

# Notch Filter Design

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## 1 Introduction

This report summarizes an investigation made into designing notch filters with the control system for seismic isolation of Advanced LIGO in mind. The notches are implemented in the control system to reduce the effect of the mechanical resonances of the plant at frequencies above the normal upper unity gain frequency. To begin, an experiment controlling a pendulum-spring system was performed as it provided a simplified model of the design process needed for active isolation of the optical tables in Advanced LIGO. Double notch, Chebyshev, and elliptical filters were then compared to the standard two-zero, two-pole notch filter in meeting given parameters. It was found that the double notch filter increased the phase margin of the system at the upper unity gain frequency without complicating the design procedure by introducing ripples in the pass-band. This method of filter design was then integrated in a program with a graphical user interface that automated the identification of resonances within the transfer function of a system and the design of notches to attenuate them.

## 2 Pendulum-Spring Control Experiment

In order to experiment with a simple system where the use of notch filters would be necessary, the problem of controlling a pendulum in one dimension was considered.<sup>1</sup> The pendulum had a photodetector which measured its position and an electromagnet which, by acting on a permanent magnet on the pendulum's side, could control the pendulum's motion in one direction. The position sensor and electromagnet were connected to a computer via dSpace hardware. In order to make the pendulum's motion harder to control, a spring was attached to the pendulum with sufficient mass such that the spring would have a resonance frequency of approximately 8 Hz. This frequency was chosen to parasitically affect the pendulum's resonance at 1 Hz. With the addition of a lead filter,

$$D(s) = 21 \frac{s + 2\pi(1.5)}{s + 2\pi(4)}$$

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<sup>1</sup>The pendulum was designed and built by William Beecher, Colin Campbell, and Daniel Wile as a precision engineering project.

in order to control the system, the resonance of the spring came above unity gain as can be seen in Figure 1. Since we wished to attenuate the resonance of the spring while leaving the other frequencies unchanged, a notch filter, i.e. a filter with a narrow bandstop frequency range, was ideal. The resulting elimination of the spring's resonance from the transfer function of the system with the use of such a notch filter can be seen in Figure 2.

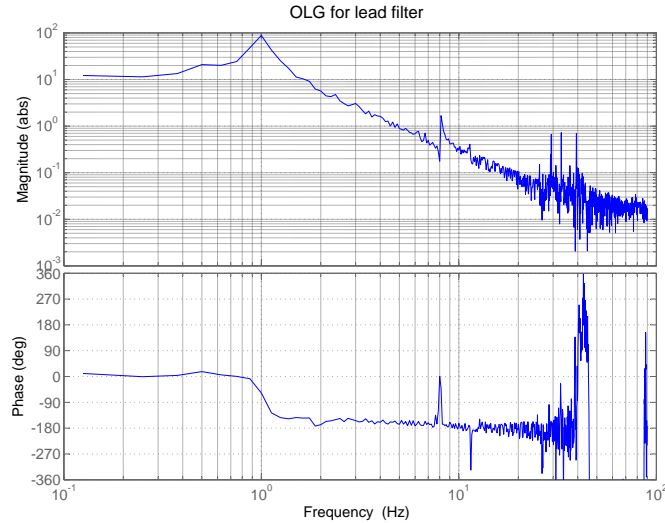


Figure 1: Open loop gain for the pendulum-spring system with a lead controller. The mode at 8 Hz rises above unity gain.

### 3 Notch Criteria and Advanced LIGO

The above example of filter design for the pendulum-spring system is in fact very similar, albeit much simpler, to the way in which controllers will be designed for the active isolation of the optical tables in Advanced LIGO. The control loop for each degree of freedom is designed by first adding a prototype controller filter and then finding and attenuating the resonances characteristic to the system using notch filters. In designing these notch filters, there are two major concerns. First, the notch should provide adequate attenuation of the resonance. Because the resonances of the system could undergo slight shifts in frequency as small adjustments were made to it, it was decided that this requirement would be formalized by requiring that a given notch  $H(s)$  fulfill a given minimal depth requirement within some given frequency band, i.e.  $H(2\pi if) \leq d_0 \forall f \in [f_1, f_2]$ , for given parameters  $d_0$ ,  $f_1$ , and  $f_2$ . Second, the notch should subtract a minimal amount of phase at the upper unity gain frequency of the system. This will

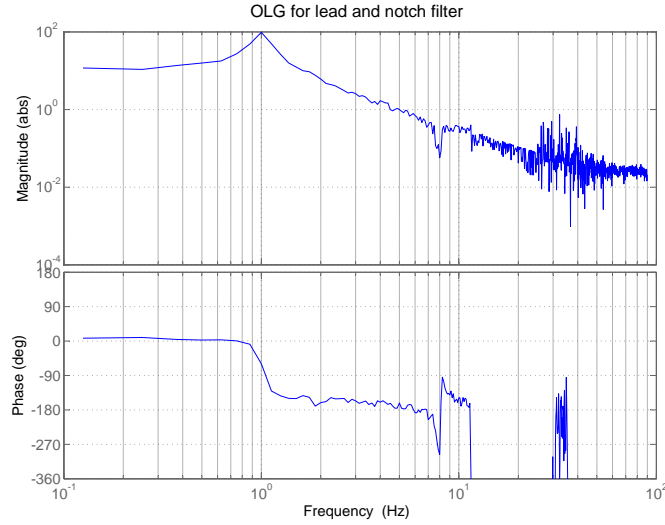


Figure 2: With a notch and lead filter in place the spring's resonance is now attenuated.

decrease the sensitivity,  $S$ , of the system to noise which is defined in terms of the open loop gain  $G$  as  $S := \frac{1}{1+G}$ .<sup>2</sup>

## 4 Notch filter comparison

With these criteria in mind, several different methods for creating notch filters were compared in order to determine if there was a better method than the one currently used.

### 4.1 Standard notch

The current method for designing for notch filters is to use a standard two-zero, two-pole notch filter of the form:

$$H_{sn}(s) = \frac{s^2 + 2\frac{d}{c}\omega_0 s + \omega_0^2}{s^2 + 2\frac{1}{c}\omega_0 s + \omega_0^2} \quad (1)$$

The parameters  $d$  and  $c$  were set by hand in order to fulfill the given requirements.

<sup>2</sup>This equation assumes a negative feedback loop.

## 4.2 Double notch

One alternative that was investigated was the possibility of placing two of the standard notches (1) next to each other so that the center frequency of the two notches were slightly offset from the center frequency that they attenuated:

$$H_{dn}(s) = \frac{s^2 + 2\frac{d}{c}\omega_1 s + \omega_1^2}{s^2 + 2\frac{1}{c}\omega_2 s + \omega_2^2} \cdot \frac{s^2 + 2\frac{d}{c}\omega_2 s + \omega_2^2}{s^2 + 2\frac{1}{c}\omega_2 s + \omega_2^2} \quad (2)$$

In general, this results in notches that have faster attenuation than the standard notches. Instead of determining the values of  $d, c, \omega_1$  and  $\omega_2$  by hand, an algorithm was written as a function in MatLab. For given requirements  $d_0, f_1$ , and  $f_2$ , the algorithm set  $d = d_0^2$  and varied the values of  $\Delta = \omega_2 - \omega_0 = \omega_0 - \omega_1$  and  $c$  in order to find the double notch filter that fulfilled the requirement that  $H_{dn}(2\pi i f) \leq d_0 \forall f \in [f_1, f_2]$  while minimizing the phase subtracted at the upper unity gain frequency.

## 4.3 Chebyshev and elliptical filters

Another method of filter design investigated was to use MatLab's built in algorithms for calculating Chebyshev and elliptical bandstop filters. These filter algorithms were given frequency ranges to insure that they fulfilled the given parameters and then an algorithm determined the maximum amount of damping that could be added (by subtracting small values from the zeros of the filter) while still allowing the filter to fulfill the given requirements. The more damping that was added, the less phase the filter subtracted at lower frequencies. In addition, the filters were normalized so that the DC gain was unity.

## 4.4 Comparing the filters

These filters were then compared using several cases from the  $x$  direction control design done for the Stanford ETF.<sup>3</sup> Figure 3 shows a comparison of the four methods of creating notch filters for an example where the standard notch was taken from the aforementioned control design. This instance is indicative of some general trends in the different types of filters. The first observation that can be made is that all three alternative methods of design offer improvement over the current method at the upper unity gain frequency (for this example, 45 Hz). The second is that, especially for the smaller depths and frequency ranges that are required for the control design example, the Chebyshev filter, and to an even greater degree the elliptical filter, outperform the double notch filter at the UUGF. However, these filters also have ripples in the passband, with the elliptical filters having more ripple than the Chebyshev. When this ripple is not taken into account, multiple instances of these filters, as in Figure 4, may build up to cause spikes in the sensitivity curve of the system as Figure 5 shows. In order to simplify the design process, it was therefore ultimately decided that the double notch filter design was the best choice.

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<sup>3</sup>Design done on 4/14/05.

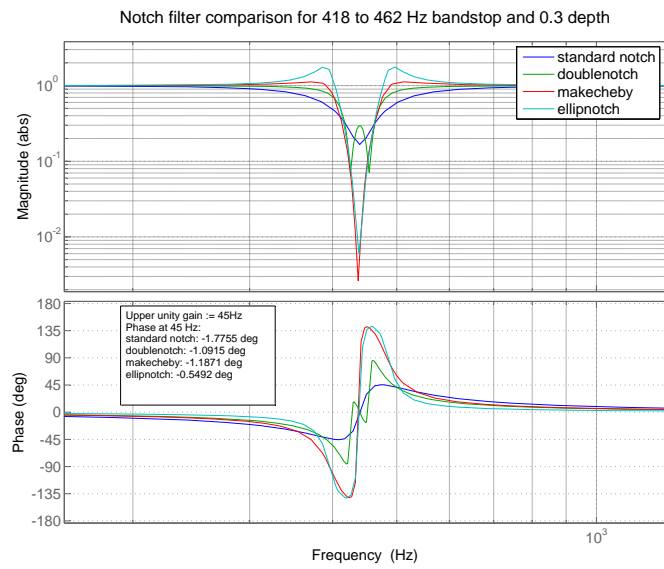


Figure 3: The double, Chebyshev, and elliptical notches have a larger phase margin than the standard notch at UUGF, however the Chebyshev and elliptical notches have passband ripple.

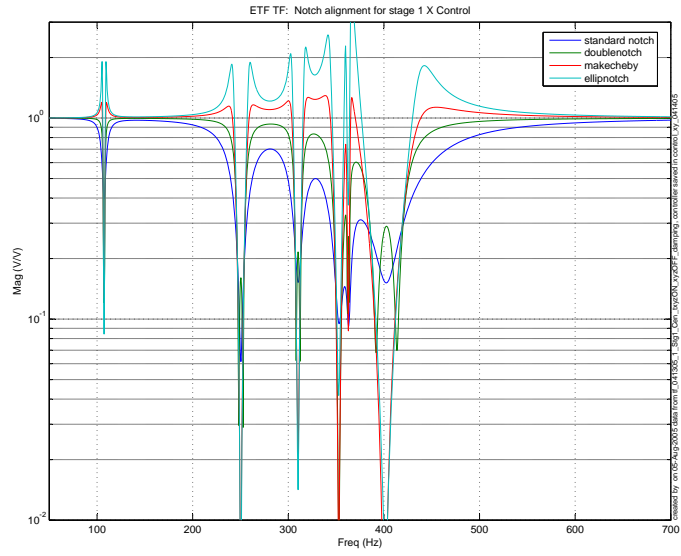


Figure 4: The combination of multiple notch filters in an example from the control design from the ETF.

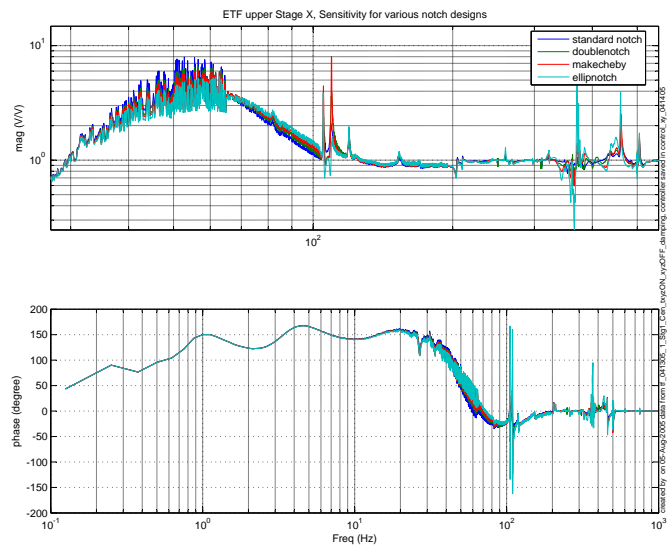


Figure 5: The sensitivity curve of the filters shown in the previous figure. Notice the spikes in the Chebyshev and elliptical filters.

## 5 Automating Filter Design

In designing the control for the active isolation of Advanced LIGO, control loops will have to be designed for each of the six degrees of freedom on both of the stages of isolation in each of the eleven chambers in each of the three interferometers. This means that 396 control loops will have to be designed, making the task very time consuming. However, the design principles outlined in the previous sections mean that designing of notch filters lends itself easily to automation. In order to increase the efficiency and accuracy with which resonances are identified in a system and notch filters to attenuate them are designed, a MatLab program with a graphical user interface was designed.

### 5.1 Identifying resonances

The program loads the transfer function of the system from a data file and applies a prototype lead controller filter so that the UUGF is 40 Hz.<sup>4</sup> The program then identifies the resonances in the resulting transfer function that rise above a user specified threshold and describes them in terms of their height, start and stop frequencies. These identified resonances can be manually altered. Additionally, the identified resonances are set to include a buffer margin that allows for the resonances to shift frequencies slightly. This is done by multiplying the upper frequency and dividing the lower frequency of the resonance by a user specified expansion factor.

### 5.2 Designing filters

Once all the resonances are properly identified, a filter consisting of a series of notches is designed that meets the given specifications will minimizing the sensitivity at the UUGF. The filter (digitized using the ‘match poles and zeros’ algorithm in MatLab) as well as the resulting open loop gain is displayed. Optionally, the sensitivity and resulting phase plot may be viewed. Again, the notches can be manually altered using the parameters  $c$ ,  $d$ ,  $\omega_1$ , and  $\omega_2$  of equation 2. The filter can currently be saved as a ZPK system in a MAT-file so that it can be loaded during the setup of a control loop. When Advanced LIGO establishes a standard for its control system, the program can be easily altered to use this format.

### 5.3 Possible future features

There are several features that could possibly improve a future version of this program. The data file used to load the original transfer function could be made selectable from within the program itself. A faster algorithm for generating the filter design could also be used. Additionally, the program could be made to

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<sup>4</sup>The prototype controller used was:  $D(s) = G \frac{s}{(s+2\pi(\frac{1+i}{\sqrt{2}}))(s+2\pi(\frac{1-i}{\sqrt{2}}))}$  where  $1/G$  was the open loop gain of the system at 40 Hz.

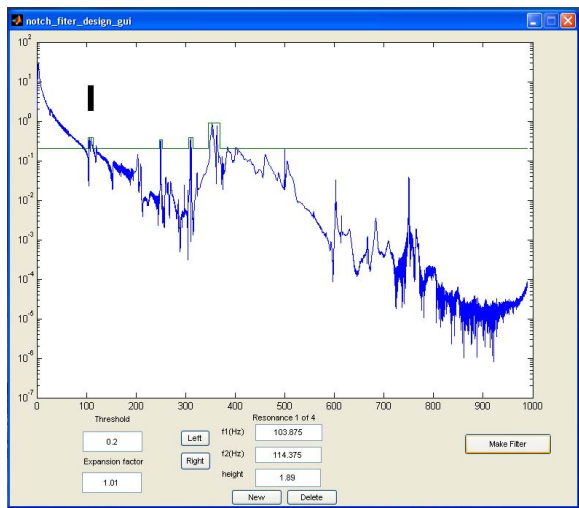


Figure 6: Notch filter design program identifying resonances

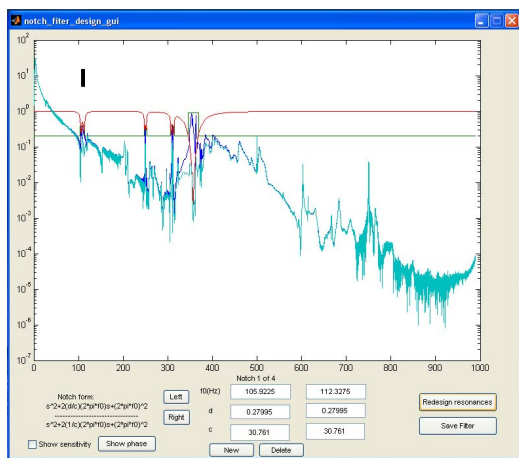


Figure 7: Notch filter design program designing filter



allow the option of selecting different types of notches (e.g. elliptical) when doing so would be desirable.

## 6 Conclusion

In conclusion, it was decided that using double notch filters in the place of the standard notches would reduce the sensitivity of the system while still maintaining the simplicity of the design process. An algorithm for designing these double notch filters so that they had a minimal phase decrease at the UUGF was integrated with a program that identified resonances in a transfer function of a given system and designed filters to attenuate them. This program has a graphical user interface that ensures flexibility in designing these notches. It is hoped that this program will significantly increase the speed with which the control systems for Advanced LIGO can be set up.