Thermally distorted Modecleaner analysis: Fused silica vs. Silicon thermal limits

September 24, 2004

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LIGO-T040191-00-Z

We wish to look at the effect a thermally distorted modecleaner would have on the power coupled into the interferometer. For this crude analysis we consider the change in mode overlap of the eigenmode exiting a thermally distorted (hot) mode cleaner with the eigenmode of the undistorted (cold) modecleaner. Assuming the eigenmode of the cold modecleaner is perfectly mode matched into the interferometer, this overlap gives the mode matching of the modecleaner output to the interferometer input.

Before examining the effect of a thermal load on the modecleaner's eigenmode, we consider separately the effect that an increase in the waist size of the output beam has on the mode overlap, and the effect that a shift in the waist position of the output beam has on the mode overlap. We use a triangular modecleaner geometry with length 12m and width 1m. We assume the input-output couplers, separated by 1m are nominally flat and that a 2.6mm waist is located symmetrically between them. Propagating this beam to the far mirror we find a curvature of 44.4m is necessary on the far mirror to match the curvature of the beam and reimage the waist onto itself.

Assuming thermal distortions effect each mirror in the mode cleaner identically, the eigenmode in the hot modecleaner should still have its waist located symmetrically between the input and output coupler, but because of a change in the focusing of the thermally distorted mirrors, the waist size may be different. We look at the mode overlap between a mode with the ideal 2.6mm waist and a beam with a collocated waist of varying sizes. Figure 1 shows the mode overlap as a function of the waist size in the hot modecleaner.

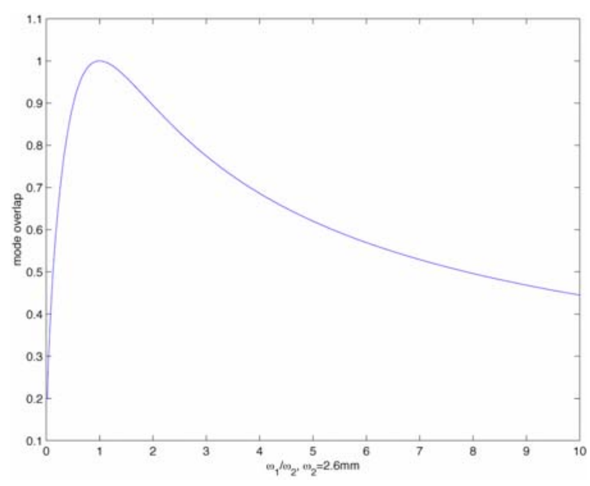

Figure 1. Mode overlap for beams with mismatched waist sizes.

The propagation of the eigenmode through the distorted output-coupler will be like propagating through a thick lens. As such it will lead to focusing causing a change in the waist location as well as the waist size. Figure 2 shows the overlap of beams with identical waist sizes, but with a longitudinal offset between the waists' positions.

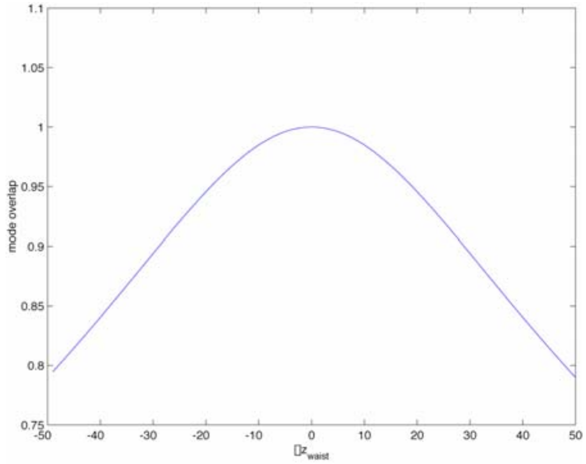

Figure 2. The mode overlap for two beams with identical 2.6mm Gaussian beam waists with a longitudinal separation of the waist positions.

We now consider the thermal distortions produced by power absorption. We use the general results for thermal distortions in an optic illuminated with a Gaussian beam given by Traeger [traeger2004]. The deformation

$$u_z(r,z) = u_c\,\bar{u}_z(\bar{r},\bar{z})$$

is written in terms of a material dependant characteristic displacement

$$u_c = \frac{\alpha \varepsilon P}{\pi \kappa}(1+\nu)$$

and a material independent part

$$\bar{u}_{\bar{z}}(\bar{r}) \approx \frac{1}{8}\left[\mathrm{E}_1\left(2\bar{r}^2\right)+\gamma+\ln\left(2\bar{r}^2\right)\right]-\frac{1}{\bar{l}_{th}}\frac{\sqrt{\pi/2}}{8}\left\{2\bar{r}^2 e^{-\bar{r}^2}\left[\mathrm{I}_0\left(\bar{r}^2\right)+\mathrm{I}_2\left(\bar{r}^2\right)\right]+\left[e^{-\bar{r}^2}\mathrm{I}_0\left(\bar{r}^2\right)-1\right]\right\}$$

where α is the coefficient of thermal expansion, ε is the fraction of power absorbed, P is the incident optical power, κ is the thermal conductivity and ν is Poisons ratio, E_1 is the exponential integral function and the I_n are the modified Bessel functions of order n. The radial coordinate $\bar{r} \equiv r/\omega$ is scaled by the Gaussian beam width ω, and the parameter $\bar{l}_{th} = \kappa/\left(4\omega\sigma' T_{ext}^3\right)$ with σ' the Stefan-Boltzmann constant corrected for an emissivity<1 and T_{ext} the external temperature, characterizes the relative importance of radiation versus conduction for removing heat from the illuminated. We evaluate the thermal distortion for fused silica substrates and for silicon.

Material Properties		
	Silicon	Fused Silica
α [1/K]	$2.6\mathrm{x}10^{-6}$	$0.54\mathrm{x}10^{-6}$
k [w/m-K]	141.2	1.38
ν	0.2154	0.17
$\sigma'\,[W/m^2K^2]$	$2.8\ 10^{-8}$	$2.8\ 10^{-8}$

Table 1: This table contains the materials constants used for calculating thermal distortions in fused silica and Silicon.

We calculate the eigenmode for the modecleaner for optics with varying levels of distortions corresponding to the power absorbed in each substrate. Figure 3 shows the waist size in the hot mode cleaner with fused silica substrates as a function of the power absorbed on each mirror. As the absorbed power reaches about 1.2W the defocusing caused by the thermal distortions is enough to make the cavity unstable.

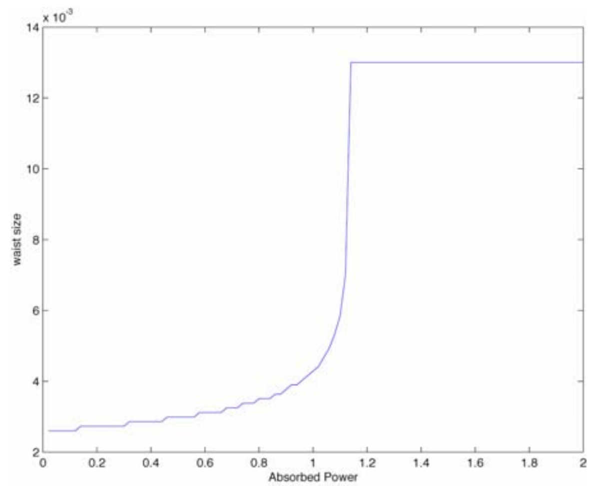

Figure 3 the waist size in the interferometer as a function of absorbed power in each mirror. The jaggy curve is due to the numerical resolution used in the simulation. The waist size diverges at about 1.2W. At higher powers the cavity is unstable and the curve plotted is meaningless.

Finally we combine all of this to give the coupling to the interferometer as a function of the power absorbed in the modecleaner. We consider the eigenmode of the hot modecleaner and the propagation of this mode through the thermal distortion on the output coupler (we ignore the dn/dt effects in the output coupler, but since we expect it o be of similar magnitude as the thick lens due to thermal distortion, and we have found that the coupling degradation is dominated by the change in size of the eigenmode's waist we believe this is negligible). Figure 4 shows the coupling of the modecleaner output to the interferometer input as a function of absorbed power for both fused silica and Silicon.

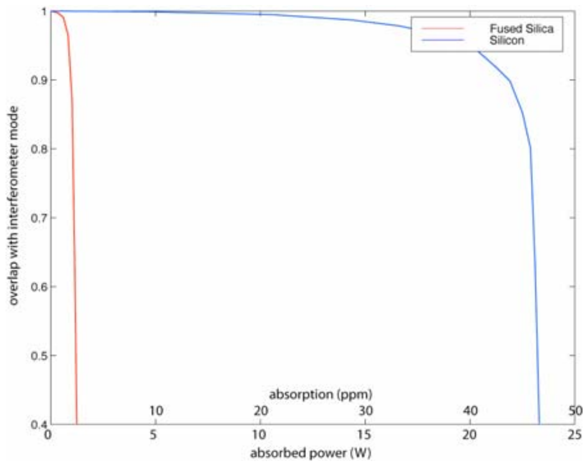

Figure 4. The coupling of power from the output of the thermally distorted modecleaner to the input of the interferometer. The Advanced LIGO modecleaners will have 500kW of circulating power thus an absorbed power of 1W is equivalent to 2ppm of absorption.

We find that for the conditions of this analysis, without prefiguring the optics to offset the thermal distortions, and without active thermal compensation, the modecleaner with fused silica optics only operates with absorption below about 2ppm per mirror, but an all-reflective silicon design will operate up to about 40ppm. Beyond these levels the coupling to the interferometer decreases as the modecleaner becomes unstable. This provides motivation for considering all-reflective modecleaners using silicon optics in Advanced LIGO.

References

[Traeger 2004] Stefan Traeger, Peter T. Beyersdorf, Patrick Lu, Justin D. Mansell, R.G. Beausoleil, E.K. Gustafson, R.L. Byer and M.M. Fejer, *"Wavefront distortion of the specular and diffracted beams produced by thermo-elastic deformation of a diffraction grating heated by a Gaussian laser beam"*, submitted to Classical and quantum gravity (2004)