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Laser Interferometer Gravitational Wave Observatory (LIGO) Project

To:	Advanced LIGO Systems Group
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Refer to:	T040199-00-R
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Subject: **Dimensions for Advanced LIGO Fused Silica Test Masses**

Summary:

The mass for fused silica test masses in Advanced LIGO would be 40 kg, identical to that for the sapphire test mass option. To date, the working dimensions of a fused silica test mass have been 340 mm diam × 200 mm thick. This note explores the performance impacts of varying the aspect ratio around this nominal design, asking the question: is there a better choice of aspect ratio, from the standpoint of interferometer performance? The conclusion is that in general there are benefits in having a thinner, larger radius test mass, but these benefits are small, and they appear to be insufficient to cause a change in the baseline dimensions.

Performance effects of aspect ratio

The aspect ratio a is the ratio of test mass radius to thickness; for the baseline design $a = 0.85$. The range considered here is $a = 1.19 – 0.65$, corresponding to a thickness range of 0.16–0.24 m, and a radius range of 0.155–0.19 m. The following effects of the aspect ratio are considered, some of which are shown in Figure 1.

Diffraction loss. For a larger aspect ratio, and a fixed beam size, the diffraction loss will be smaller. Since the total test mass optical loss budget of 37.5 ppm will be difficult to meet, this could be an important effect. The estimated diffraction loss versus aspect ratio is shown in Figure 1. The diffraction loss for the baseline aspect ratio is already quite low, approximately 0.3 ppm (in contrast to the sapphire design), and further increasing the aspect ratio is not terribly useful.

Angular instability. The critical power level for the cavity angular instability is proportional to the test mass moment of inertia (LIGO-T030120-00). The moment of inertia is actually minimized at an aspect ratio $a = (2/3)^{1/2} = 0.8165$, very close to the baseline aspect ratio. So, going to either a smaller radius (aspect ratio) or larger radius (aspect ratio) would increase the moment of inertia. However, as Figure 1 shows, the potential gain over the plausible range of radii is very small (at most 6%).

Thermal noise. Thermal noise (Brownian plus coating) is minimized at an aspect ratio of approximately $a = 0.6$ (only weakly dependent on the assumed coating parameters); this corresponds to a mirror thickness of 250 mm, significantly thicker than the baseline dimension. However, as Figure 1 shows, even at the thin end of the plausible thickness range, thermal noise motion is increased by only 2%. Another factor to consider is that a thinner, larger radius optic could better support a design change to a larger radius beam, and a larger beam would experience lower thermal noise (coating thermal noise scales inversely with beam size). For example, thermal noise for a 6.5 cm radius beam is about 7% smaller than for the baseline size, but diffraction loss would be about 3.5 ppm if the optic radius were kept at 0.17 m.

Loss from small angle scatter. Brett Bochner's FFT modeling showed that the effective optical loss per arm cavity mirror depended not only on the mirror surface quality, but also on the mirror diameter (see Bochner's Ph.D. thesis, p 159, Fig. 4.10). A larger diameter mirror will capture more light scattered at small angles (by the other arm mirror), and feed it back into the arm cavity mode. Bochner's analysis was for initial LIGO beam sizes, starting with the initial LIGO mirror radius and increasing it from there. It is not clear how exactly to scale his results to the advanced LIGO design, but it is worth reviewing what he saw: for mirrors with 1.3 nm-rms distortion, the stored arm cavity power increased by 30% when the mirror radius was increased from 0.12 m to 0.2 m (with most of the increase coming by 0.18 m radius). A couple of factors are different for advanced LIGO: the rms mirror distortion should be smaller (sub-nm); given the larger beam size, larger spatial scale distortions will be important for scattering. Both of these would tend to reduce the effect of increasing the mirror radius: for smaller distortion there is less scattered light to recover; and larger scale distortions scatter light into smaller angles. Best would be to directly model the effect of mirror radius using adanced LIGO parameters, but I would guess that increasing the radius from the baseline value would increase the arm power by no more than 10%.

Thermal loading. A thinner optic will obviously absorb less power, but the variation of total absorbed power over the thickness range is not large. The coating absorption is expected to be: 800kW×0.6 ppm = 480mW; and the expected substrate absorption is: 2×1kW×3ppm/cm×20cm = 120mW. Reducing the optic thickness to 0.16m would only reduce the total absorbed power from 600mW to 574mW (about 5%).

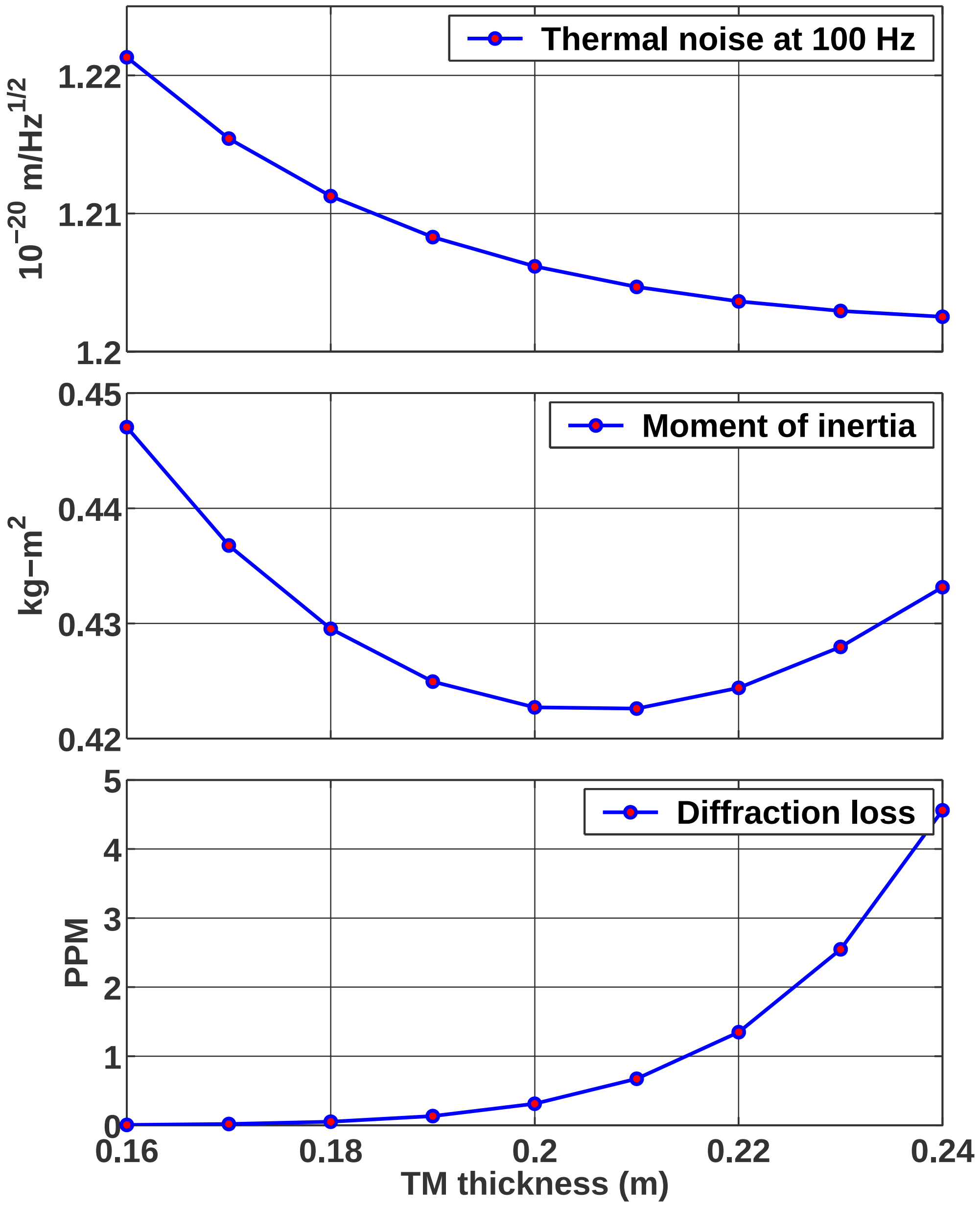

Figure 1. Thermal noise, moment of inertia, and diffraction loss as a function of test mass thickness, for 40 kg fused silica test masses. Test mass radius ranges from 0.19 m (far left) to 0.155 m (far right). Thermal noise is for 6 cm beam radius, nominal bulk and coating mechanical parameters. The diffraction loss is estimated to be 3 times the geometrical clipping loss value.