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TCS Maintenance of Unlocked Interferometer

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INTRODUCTION

This note describes a purely mathematical insight in the heating and cooling of cylindrical test masses (TM) by Gaussian heating beams. This is the classic thermal lensing (TL) phenomenon of the LIGO interferometers. The inferences of this insight for the very practical problem of cool off when the interferometers "drop lock" is discussed. It has been established that the TCS auxiliary TM surface heating system could *compensate* for the interferometer beam TL[1]. However it would also be desirable to maintain the steady state (SS, compensated) TL when the interferometer spontaneously drops out of lock (until it re-locks some uncertain time later). It has not been clear how the evolution of the TL profile in cooling is related to either the SS or to the heating evolution, so that a related out of lock compensation might be applied. The relations between heating and cooling profiles derived here help toward that design. This work starts where the work of Hello and Vinet[2] leaves off and borrows their same notation.

1. Hello and Vinet Analysis

Hello and Vinet[H/V:2] describe axi-symmetric *heating* T(t,r,z) of right cylindrical masses by Gaussian beams. We wish to extend that to *cooling* from the SS $\mathrm{T}_\infty(\mathrm{r,z}) \equiv \mathrm{T}(\mathrm{t} \to \infty,\mathrm{r,z})$. Use will be made of the surprising near identity (as illustrated in Fig. 10 of [2]) of the lensing caused by bulk and [HR]surface coating heat, *at every t*[3].

The heat diffusion ("Fourier" in [2]) equation is linear. H/V take advantage of the fact that the radiative B.Cs can also be linearized in the regime (small T swings relative to ~300$^{\mathrm{o}}$K) of interest. In this situation the net absolute temperature field in the TM may be usefully expressed as $\mathrm{T(t,r,z)} \equiv \mathrm{T_e} + \mathrm{T}_\infty(\mathrm{r,z}) + \mathrm{T_{tr}(t,r,z)}$. Here $\mathrm{T_e}$ is the constant, *environmental* temperature that drives radiative cooling, and $\mathrm{T}_\infty(\mathrm{r,z})$ is the asymptotic SS rise due to beam heating. Then $\mathrm{T_{tr}(t,r,z)}$ is the residual *transient* which connects $\mathrm{T_e}$ to $\mathrm{T_e} + \mathrm{T}_\infty(\mathrm{r,z})$ during heat up (and vice versa during cool down). If we start heating at t=0 from a uniform "cold" $\mathrm{T_e}$ then it is an identity that $\mathrm{T_{tr}(0,r,z)} = -\mathrm{T}_\infty(\mathrm{r,z})$, and that $\mathrm{T_{tr}}(\infty,\mathrm{r,z}) = 0$.

What is the relation, in this syntax, between heating and cooling? First, we assume that heating will always mean "*from* $\mathrm{T(0,r,z)} \equiv \mathrm{T_e}$ at t=0", and that cooling will be "*from* $\mathrm{T}(\mathrm{t} \to \infty,\mathrm{r,z}) = \mathrm{T_e} + \mathrm{T}_\infty(\mathrm{r,z})$". We also limit to the other assumptions of H/V: exact axial symmetric Gaussian [beam] heating, and complete homogeneity of optical properties.

Our LIGO TL situation involves *both* bulk (B) and surface (S) heating. In the H/V analysis these are distinguished by [Fourier] *equation* inhomogeneity for B, and *B.C.* inhomogeneity for S. In either case the inhomogeneity is expressed in the full solution entirely through T_∞. Then, for *either* B or S heating, the T_{tr} part is a solution of the same [*homogeneous*] equation and B.Cs. But such a homogeneous equation and B.Cs. are just those pertaining to cooling.

2. Cooling off from the Hello Vinet solutions

Since everything (equations and B.Cs.) is linear we can separately treat the B and S contributions. First consider cooling from an S contribution. From the last section we have that the SS *heated* solution is $T(t \to \infty,r,z)=T_e + T^S_\infty(r,z)$, attained via a heat up transient $T^S_{tr}(t,r,z)$ such that $T^S_{tr}(0,r,z) = -T^S_\infty(r,z)$ and $T^S_{tr}(\infty,r,z) = 0$. For cooling we want the homogeneous solution T(t',r,z) starting at $T(t'=0,r,z)=T_e + T^S_\infty(r,z)$ where $t\text{-}t' \to \infty$. The cooling transient which satisfies this initial temperature field, the homogeneous equation and B.Cs, as well as $T(t' \to \infty,r,z)=T_e$ is *uniquely* $-T^S_{tr}(t',r,z)$. That is, the essential temperature transient field is the *same* for cooling and heating. Note also that we are always physically describing diffusion *forward* in time. An initial temperature field is always *decaying* away. There is no sense of cooling/heating identity based on *reversal* of time.

Now consider cooling from a B contribution. But, once the heating source is turned off, *any* cooling transient is specified by the same homogeneous equation and B.Cs, as in the above S cooling. Only in the initial t'=0 field $T^B_\infty(r,z)$ differs. Therefore the bulk cooling must uniquely be described by $T(t',r,z)=T_e\text{-}T^B_{tr}(t',r,z)$.

3. Lensing during cooling

We now have the general temperature field during cooling from an asymptotic SS beam surface and bulk absorbing state:

$$T(t',r,z)=T_e\text{-}T^S_{tr}(t',r,z)\ \text{-}T^B_{tr}(t',r,z)$$

Now consider the numerical results of H/V compiled in their Figure 10. This shows, that for a typical TM geometry, the first five (and implicitly all) relevant Zernike aberration polynomials for TL are nearly the same for either S or B *heating* (when normalized to unit absorbed power). This was previously recognized as a fact making it difficult to distinguish the actual ratio of S to B absorption in LIGO interferometers [4]. Here we use it to our advantage, taking it to be an *exact* identity for our TM geometry. For any temperature field T(r,z) we denote the resultant TL by $\mathcal{L}[T(r,z)]$ (at least this represents the beam *self* lens, i.e. on axis). This functional will be *linear* since $(T(r,z)\text{-}T_e)\partial n/\partial T$ is always <<1. Then for *any* S and B absorption the identity means $\mathcal{L}[T^B_\infty(r,z)]/\beta = \mathcal{L}[T^S_\infty(r,z)]/\sigma \equiv \tilde{\mathcal{L}}(\infty)$ where β and σ are the bulk and surface absorption coefficients respectively. Linearity then implies also

$\mathcal{L}[T^B_{tr}(t,r,z)]/\beta = \mathcal{L}[T^S_{tr}(t,r,z)]/\sigma \equiv \tilde{\mathcal{L}}(\infty) - \tilde{\mathcal{L}}(t)$ since the identity holds at every time. During heating then the net lens is

$$\mathcal{L}[T^B_{tr} + T^B_{\infty} + T^S_{tr} + T^S_{\infty}] = (\beta+\sigma)\tilde{\mathcal{L}}(t)$$

and during cooling the net lens is

$$\mathcal{L}[-T^B_{tr} - T^S_{tr}] = (\beta+\sigma)(\tilde{\mathcal{L}}(\infty) - \tilde{\mathcal{L}}(t'))$$

4. **Maintenance**

So far we have discussed only lensing due to S and B [ifo] beam heating. For compensation or maintenance we include also the effect of a TCS beam. This is assumed to deposit heat identically to that which the ifo beam does in S, except for a uniform intensity scale factor. Therefore, when switched on the TCS heating will produce an incremental temperature field $\propto (T^S_{tr}(t',r,z) + T^S_{\infty}(r,z))/\sigma$. If the proportionality constant is μ, then, if t'=0 is the instant lock drops and this TCS central heating is switched on, the net lens will evolve as

$$\mathcal{L}[-T^B_{tr} - T^S_{tr} + \mu(T^S_{\infty} + T^S_{tr})] = (\beta+\sigma)(\tilde{\mathcal{L}}(\infty) - \tilde{\mathcal{L}}(t')) + \mu\tilde{\mathcal{L}}(t')$$

We see that the entire transient lensing vanishes when μ=β+σ and the asymptotic steady state ifo beam TL is preserved. This is the basic result for TCS maintenance of SS TL at lock drop.

Of course, the practical situation is more complex. The TCS exists to compensate out of specification "hot" SS TL, so that it is normally on *during lock* at some strength for asymptotic $t \to \infty$ described previously. However, again if the context is purely axi-symmetric and homogeneous in optical properties, this will merely alter the coefficient $\sigma_{effective} = \sigma + \sigma_{TCS}$ in our equations. The possibility of σ_{TCS} being <0 (annular heating) is accommodated. Also, there are inevitable departures from the strict geometry prescribed by H/V (inhomogeneous absorption, TCS beam not Gaussian, centered, or same Gaussian as ifo beam, etc.). Therefore the reference solutions of H/V and the heating/cooling identity will not hold so well. However it is likely that the mechanism described here will *approximately* hold, especially in the complementary relation between time evolution in heating up vs cooling down, which is fundamental. An *approximate* aid in maintaining the "hot" state is all we need. It has been demonstrated that the TCS servos and stable locking can be restored with no such maintenance, if the lock drop is reasonably short (<5 minutes).

One flaw, in principle, is that the ideal maintenance will only work if lock drops in the asymptotic SS, i.e. when $\mathrm{T(t)} \to \mathrm{T_e} + \mathrm{T_\infty}$. This is *not* just a matter of choosing μ correctly according to the finite time of the lock (from cold). The solutions are not time reversal invariant. For such finite heating times D, one could only achieve maintenance via TCS if $\mathcal{L}[\mathrm{T^S_{tr}}(\Delta,\mathrm{r,z}) + \mathrm{T^S_{tr}}(\Delta,\mathrm{r,z}) + \mathrm{T^S_\infty}(\mathrm{r,z}) + \mathrm{T^B_\infty}(\mathrm{r,z})] \propto \mu\mathcal{L}[\mathrm{T^S_\infty}(\mathrm{r,z})]$ or that $\tilde{\mathcal{L}}(\Delta) \propto -\tilde{\mathcal{L}}(\infty)$ which cannot hold for any strength of the TCS beam. On the other hand, a TL maintenance scheme is probably more useful the longer a lock is held.

REFERENCES:

[1] LHO e-log entries, S. Ballmer, et. al. and LIGO I commissioning reports, 9/04-1/05.

[2] P. Hello and J-Y. Vinet, J. Phys. France **5**, 1267 (1990).

[3] It begs to be investigated why these two heating sources give the same lens. It is clear that their temperature fields are qualitatively different. Is the difference in the numerical result even correct? Perhaps it might become a true identity in an appropriate limit (say transverse dimension of mass->infinity).

[4] W. Kells, “The Thermal Lens in LIGO I” talk given at GWADW, Aspen, February, 2003.