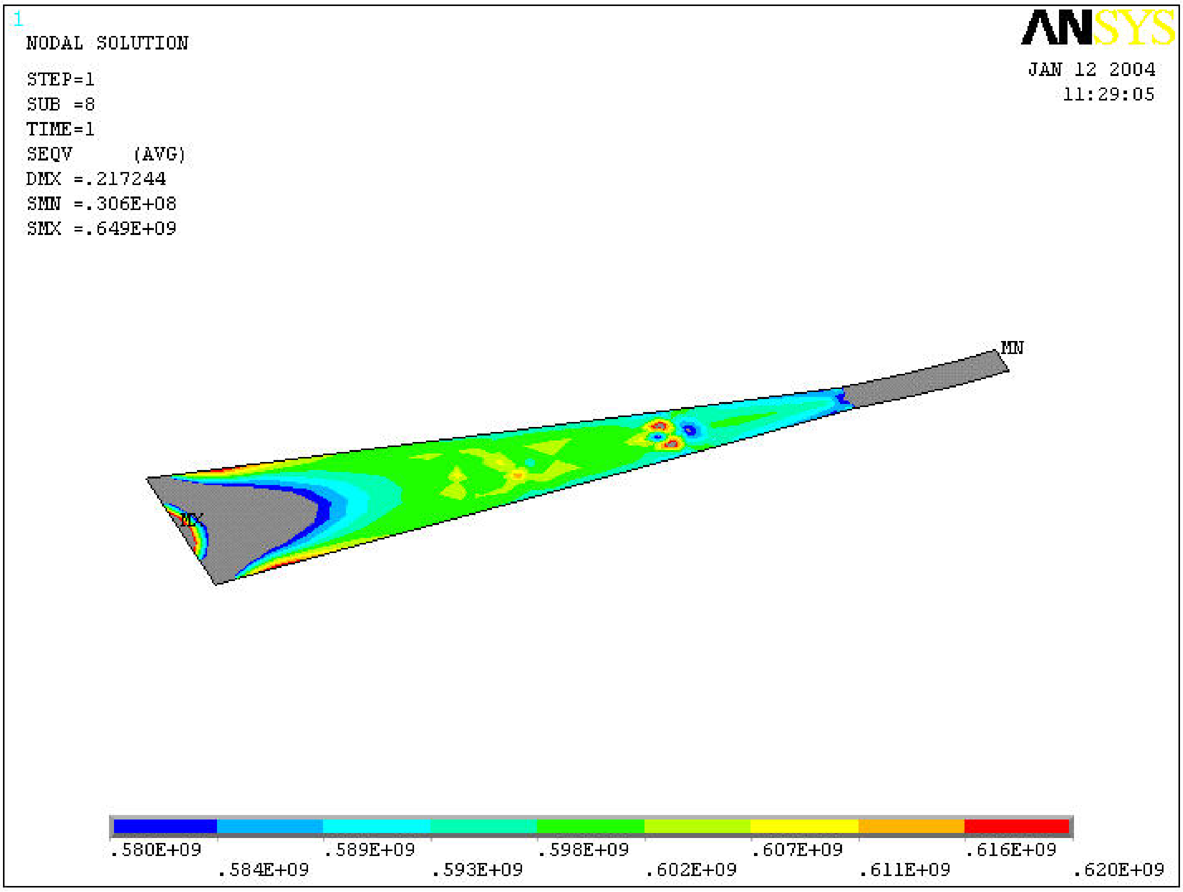
And for a bend radius of 293.5mm here:



The second result looks much flatter. Note that the tip is turned up – this is as expected because the region near the tip is not part of the triangle. Therefore it does not have a constant bending radius (or stress). This is clearer in a stress plot:



The plot also confirms maximum stress in the region of 600Mpa.



4 Measured results

We have a set of measured results for the MC blades. See T030107-00-D by Mike Plissi.

Once again the shape is essentially triangular with a feature at the end (see drawing in Mike's paper). The measurements were that with a load of 4.42kg and blades formed to a target radius of 194mm[1] (but with a 4mm error in pre-formed shape), the tip came to about 1mm above "flat".

Applying the simple formulae above, we have

Applying the first method in a simple way, with α =1.5 for a triangle:

$$\lambda = \alpha \frac{4Pl^3}{Eah^3}$$

$$\lambda = 1.5 \frac{4 \times 4.42 \times 9.81 \times 250^3}{176000 \times 39.878 \times 1.509^3}$$

$$\lambda = 169\text{mm}$$

And the radius comes to 140mm.

[1] Not directly given in the paper. The 194mm can be inferred from the 140mm quoted deflection. Thanks to Mike Plissi for supplying the 194mm directly and for giving the working load.

But when the formula was actually used in this case, there were three differences for the above calculation. Firstly, the value of Young's modulus was assumed to be 186Gpa (the direct measurement reported in T030107 had not been made at that time). Second, a value of alpha was used, based on experience, of order 1.35. Finally, the nominal thickness 1.5mm rather than the measured value 1.509 was used. It was in that way that the bend radius of 194mm was set.

Second method:

$$R = \frac{EI}{M}$$

$$R = \frac{176000 \times 39.878 \times 1.509^3}{12 \times 250 \times 4.42 \times 9.81}$$

$$\mathbf{R} = 185.4\,\mathbf{mm}$$

5 Conclusion so far

Using formula A with a modified value of alpha based on experience gives the best results. Without the benefit of such experience and taking the formulae at face value, formula B gives a better match to practical measurements and FE than formula A.

6 Bend radius for RAL test blades

We will shortly (thanks to supply of material from the US!) be making some blades based on the current conceptual design for the quads (T010103-03; Norna Robertson et al). These will not be used on the control prototype but we should nonetheless try to get the correct bend radius.

The design is:

Length (*lnb*) = 480mm

Root width suggested by Mike Plissi = 95mm

Thickness suggested by Mike Plissi = 4.4mm

Shape factor suggested by Mike Plissi = 1.36

The shape Factor Mike suggested would give a value of β (ratio of widths of ends, see T030107) of 1.37, and a tip width of 13mm.

Using a spreadsheet developed earlier, with Method B added:

l (length)	0.48	m
a (root width)	0.095	m
h (thickness)	0.0044	m
E (young's modulus)	1.86E+11	Mpa
alpha (shape factor)	1.36	
mt (total mass on spring)	61.936	kg
m (mass of next stage, per spring)	10.95	kg
g (gravitational acceleration)	9.81	m/s^2
elastic limit of Marval 18	1.60E+09	Mpa

calculated values		
I (2nd moment of area)	6.74373E-10	
lambda (tip deflection)	2.43E-01	m
k (spring constant)	2501.911637	n/m
f (uncoupled vertical frequency)	2.40574203	hz
SigmaMAX (max blade stress)	9.51E+08	Mpa
does SigmaMAX exceed elastic limit?	NO	
ratio of elastic limit to SigmaMAX	0.59	
undeflected radius (read from graph)	0.426323713	m
Internal mode		
Measured result length	0.37	m
Measured result thickness	0.002	m
Measured frequency	55	Hz
Inferred result for this design	71.90	Hz
Method B gives	0.43009	m

In this instance the two methods give very close results (in bold above). (Although note that according to the second method the radius should progressively reduce along a trapezoidal blade so this method predicts that the tip will be too high). Since these are experimental blades, we will make the blade at 0.430 and see what happens.

Appendix 1. ANSYS macro.

```
finish
/CLEAR,START
*abbr,doit,doit
/input,start71,ans,'C:\Program Files\Ansys
Inc\v71\ANSYS\apdl\',,,,,,,,,,,,,,,,1
/PREP7
!*
! values of parameters
blength=0.37
taperl=0.30
rootwidth=0.082
hroot=rootwidth/2
tipwidth=0.016
htip=tipwidth/2
bthick=0.002
maryoung=1.76e11
marpoiss=0.3
mardens=7800
dampratio=1e-4
tipload=88.6
bendrad=0.2935
!*
raddeg=180/3.1415926
thtip=blength/bendrad*raddeg
thwaist=taperl/bendrad*raddeg

!*
ET,1,SHELL93
R,1,bthick, , , , , ,
!*
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,maryoung
MPDATA,PRXY,1,,marpoiss
MPTEMP,1,0
MPDATA,DENS,1,,mardens

/VIEW,  1,  -0.361338990165    ,  -0.590998239660    ,
0.721217869236
/ANG,  1,  -68.0920680969
/DIST,1,1.08222638492,1

csys,1
k,1,bendrad,0,-hroot
,2,bendrad,thwaist,-htip
,3,bendrad,thtip,-htip
,4,bendrad,thtip,htip
,5,bendrad,thwaist,htip
,6,bendrad,0,hroot

L,1,2
,2,3
,3,4
,4,5
,5,6
,6,1

AL,1,2,3,4,5,6

aplot
ESIZE,hroot/4,0
amesh,1,2

csys,0

DL,6,,all,0
FK,3,FX,tipload/2
FK,4,FX,tipload/2

FINISH
/SOL
!*
ANTYPE,0
ANTYPE,0
NLGEOM,1
NSUBST,10,0,0
/STATUS,SOLU
SOLVE
FINISH
/POST1
PLDISP,0
PLDISP,1
```

Old results:

l (length)	0.48	m
a (root width)	0.095	m
h (thickness)	0.0045	m
E (young's modulus)	1.76E+11	Mpa
alpha (shape factor)	1.36	
mt (total mass on spring)	61.936	kg
m (mass of next stage, per spring)	10.95	kg
g (gravitational acceleration)	9.81	m/s^2
elastic limit of Marval 18	1.60E+09	Mpa
calculated values		
I (2nd moment of area)	7.21406E-10	
lambda (tip deflection)	2.40E-01	m
k (spring constant)	2532.510196	n/m
f (uncoupled vertical frequency)	2.420408522	hz
SigmaMAX (max blade stress)	9.10E+08	Mpa
does SigmaMAX exceed elastic limit?	NO	
ratio of elastic limit to SigmaMAX	0.57	
undeflected radius (read from graph)	**0.433**	m
Internal mode		
Measured result length	0.37	m
Measured result thickness	0.002	m
Measured frequency	55	Hz
Inferred result for this design	73.53	Hz
Method B gives	**0.435**	m