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Ribbon tolerances and alignment requirements for Advanced LIGO optics

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1 Introduction

The dimensional tolerances and welding alignment requirements for the Advanced LIGO optics are calculated.

2 The model

The geometrical errors here considered are errors in the fibre diameter that lead to a variation on the equivalent spring constant, the length at rest and the position of the welding points.

In the calculation the fibres have been considered as extensible strings (no bending strength hence the bending length is zero). Therefore the positions of the welding points have to be considered as the position of the bending point in real suspensions.

The relevant dimension and degree of freedoms are reported in the following picture.

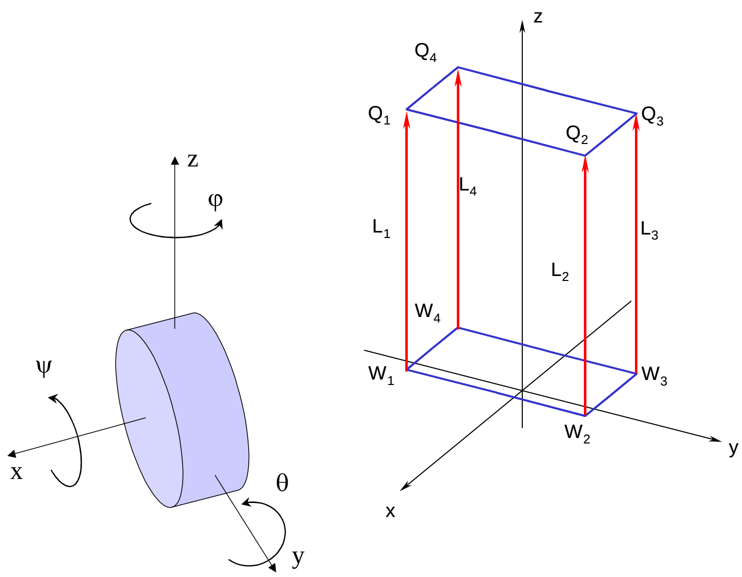


Figure 1 Degrees of freedom and relevant lengths

The static position of the mass is given by the variables x, y, z, ψ, θ, φ. The positions of the welding points Ws are referred to the centre of mass of the mirror (in Figure 1 above you can imagine it at the origin of the axes). The 4 length vectors are referred to the respective welding points. This means that a vertical fibre has a vector length of the form (0,0,L).

The inputs are the W and the L vectors and the frequencies fs of the uncoupled bouncing mode for each fibre. For a perfect suspension the parameters are:

```
Wx[1] := 15e-3:
Wx[2] := 15e-3:
Wx[3] := -15e-3:
Wx[4] := -15e-3:

Wy[1] := -151e-3:
Wy[2] := 151e-3:
Wy[3] := 151e-3:
Wy[4] := -151e-3:

Wz[1] := 1e-3:
Wz[2] := 1e-3:
Wz[3] := 1e-3:
Wz[4] := 1e-3:

Lx[1] := 0e-3:
Lx[2] := 0e-3:
Lx[3] := 0e-3:
Lx[4] := 0e-3:

Ly[1] := 0e-3:
Ly[2] := 0e-3:
Ly[3] := 0e-3:
Ly[4] := 0e-3:

Lz[1] := .6:
Lz[2] := .6:
Lz[3] := .6:
Lz[4] := .6:

f[1] := 6.14:
f[2] := 6.14:
f[3] := 6.14:
f[4] := 6.14:
```

Table 1 Suspension parameters.

A mass of 39.6 kg is considered.

3 Method of calculation and approximations

The upper welding points are maintained as fixed throughout the calculation. The lower welding points are not fixed because they are linked to the position of the centre of mass, that is a variable, and they are dragged by the rotation of the mirrors.

The exact theory says that one has to use the Euler angles and the most general transformation if a multiplication of the 3 basic rotations. This method produces immediately non linear terms (product of the three angles). Since the rotations we are considering are small (< 0.05 rad) the first approximations that have been applied is to write the most generic rotation as $1 + R_\theta + R_\varphi + R_\psi$ and replace the trigonometric functions with their first term in the Taylor expansion (i.e. cos(x)~1 and sin(x)~x).

The potential energy is then computed as the sum of the gravitational component and the longitudinal elastic component for each of the fibres. The equilibrium point corresponds to the minimum of the potential energy.

Another approximation that has been used is to reduce artificially the number of variables and equations. For example in the Section 4 “Effect of the fibre diameter variation” the resulting equilibrium x and y in each trial was of the order of 1e-8 m. To speed up the calculation the x and y variables were set to 0.

The variation of the fibre diameter was considered through its effect on the equivalent longitudinal spring constant k_i. This parameter with the mass m of the suspended mirror determine the uncoupled vertical bouncing frequency f_i

$$f_i = 1/(2\pi)\mathrm{sqrt}(4k/m)$$

The program takes as input the uncoupled vertical bouncing frequency.

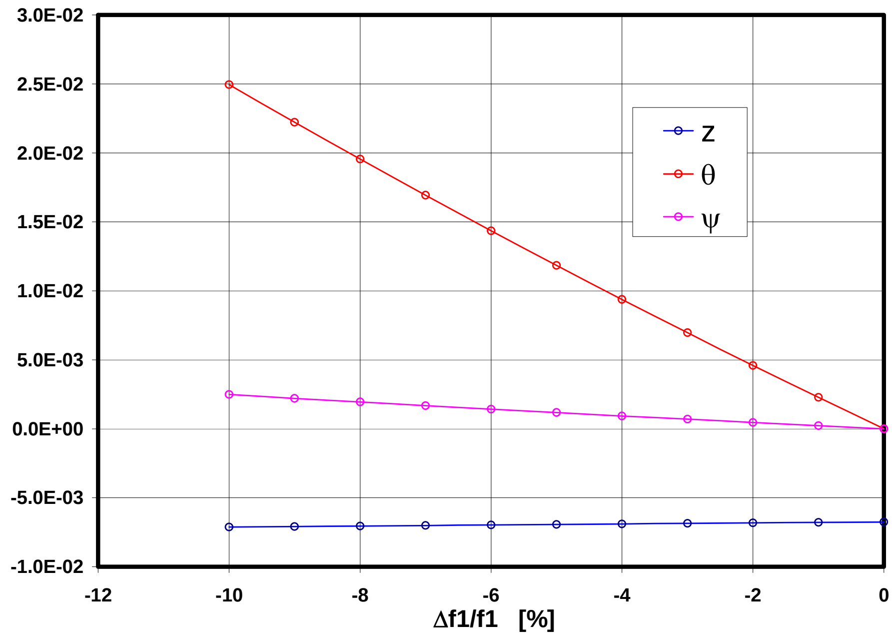
4 Effect of fibre diameter variation

In order to calculate the static position in all possible cases one can use the symmetry of the problem and the superposition effect. The superposition effect is a consequence of the linearization of the problem but both superposition and symmetry in the solution have been checked.

The parameters used are the same as in the table in Section 2 “The model”, apart from one of the frequencies that was altered by successive increments (or decrements) of 1%.

It has been chosen to decrease f_1 and to increase f_3. From these results by symmetry and superposition the equilibrium position with random values for the 4 vertical bouncing frequencies can be calculated.

The results are plotted in the graphs of Figure 2.



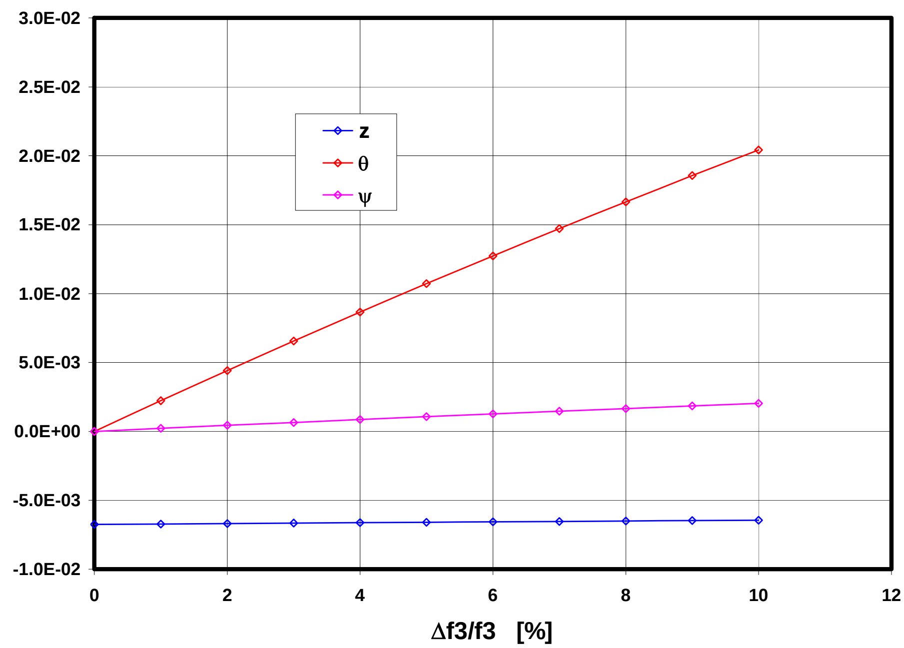


Figure 2 In graph 1 there is the effect of making fibre N.1 weaker and in graph 2 when the opposite fibre N.3 becomes stronger

All the other variables are set to zero or like φ are comparable to zero.

The two plots are not mirror images and this is intuitive because the situation with one fibre weaker than the others is not exactly the same as having the diametrically opposite fibre stronger.

The effect of the variation of f1 is then:

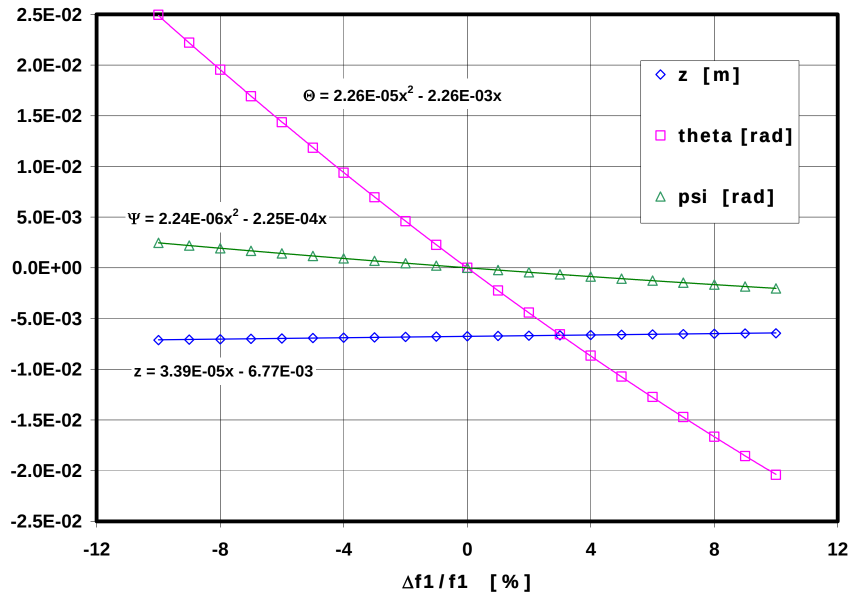


Figure 3 Equilibrium positions for different values of the vertical bouncing frequency of fibre N.1. f1 is 6.14 Hz. Best fit polynomials are reported as well.

Summing the effects of the 4 fibres one has the following relations:

$$z = -6.77e-3 + 3.39e-5 \cdot \left(\frac{\delta f_1}{f_0} + \frac{\delta f_2}{f_0} + \frac{\delta f_3}{f_0} + \frac{\delta f_4}{f_0} \right)$$

$$\psi = -2.25e-4 \cdot \left(\frac{\delta f_1}{f_0} - \frac{\delta f_2}{f_0} - \frac{\delta f_3}{f_0} + \frac{\delta f_4}{f_0} \right) + 2.24e-6 \cdot \left[\left(\frac{\delta f_1}{f_0} \right)^2 - \left(\frac{\delta f_2}{f_0} \right)^2 - \left(\frac{\delta f_3}{f_0} \right)^2 + \left(\frac{\delta f_4}{f_0} \right)^2 \right]$$

$$\theta = -2.26e-3 \cdot \left(\frac{\delta f_1}{f_0} + \frac{\delta f_2}{f_0} - \frac{\delta f_3}{f_0} - \frac{\delta f_4}{f_0} \right) + 2.26e-5 \cdot \left[\left(\frac{\delta f_1}{f_0} \right)^2 + \left(\frac{\delta f_2}{f_0} \right)^2 - \left(\frac{\delta f_3}{f_0} \right)^2 - \left(\frac{\delta f_4}{f_0} \right)^2 \right]$$

Frequency variations are in %.

The reference value f_0 is 6.14Hz. As stated before x, y and φ are negligible as compared to z, θ and ψ.

5 Effect of positioning errors of the lower welding points

In this section the effect of the position errors in x and y of the welding points are investigated.

Varying Wx[1] from 14mm to 16mm by steps of 0.1mm the equilibrium position was found and the results are represented in Figure 4.

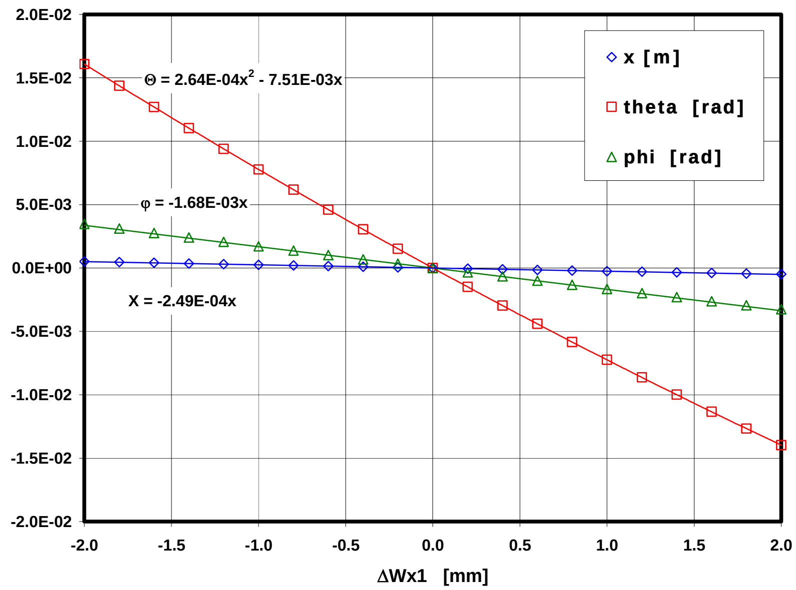


Figure 4 Equilibrium positions for different values of Wx[1]. In the x axis the difference between Wx[1] and its reference value of 15mm is reported. The dimensional units of the different parameters are given in the legend. Y has values negligible as compared to x, z was maintained constant at -6.585mm. ψ is of the order of 5e-5 or less.

Summing the effects of all the 4 welding points one has:

$$x = -2.49e-4 \cdot (\Delta Wx_1 + \Delta Wx_2 + \Delta Wx_3 + \Delta Wx_4)$$
$$\varphi = -1.68e-3 \cdot (\Delta Wx_1 - \Delta Wx_2 - \Delta Wx_3 + \Delta Wx_4)$$
$$\theta = -7.51e-3 \cdot (\Delta Wx_1 + \Delta Wx_2 + \Delta Wx_3 + \Delta Wx_4) + 2.64e-4 \cdot \left[(\Delta Wx_1)^2 + (\Delta Wx_2)^2 - (\Delta Wx_3)^2 - (\Delta Wx_4)^2\right]$$

Positions are in mm.

Varying Wy[1] from -153mm to -149mm by steps of 0.2mm the equilibrium position was found and the results are represented in Figure 5.

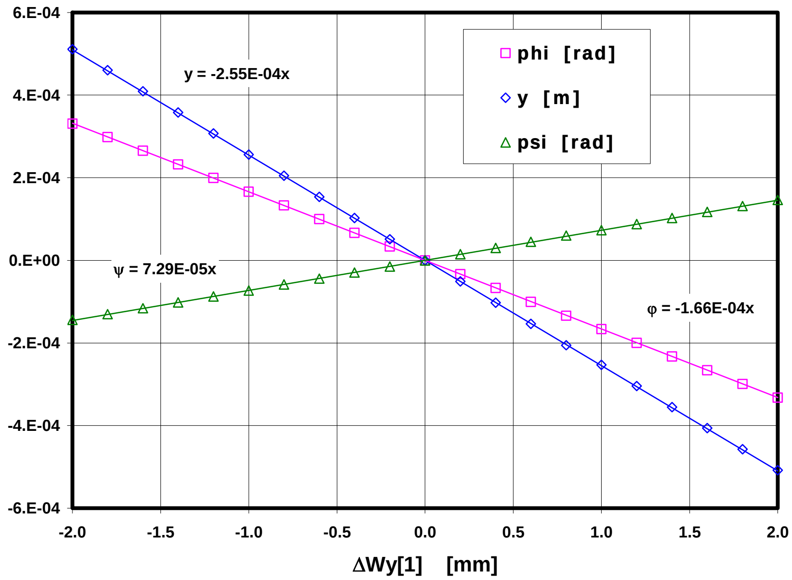


Figure 5 Equilibrium positions for different values of Wy[1]. In the x axis the difference between Wy[1] and its reference value of -151mm is reported. The dimensional units of the different parameters are given in the legend. X has values negligible as compared to y, z was maintained constant at -6.585mm. θ is of the order of 3e-5 or less.

Summing the effects of all the 4 welding points one has:

$$
\begin{aligned}
y &= -2.55e-4 \cdot (\Delta Wy_1 + \Delta Wy_2 + \Delta Wy_3 + \Delta Wy_4) \\
\varphi &= -1.66e-4 \cdot (\Delta Wy_1 + \Delta Wy_2 - \Delta Wy_3 - \Delta Wy_4) \\
\psi &= 7.29e-5 \cdot (\Delta Wy_1 + \Delta Wy_2 + \Delta Wy_3 + \Delta Wy_4)
\end{aligned}
$$

Positions are in mm

6 Results

6.1 Defining equations

If all the errors are considered at once the equilibrium position is given by the following 6 equations:

$$x = -2.49e-4 \cdot (\Delta Wx_1 + \Delta Wx_2 + \Delta Wx_3 + \Delta Wx_4)$$

$$y = -2.55e-4 \cdot (\Delta Wy_1 + \Delta Wy_2 + \Delta Wy_3 + \Delta Wy_4)$$

$$z = -6.77e-3 + 3.39e-5 \cdot \left(\frac{\delta f_1}{f_0} + \frac{\delta f_2}{f_0} + \frac{\delta f_3}{f_0} + \frac{\delta f_4}{f_0}\right)$$

$$\varphi = -1.68e-3 \cdot (\Delta Wx_1 - \Delta Wx_2 - \Delta Wx_3 + \Delta Wx_4) - 1.66e-4 \cdot (\Delta Wy_1 + \Delta Wy_2 - \Delta Wy_3 - \Delta Wy_4)$$

$$\theta = -2.26e-3 \cdot \left(\frac{\delta f_1}{f_0} + \frac{\delta f_2}{f_0} - \frac{\delta f_3}{f_0} - \frac{\delta f_4}{f_0}\right) + 2.26e-5 \cdot \left[\left(\frac{\delta f_1}{f_0}\right)^2 + \left(\frac{\delta f_2}{f_0}\right)^2 - \left(\frac{\delta f_3}{f_0}\right)^2 - \left(\frac{\delta f_4}{f_0}\right)^2\right] +$$
$$-7.51e-3 \cdot (\Delta Wx_1 + \Delta Wx_2 + \Delta Wx_3 + \Delta Wx_4) + 2.64e-4 \cdot \left[(\Delta Wx_1)^2 + (\Delta Wx_2)^2 - (\Delta Wx_3)^2 - (\Delta Wx_4)^2\right]$$

$$\psi = -2.25e-4 \cdot \left(\frac{\delta f_1}{f_0} - \frac{\delta f_2}{f_0} - \frac{\delta f_3}{f_0} + \frac{\delta f_4}{f_0}\right) + 2.24e-6 \cdot \left[\left(\frac{\delta f_1}{f_0}\right)^2 - \left(\frac{\delta f_2}{f_0}\right)^2 - \left(\frac{\delta f_3}{f_0}\right)^2 + \left(\frac{\delta f_4}{f_0}\right)^2\right] +$$
$$+7.29e-5 \cdot (\Delta Wy_1 + \Delta Wy_2 + \Delta Wy_3 + \Delta Wy_4)$$

From the document "Advanced LIGO Quad Installation & Alignment Fixtures Product Design Specification" (LIGO-T040151-02) the tolerances on x, y and z are large (+/- 1 mm at least). Stronger requirements are in the angular positioning.

From "Monolithic suspension workshop (Glasgow January 2005): Minutes and Action" (LIGO-T050010-00-K) the roll range (ψ) is +/- 2mrad; pitch range (θ) is +/- 20mrad; for yaw just the interferometer requirements are quoted (0.1mrad, the same as for the pitch) but nothing is stated about the range of adjustability. Let's work with the inputs on ψ and θ only.

6.2 Errors in bounce frequency and x, y position of welding point

Pitch and roll depend on 8 variables each. We can consider the 8 variables to be uncorrelated, hence we sum their contribution in quadrature and we can assume that each variable contributes to the total error with the same amount. Approximating sqrt (8) ~ 3, using the previous equations, one can work out the maximum errors on the bouncing frequency, on x and y position of the welding points. The results are summarized in the following Table 2.

	Maximum error on bouncing frequency $\Delta f/f$ [%]	*Maximum error on x position of welding point* ΔW_x [mm]	*Maximum error on y position of welding point* ΔW_y [mm]
Pitch, θ Range = ± 20mrad	± 2.8	± 0.8	—
Roll, ψ Range = ± 2mrad	± 2.8	—	± 9

Table 2 Maximum errors on bouncing frequency and x, y position of welding point.

With these errors on the welding positions, the expected error on yaw is φ = ± 8.5mrad. This could be considered acceptable considering the range in the pitch.

6.3 Errors in dimensions of fibres

From Table 2, taking the maximum allowable error in bounce frequency of ± 2.8 %, the allowable errors on fibre dimension can be calculated.

For cylindrical fibres,

$$f = \frac{1}{2\pi}\sqrt{\frac{E\pi r^2}{Lm}}$$

With

f	=	frequency (Hz)
E	=	Young's Modulus (N/m2)
r	=	fibre radius (m)
L	=	fibre length (m)
m	=	mass of suspended optic (kg)

Similarly, for ribbons,

$$f = \frac{1}{2\pi}\sqrt{\frac{Ewt}{Lm}}$$

With

w	=	ribbon width (m)
t	=	ribbon thickness (m)

Assuming errors in the three dimensions contribute equally (r_x, r_y and L in the case of fibres and w, t and L in the case of ribbons) then,

$$\varepsilon_{\dim ension} = \frac{2}{3}\varepsilon_{frequency}$$

This yields an error of ± 1.9 % in each dimension.

7 Conclusions

The dimensional tolerances required for Advanced LIGO of ± 1.9 % in each fibre/ribbon dimension can be compared with the dimensional errors typically measured for the GEO 600 silica fibres.

In GEO 600 the error in bounce frequency was ± 3.1 %. Using similar arguments this yields dimensional tolerances of ± 2.1 %.

These are very close to each other with the requirements for Advanced LIGO only slightly more stringent.

The implication of this is that flame fabrication and welding could confidently be used as a fall back to laser fabrication and welding assuming we are selective within the defined limits whilst characterizing pulled fibres and assuming suitable flame welding jigs are designed.