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LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

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Visibility of the Optical Spring Effect in the Spectrum of the 40m RSE Interferometer

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1 Introduction

The Caltech 40m interferometer has the same configuration as the next generation interferometer Advanced LIGO, which is a power-recycled detuned RSE interferometer with high-finesse arm cavities. The main purpose of the 40m is to develop a lock-acquisition sceme for the Advanecd LIGO and to observe the two-peak optical spring effect by the transfer function measurement. Moreover, the 40m will be the first experiment that can see one of the peaks, or the dips in the spectrum measurement, in its sensitivity spectrum.

The RSE technique has been developed by several table-top and prototype experiments [1][2][3]. The peak at the higher frequency has been observed by transfer function measurement in those experiments, but no one has observed it in the noise spectrum. The peak at the lower frequency has been barely observed by transfer function measurement in the Japan RSE experiment [4], but the 40m will observe it much more clearly.

In the Conceptual Design Report of the 40m [5], the lower dip is supposed to appear at around 20Hz where many other noise may hide the peak in the transfer function measurement. But recently it is turned out that the peak can be at around 200Hz and possibly observable in the spectrum measurement even with the same detune phase, which will be shown in this report briefly.

2 Laser power and the shot noise level

Figure 1 (a) shows the calculated noise spectrum almost same as is shown in the Conceptual Design Report. The finesse of the arm cavity is about 1200, the signal recycling mirror has 93% reflectivity, and the detune phase is 90-21.186 degree. While the lower peak is hidden below thermal noise and seismic noise, the higher peak is clearly observable beyond other noise curves. However, the laser power used in this calculation is only 100mW before the power recycling mirror, which is more than 20 times smaller than the value that we currently use and too small to control all the mirrors robustly. The reason why this value has been used is to make the shot noise level higher than the thermal noise curve by decreasing the power.

Figure 1 (b) shows the result with the highest power anticipated in the 40m: the incident power of 6W and the power recycling gain of 16. With these parameters, the shot noise level is so improved that the peak at the higher frequency cannot be observed in the spectrum. Figure 1 (c) is the result with 1W times power recycling gain of 10, which will be a reasonable minimum value to control the interferometer robustly. The peak is barely seen but it will be hard to identify from internal-mode thermal noise.

3 Additional vacuum injection

There is alternative way to increase shot noise other than decreasing the incident power. As is shown in Fig. 2, the installation of an additional pickoff mirror at the dark port invites additional vacumm fluctuation and increases shot noise.

Input-output relation of the vacuum in the detuned RSE system [6] can be rewritten to the following form with a pickoff mirror of the power reflectivity $R_{\rm po}$:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \frac{\sqrt{R_{\rm po}}}{M}\left[e^{2i(\beta+\Phi)}\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \sqrt{2\kappa}\tau e^{i(\beta+\Phi)}\begin{pmatrix} D_1 \\ D_2 \end{pmatrix}\frac{h}{h_{\rm SQL}}\right] + \sqrt{1-R_{\rm po}}\begin{pmatrix} a'_1 \\ a'_2 \end{pmatrix}$$

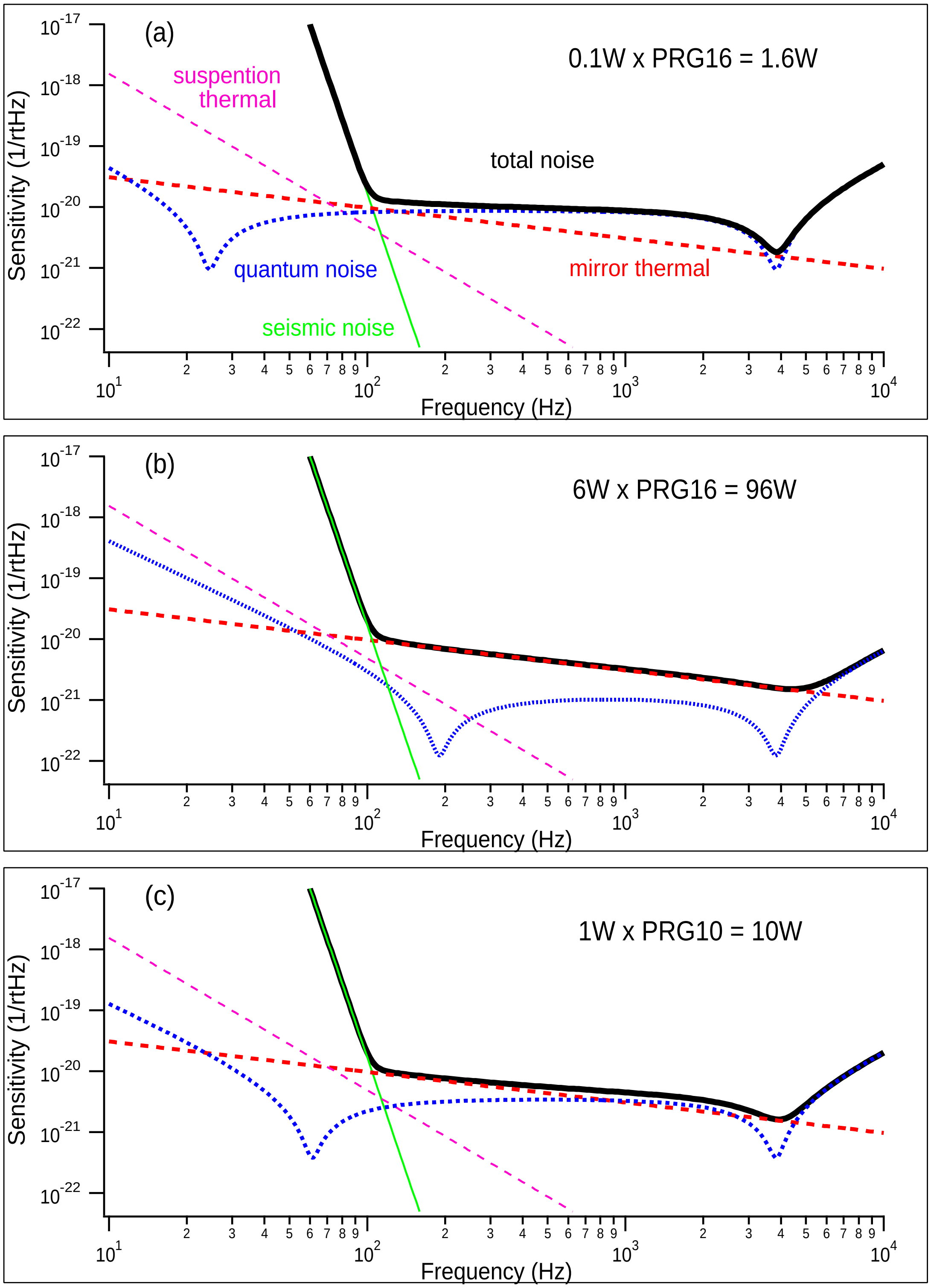

Figure 1: The calculated 40m noise spectra with different incident powers

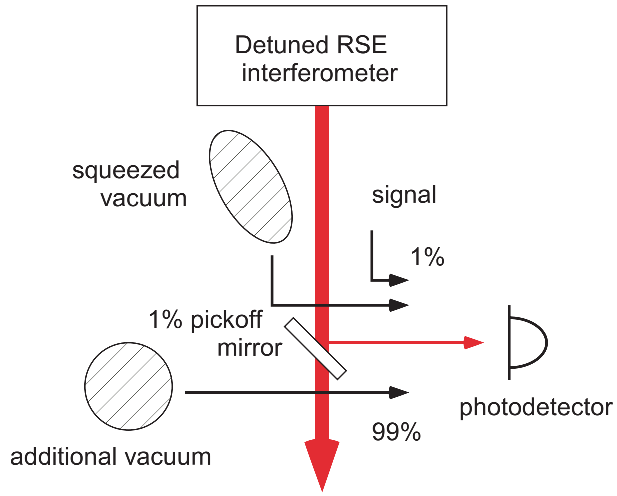

Figure 2: Additional vacuum fluctuation is injected from the other side of the pick-off mirror.

$$M = 1 + \rho^2 e^{4i\beta} - 2\rho e^{2i\beta}\left(\cos 2\phi + \frac{\kappa}{2}\sin 2\phi\right),$$

$$C_{11} = C_{22} = (1+\rho^2)\left(\cos 2\phi + \frac{\kappa}{2}\sin 2\phi\right) - 2\rho\cos[2\beta],$$

$$C_{12} = -\tau^2(\sin 2\phi + \kappa\sin^2\phi), \quad C_{21} = \tau^2(\sin 2\phi - \kappa\cos^2\phi),$$

$$D_1 = -(1+\rho e^{2i\beta})\sin\phi, \quad D_2 = (1-\rho e^{2i\beta})\cos\phi\,,$$

where the parameters to be used are

$$\kappa = \frac{(I_0/I_{\rm SQL})2\gamma^4}{\omega^2(\gamma^2+\omega^2)}, \quad \beta = \arctan\frac{\omega}{\gamma}$$
$$I_{\rm SQL} = \frac{mL^2\gamma^4}{4\Omega}, \quad h_{\rm SQL} = \sqrt{\frac{8\hbar}{m\omega^2 L^2}}\,.$$

Here $a_j(j=1,2)$ is the vacuum fluctuation in two quadrature phases injected from the dark port, which is composed of the creation and annihilation operators, a'_j is also the vacuum fluctuation that is injected from the additional pickoff mirror, b_j is the output vacuum that causes quantum noise at the signal extraction, ρ and τ are the reflectivity and transmittance of the signal recycling mirror, ϕ is the detune phase, γ is the cavity pole, and ω is the signal frequency. All the parameters are same as defined in reference [6]. The input-output relation gives the quantum noise sensitivity with readout phase ζ as

$$h_{\rm n} = \frac{h_{\rm SQL}}{\sqrt{2\kappa}}\sqrt{\frac{(C_{11}\sin\zeta + C_{21}\cos\zeta)^2 + (C_{12}\sin\zeta + C_{22}\cos\zeta)^2 + |M|^2\cdot(1-R_{\rm po})/R_{\rm po}}{\tau^2|D_1\sin\zeta + D_2\cos\zeta|^2}}\,.$$

The optical spring is the classical effect with the signal being enhanced at around the resonant frequencies of the optical spring. The vacuum coming out from the interferometer is squeezed at the low frequencies but not at the high frequencies so that the shot noise level around the higher peak goes straight up when the additional non-squeezed vacuum is injected from the pickoff mirror.

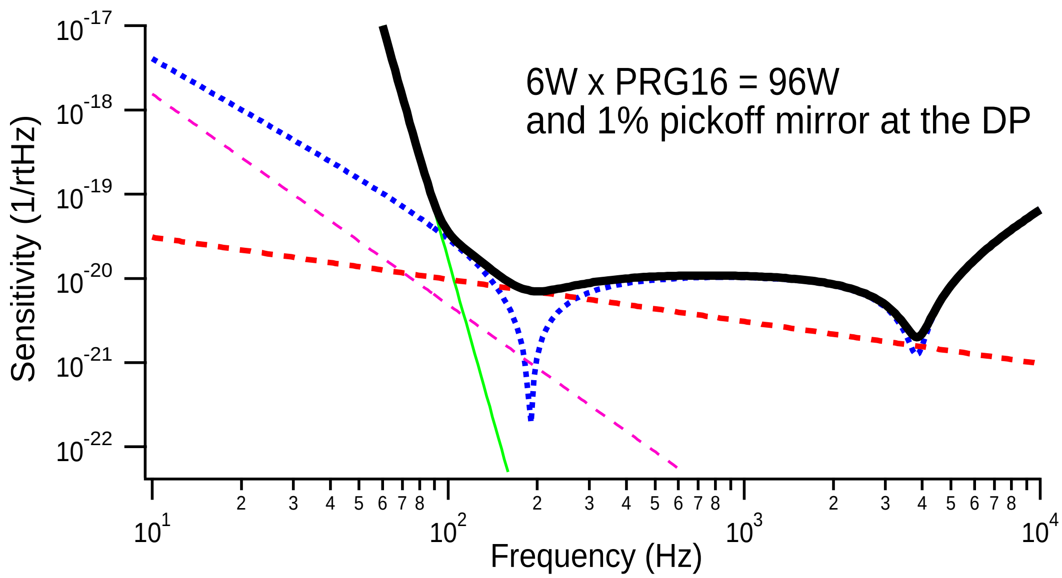

Figure 3: The calculated 40m noise spectrum with additional pickoff mirror at the dark port. Here the DC readout scheme is assumed for the calculation and the readout phase is set to be $\pi/2$.

The calculated result is shown in Fig. 3 and we can see an interesting fact here. We can not only see the higher peak clearly but also a fraction of the lower peak beyond thermal noise. It might be too challenging to try to see this radiation pressure peak in our noise spectrum, but it is worth trying.

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