LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T050057-00-D *Advanced LIGO* 7 Apr 2005

Beamsplitter
Coating Strain Induced Radius of Curvature
(Advanced LIGO)

Dennis Coyne

Distribution of this document:
COC subsystem group

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project – MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW17-161
175 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

1 Introduction

There are many factors that need to be considered in choosing appropriate dimensions for the beamsplitter for advanced LIGO (e.g. manufacturability, coating strain induced distortion, physical limits to fitting in the optical layout, clipping/diffraction loss, vibration isolation performance, etc.). This memo addresses the effect of coating strain on the radius of curvature (ROC) as a function of the beamsplitter diameter and thickness. The effect of coating strain can be compensated by various approaches such as pre-figuring, balancing with opposite face, or barrel, coating stress, or thermal compensation.

2 Coating Strain Calculation

Kirchoff thin plate bending analysis has been shown[1] to calculate the change in radius of curvature (ROC) due to coating stress, for even the low aspect ratios (diameter over thickness) of LIGO test masses. When applied to the initial LIGO test masses (25 cm diameter and 10 cm thickness fused silica), plate bending theory calculates a ROC for the central region (6 cm diameter) which is within 15% of the value calculated with a finite element analysis. Better accuracy is expected for the BS due to it's higher aspect ratio (i.e. more plate like).

The analytical formulation in reference 1 indicates (as one might suspect) that the ROC change is independent of the optic radius and only dependent upon the substrate thickness, substrate elastic modulus and differential (HR minus AR) coating stress.

The largest uncertainties in the calculation of the coating induced ROC change are the coating process (ion beam sputtering) induced strain and the effect of high temperature annealing on reduction of the built-in coating stress. The origin of the approximate coating strain values (for Ta_2O_5 and SiO_2) are given in reference 1. Limited experimental data from which one can derive an annealing stress reduction factor, is discussed in the next section.

3 Coating Induced ROC Change Data

In the course of the development of low absorption and low mechanical loss coatings for advanced LIGO, a number of small "coupons" were coated. The surface figure of these coupons (3 in diameter by either 1 inch or 0.1 inch thickness) was measured before coating, after coating and then after annealing. Using this data one can derive an approximate factor for the stress reduction effect of the annealing process (Tables 1 and 2). Sapphire results are shown just for completeness; The beamsplitter substrate is fused silica. Fused Silica has also been chosen as the test mass substrate material for advanced LIGO. There were no post annealing measurements for sapphire substrates so no annealing stress reduction factors are derived for sapphire substrates.

The coating R&D to date[2] suggests that a doped tantalum and silica coating is likely to be used for advanced LIGO. For the Ta_2O_5 / SiO_2 coatings on fused silica, the apparent annealing stress reduction factor is ~0.10 for the thin samples and ~0.26 for the thick samples. For use in predicting

---

[1] Kartik Srinivasan, Coating Strain Induced Distortion in LIGO Optics, LIGO-T970176-00, Oct. 1997.

[2] Greg Harry, Coating Program Status Report, LSC Meeting at LIGO Livingston Observatory, LIGO-G05pending, Mar 22, 2005.

residual coating induced ROC changes for adv. LIGO optics, I suggest that we use 0.26 to be conservative, i.e. 26% of the stress remains after annealing.

Table 1: ROC Changes for Fused Silica Substrates

				power (waves @ 633 nm)						
SN	**Dia. (cm)**	**Thickness (cm)**	**Coating**	**uncoated**	**coated**	**Δ coated**	**annealed**	**Δ annealed**	**anneal factor**	**Notes**
15	7.62	0.254	30 layers, Ta2O5/SiO2	0.569	-25.388	-25.957	-2.196	-2.765		450C, 24 hr
16	7.62	0.254	30 layers, Ta2O5/SiO2	0.798	-28.616	-29.414	-2.320	-3.118		450C, 24 hr
					avg	-27.69	avg	-2.94	0.106	
					calc	-27.72	calc	-2.94		
18	7.62	2.54	30 layers, Ta2O5/SiO2	0.003	-0.202	-0.205	-0.057	-0.060		450C, 24 hr
19	7.62	2.54	30 layers, Ta2O5/SiO2	0.020	-0.206	-0.226	-0.032	-0.052		
					avg	-0.216	avg	-0.056	0.260	
					calc	-0.277	calc	-0.029		
8	7.62	0.254	30 layers, Nb2O5/SiO2	0.110	-10.005	-10.115	-3.246	-3.356		
17	7.62	0.254	30 layers, Nb2O5/SiO2	0.855	-9.814	-10.669	-3.014	-3.869		
					avg	-10.39	avg	-3.61	0.348	
9	7.62	2.54	30 layers, Nb2O5/SiO2	-0.017	-0.148	-0.131	-0.069	-0.052		
10	7.62	2.54	30 layers, Nb2O5/SiO2	-0.021	-0.148	-0.127	-0.058	-0.037		
					avg	-0.13	avg	-0.04	0.345	
26	7.62	0.254	30 layers, SiO2/Al2O3	0.346	-8.443	-8.789	-5.518	-5.864		
27	7.62	0.254	30 layers, SiO2/Al2O3	0.330	-13.027	-13.357	-12.264	-12.594		
					avg	-11.07	avg	-9.23	0.833	
22	7.62	2.54	30 layers, SiO2/Al2O3	0.008	-0.153	-0.161	-0.134	-0.142		
28	7.62	2.54	30 layers, SiO2/Al2O3	0	-0.162	-0.162	-0.146	-0.146		
					avg	-0.16	avg	-0.14	0.892	
16	7.62	0.254	30 layers, Ta2O5/Al2O3	0.378	-10.186	-10.564	0.602	0.224		350C, 12 hr
27	7.62	0.254	30 layers, Ta2O5/Al2O3	0.332	-9.922	-10.254	0.626	0.294		350C, 12 hr
					avg	-10.41	avg	0.26	-0.025	
19	7.62	2.54	30 layers, Ta2O5/Al2O3	0.011	-0.047	-0.058	0.039	0.028		350C, 12 hr
20	7.62	2.54	30 layers, Ta2O5/Al2O3	-0.013	-0.106	-0.093	-0.001	0.012		350C, 12 hr
					avg	-0.08	avg	0.02	-0.265	

Table 2: ROC Changes for Sapphire Substrates

				power (waves @ 633 nm)					
SN	Dia. (cm)	Thickness (cm)	Coating	uncoated	coated	Δ coated	annealed	Δ annealed	Notes
A01	7.62	2.54	30 layers, Ta2O5/SiO2		-0.087	-0.087			65% aperture
M05	7.62	2.54	30 layers, Ta2O5/SiO2		-0.081	-0.081			65% aperture
					avg	-0.08			
					calc	0.05			

4 Beamsplitter ROC Change

The nominal dimensions of the beamsplitter for advanced LIGO are defined in the COC Design Requirements Document[3] as 350 mm diameter and 60 mm thick. We do not yet have coating designs for the advanced LIGO beamsplitter; In lieu of advanced LIGO coating designs, we'll use the initial LIGO beamsplitter coating design for estimation[4] (8 layer high reflectance (HR) coating and 3 layer anti-reflection (AR) coating). What really matters is the difference in number of layers between the HR and AR coatings (in the nominal case 5) and not the absolute number of layers.

The variation in ROC for changes in the HR-AR coating layer difference and for variation in the BS fused silica substrate thickness are indicated in Figure 1. For the nominal case of a 60 mm thick BS with a 5 layer difference between HR and AR coatings, the ROC resulting from residual coating strain is estimated to be 533 km (without measures taken to compensate for the ROC

---

[3] G. Billingsley, G. Harry, W. Kells, Core Optics Components Design Requirements Document, LIGO-T000127-01, Jun 20, 2004.

[4] The BS initial LIGO coating is defined in reference 1 as 7 layers for the HR and 2 layers for the AR. The AR coating was prone to change if/as the outer layer was etched. Here I've assumed a design which is made insensitive to the thickness of an added top or cap layer.

change). If the acceptable ROC change[5] is 200 km, then the following layer difference and thickness combinations are acceptable, without ROC compensation:

Table 3: Combinations of Coating Layer Difference Number (HR – AR) and thickness to meet a 200 km minimum ROC

# HR Layers - # AR Layers	Minimum Thickness (mm)
5	37
10	52
15	64
20	74
25	82

Other considerations, such as manufacturability, are likely to require the thickness to be at least 60 mm. If the difference in the number of coating layers (HR – AR) is as few as for initial LIGO (5), then it would appear that a 200 km ROC can be achieved without resorting to means to compensate for the coating strain induced ROC change.

5 Parameters

The parameters used in the calculations for fused silica and sapphire are given in Tables 4 and 5.

---

[5] Bill Kells, personal communication: Bill made a very rough estimate that 200 km ROC might be acceptable from the standpoint of degradation to the gravity-wave sidebands, 4/7/2005.

Figure 1: ROC Change versus Thickness and Difference in HR and AR Layer Numbers

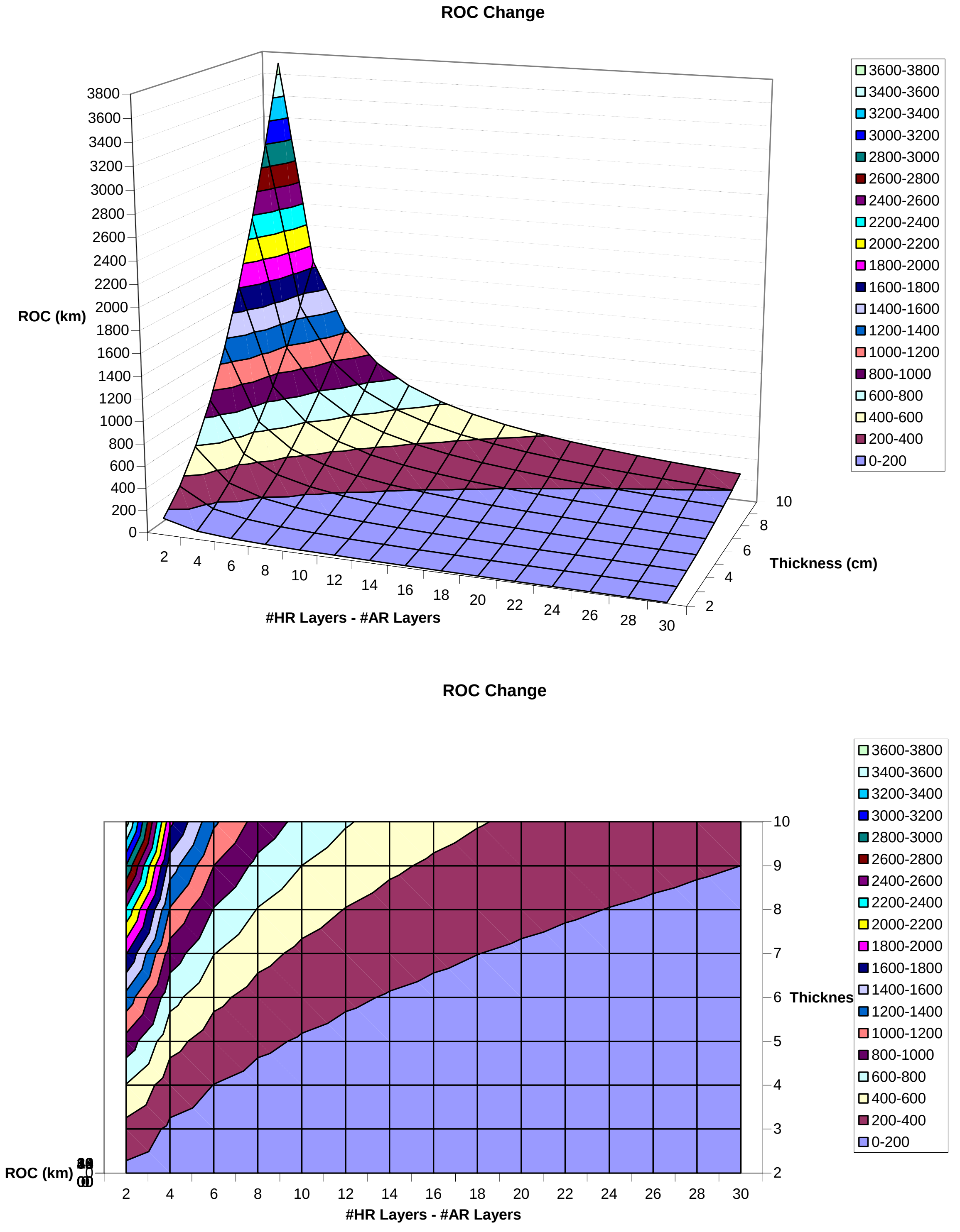

Table 4: Fused Silica and Coating Properties

Material Parameters							
value	**units**	**symbol**	**name**				
72	Gpa	Es	substrate (fused silica) elastic modulus				
0.17	--	ns	substrate (fused silica) Poisson's ratio				
72	GPa	E1	SiO2 film elastic modulus				
0.17	--	n1	SiO2 film Poisson's ratio				
140	GPa	E2	Ta2O5 film elastic modulus				
0.23	--	n2	Ta2O5 film Poisson's ratio				
5.10E-07	1/C	as	substrate (fused silica) thermal coeff. of expansion				
5.10E-07	1/C	a1	SiO2 thermal coeff. of expansion				
2.42E-06	1/C	a2	Ta2O5 thermal coeff. of expansion				
22	C	dT	Temperature change				
Locked-in Coating Strain				**Thermal Expansion Strain**			
Coating process parameters							
0.28875	Gpa	s1	SiO2 film stress (unannealed)	0.00E+00			SiO2 thermal expansion stress (restrained)
0.168	Gpa	s2	Ta2O5 film stress (unannealed)	5.88E-03			Ta2O5 thermal expansion stress (restrained)
0.26	--	sr	stress reduction factor due to annealing				
5	--	nhr - nar	#HR layers - #AR layers				
8	--	nhr	total number of HR layers				
1.064	microns	t1hr	HR: SiO2 film total thickness				
1.064	microns	t2hr	HR: Ta2O5 film total thickness				
2.128	microns	tfhr	HR: film total thickness				
3	--	nar	total number of AR layers				
0.399	microns	t1ar	AR: SiO2 film total thickness				
0.399	microns	t2ar	AR: Ta2O5 film total thickness				
0.798	microns	tfar	AR: film total thickness				
optic parameters							
17.5	cm	r	radius				
6	cm	t	thickness				
HR Coating:							
106	Gpa	Efhr	Film effective elastic modulus	1.29E-06	1/C	ahr	effective coating coeff. of thermal expansion (restrained by substrate)
0.2092846	--	nfhr	Film effective Poisson's ratio				(compares to 1.8e-6 unconstrained by the substrate)
5.48E-04	--	e0hr	average coating strain	2.84E-05	--	ethr	average thermal expansion strain
3.004E-06	1/m	Khr	curvature	1.56E-07	1/m	Kthr	curvature
332929.28	m	Rhr	ROC	6413642	m	Rthr	ROC
45.993251	nm	dhr	sagitta	2.387489	nm	dthr	sagitta
AR Coating:							
106	Gpa	Efar	Film effective elastic modulus				
0.2092846	--	nfar	Film effective Poisson's ratio				
0.000548	--	e0ar	average coating strain	2.84E-05	--	etar	average thermal expansion strain
1.126E-06	1/m	Kar	curvature				
887811.42	m	Rar	ROC				
17.247469	nm	dar	sagitta				
HR & AR Coating							
28.745782	nm	d	sagitta due to locked-in coating strain (from both HR and AR coatings)	2.387489	nm	dt	sagitta due to thermal expansion (from both HR and AR coatings)
0.0270167	waves		at 1064 nm	0.108522	nm/C		sagitta per deg. C
0.045412	waves		at 633 nm				
533	km	ROC	without compensation				

T970076-00 values:		
0.55	Gpa	SiO2 film stress (unannealed)
0.32	Gpa	Ta2O5 film stress (unannealed)
0.33	--	stress reduction factor due to annealing

Table 5: Sapphire Properties

Material Parameters							
value	**units**	**symbol**	**name**				
400	Gpa	Es	substrate (sapphire) elastic modulus				
0.23	--	ns	substrate (sapphire) Poisson's ratio				
72	GPa	E1	SiO2 film elastic modulus				
0.17	--	n1	SiO2 film Poisson's ratio				
140	GPa	E2	Ta2O5 film elastic modulus				
0.23	--	n2	Ta2O5 film Poisson's ratio				
5.40E-06	1/C	as	substrate (sapphire) thermal coeff. of expansion				
5.10E-07	1/C	a1	SiO2 thermal coeff. of expansion				
2.42E-06	1/C	a2	Ta2O5 thermal coeff. of expansion				
22	C	dT	Temperature change				
Locked-in Coating Strain				**Thermal Expansion Strain**			
Coating process parameters							
0.28875	Gpa	s1	SiO2 film stress (unannealed)	-7.75E-03			SiO2 thermal expansion stress (restrained)
0.168	Gpa	s2	Ta2O5 film stress (unannealed)	-9.18E-03			Ta2O5 thermal expansion stress (restrained)
1	--	sr	stress reduction factor due to annealing				
30	--	nhr	total number of HR layers				
3.99	microns	t1hr	HR: SiO2 film total thickness				
3.99	microns	t2hr	HR: Ta2O5 film total thickness				
7.98	microns	tfhr	HR: film total thickness				
0	--	nar	total number of AR layers				
0	microns	t1ar	AR: SiO2 film total thickness				
0	microns	t2ar	AR: Ta2O5 film total thickness				
0	microns	tfar	AR: film total thickness				
optic parameters							
3.81	cm	r	radius				
2.54	cm	t	thickness				
HR Coating:							
106	Gpa	Efhr	Film effective elastic modulus	-3.597E-06	1/C	ahr	effective coating coeff. of thermal expansion (restrained by substrate)
0.20928455	--	nfhr	Film effective Poisson's ratio				(compares to 1.8e-6 unconstrained by the substrate)
2.11E-03	--	e0hr	average coating strain	-7.913E-05	--	ethr	average thermal expansion strain
4.0367E-05	1/m	Khr	curvature	-1.516E-06	1/m	Kthr	curvature
24772.73001	m	Rhr	ROC	-659844.91	m	Rthr	ROC
29.29854722	nm	dhr	sagitta	-1.099963	nm	dthr	sagitta
AR Coating:							
0	Gpa	Efar	Film effective elastic modulus				
0	--	nfar	Film effective Poisson's ratio				
0	--	e0ar	average coating strain	0	--	etar	average thermal expansion strain
0	1/m	Kar	curvature				
inf	m	Rar	ROC				
0	nm	dar	sagitta				
HR & AR Coating							
29.29854722	nm	d	sagitta due to locked-in coating strain (from both HR and AR coatings)	-1.099963	nm	dt	sagitta due to thermal expansion (from both HR and AR coatings)
0.027536229	waves		at 1064 nm	-0.0499983	nm/C		sagitta per deg. C
0.046285225	waves		at 633 nm	-0.0016666			