Mechanics calculations on Earthquake stops

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1. BACKGROUND

This note sets out some simple calculations on the forces, stiffness, and stresses in the earthquake stops for the ETM/ITM noise prototype. Whilst a full FEA of the structure with the stops and masses could in principle be done, it would have to feature non-linear gap elements and would be very time-consuming if possible. The strategy therefore is to start with much simpler calculations and use generous margins.

The general approach is

- to look at the equivalent static acceleration, assume the mass accelerates through the gap with that acceleration and hits the stop. Knowing the elasticity of the stop, to look at the motion that occurs in the stop, to compare it with allowable motion (to avoid OSEM-flag interference) and to look at the contact stresses;
- to extend this with a FEA model of a simple 1-D representation of the structure with a time-history input. This could allow for multiple bounces.

2. INITIAL MOTION OF MASS

Consider the mass to be accelerated across the gap between the stops.

The standard equations of motion with fixed acceleration apply, from which

S	0.001	m	equal to whole mot acceleration from T	ion stop-stop. 000053-03 last para in
а	4	m/s^2	2.6.1.2.1	
			time to cross gap	t = 2s
t	0.0223607	S	time to cross gap	$i = \sqrt{a}$
v	0.0894427	m/s	speed on impact	v = at

3. DECELERATION BY STOP

The kinetic energy of the mass is converted to stored energy in the stop (this assumes that the mass hits the structure, when in fact the reverse is true – return to this later). For the stiffness of the stop I have assumed the stiffness of the structure as given by T060058 (Tim Hayler).

In Tim's document a force of 400N produced a displacement of 0.2mm when applied axially or 0.15mm when applied laterally. Take a simple number of k = 400/0.0002 = 2MN/m.

Equating the kinetic energy of the motion of the mass to the stored energy in the spring:

 $\frac{1}{2}mv^2 = \frac{1}{2}k\delta^2$

Whence the deflection is

$$\delta = v \sqrt{\frac{m}{k}}$$
$$\delta = 0.09 \sqrt{\frac{40}{2e6}} = 0.4E - 3$$

In this case delta = 0.4mm

The maximum force involved is simply $F = k\delta = 2E6^*.4E-3 = 800N$

This is spread between, say, three stops.

4. STRESS ON DECELERATION

It is proposed to make the end of the stop spherical to avoid stress concentrations at the tip (if the end were flat then a small tilt would bring all the force onto a near-point contact). The stress between a sphere and a plane of the same material is (Roark & Young 5th ed table 33 case 1a)

$$\sigma_{comp} = 1.5 \frac{F}{\pi a^2}$$

where

$$a = 0.721 \times \sqrt[3]{FD_2C_E}$$

and

$$C_E = \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2}$$

Typical results are:

F	800	Ν	max force on mass			
n	3		number of stops			
Fstop	267	Ν	Force per stop			
Boork 5th	ad table 22 apr	0.10				
RUAR SITI EU IADIE SS CASE TA						
D2	0.2	m	diameter of snub piece			
E	7.00E+10	Ра	modulus of fused silica			
nu	0.17		poisson (Coyne, private com)			
CE	2.77E-11					
а	0.000821612	m	radius of contact zone			
sigma_c	1.89E+08	Ра				
or	189	MPa	comressive stress			
sigma_t	25	MPa	tensile stress			
Tau	63	MPa	shear stress			

Matweb gives (<u>http://www.matweb.com/search/SpecificMaterial.asp?bassnum=CCERDN28</u>) tensile and compressive strengths of ~20MPa and ~50MPa respectively for "HS" and ~50 MPa and 170-240MPa for "UHS". Opticsland (<u>http://www.sciner.com/Opticsland/FS.htm</u>) gives 50 and 1100 MPa. So there is clearly big variety – almost certainly down to surface finish.

5. FINITE ELEMENT MODELLING

We model the system of the test mass with gaps, the polymer buttons, and the structure as follows:



The values of the spring constants are found thus:

For the structure, spring constant is 2E6 as before

To find the dynamic mass of the structure, note that the frequency is of order 65 Hz and use

 $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ whence *m* comes to around 12 kg.

To find the stiffness of the polymer elements take

$$k = \frac{Ea}{l}$$

Which gives stiffness of polymer button r 0.005 m radius t 0.003 m thickness E 5.00E+09 Pa modulus k 1.31E+08 N/m stiffness

Our model is then



The ANSYS macro in appendix 1 will generate such a model.

To add damping to the polymer, use the usual equations (thanks to Norna and Dennis here):

 $F = m\ddot{x} + b\dot{x} + kx$ gives natural frequency

 $\omega_0^2 = \frac{k}{m}$

Define

 $\gamma = \frac{b}{m}$

$$Q = \frac{\omega_0}{\gamma}$$

whence

$$Q = \frac{\sqrt{km}}{b}$$
or

$$b = \frac{\sqrt{km}}{Q}$$

For our case use the stiffness of the polymer and the mass of the test mass; assume a Q of say 30 for the polymer material and the damping constant is

B= sqrt(1.4E8*40)/30 = 2.4E3 N/m/s

Strategy:

Explore stiffness/damping space to find a region in which the maximum deflection and stress are both acceptable. (high stiffness gives high stress and low movement; high damping the

same). If there is a region with acceptable answers for both, try to find a tip design that gives it. A possibility is a double-acting tip such as this:



Which should give low stiffness in mild earthquakes but gives higher stiffness (and polymersilica contact) in more severe earthquakes.

6. INITIAL ANSYS RESULTS

The macro in Appendix 2 has the following features:

Masses and stiffnesses as shown above. No damping. During first 0.1sec, move nodes 1 and 7 to +0.001m During next 0.1 sec, move them to -0.001m Observe until t = 1 sec.

Results look sensible at first. The traces are: UX_1 = movement at node 1 (left-hand-most node) Etc. So we see the "ground" move as expected (UX_1 overwritten by UX_7), we see the test mass move (UX_4) and we see the structure moving with high-frequency wiggles (UX_2, UX_3, UX_5, UX_6):



But become unstable as time progresses, presumably with the lack of damping:



7. ADD DAMPING

The ANSYS documentation (Theory reference chapter 14.14) gives

k = stiffness (input as K on **R** command)

 $C_v = C_{v1} + C_{v2} \left| v \right|$

 C_{v1} = constant damping coefficient (input as CV1 on **R** command)

 C_{v2} = linear damping coefficient (input as CV2 on **R** command)

v = relative velocity between nodes computed from the nodal Newmark velocities

So we need C_{v2}

So insert the following:

bPad = 2.4E3 !damping constant of pad

and change this: !For the polymer pad ! !R,n,K,CV1,CV2 R,3,KPad,

To this: !For the polymer pad ! !R,n,K,CV1,CV2 R,3,KPad,,bPad

Add this: ET,4,COMBIN14 !spring for the pads !* KEYOPT,4,1,1 !0=linear spring; 1 required for damping KEYOPT,4,2,1 !1 = 1D longitudinal in UX KEYOPT,4,3,0 !2 = 2D rather than 3D; Z=0 throughout !*

And change this: !pads type,4 real,3 E,5,6 E,3,2

Solution failed to converge after 0.6 secs. Initial results suspiciously similar to before:



To compare without/with damping:



The movement of point 6 (the structure) has decreased somewhat, but the movement of the mass is if anything larger.

Try increasing damping by a factor 100: bPad = 2.4E5 !damping constant of pad

solution now gets to 0.9 sec before failing.



So this makes sense but the damping is way too high and the solution fails after 0.9 secs. Try tweaking the substeps in step 3:

nsubst,1000 autots,off TIME,1 Solve

Better, but now runs out of space on solution file! (max number of results defaults to 1000)



Incidentally, a frequency of 65 Hz has a period of .015 secs, or ten periods should last about 0.15 secs. This is about what the structure (UX_6) is doing.

Force an even shorter timestep: nsubst,2000 autots,off TIME,1 Solve

And things look better:



Note that the movement of the structure seems to be decaying – effect of damping?

So go back to the more sensible damping number. bPad = 2.4E3 !damping constant of pad

The reduced damping changes the frequency, but not the magnitude, of the TM motion:



8. SOLUTION ROBUSTNESS

Try varying the time steps to verify solution robustness.

Here is the time step size history from the solution above:



Try removing the autots on steps 1 and 2. Results seem little affected in steps 1 and 2 but the results for step 3 are a little







Hmmm. This is worrying because presumably the contact forces are related to the bouncing frequency of the mass. It also seems odd that the test mass shows small straight-line portions during its peaks – reminiscent of digitising error on the timestep (but we have a very small timestep). The time substeps are, in steps 1 and 2 1000 substeps in 0.1 sec or 1E-4 secs long. Examine a few bounces in detail. For the bounce around 0.16 secs there is plenty of information with time:

TI ME 4 UX UX_4 -0. 923374E-03 -0. 924005E-03 0.16410 0.16420 0.16430 -0.924636E-03 0.16440 -0.925267E-03 -0. 925898E-03 0.16450 -0. 926529E-03 0.16460 0.16470 -0.927160E-03 0.16480 -0.927791E-03 -0. 928422E-03 0.16490 -0. 929053E-03 0.16500 -0.929684E-03 0.16510 0.16520 -0.930316E-03 0.16530 -0.930947E-03 0.16540 -0.931578E-03 -0.932210E-03 0.16550 0.16560 -0. 932841E-03 0.16570 -0.933472E-03 0.16580 -0.934104E-03 0.16590 -0.934735E-03 0.16600 -0.935366E-03



***** ANSYS POST26 VARIABLE LISTING *****

What is going on here? The straight-line part of the test mass trace is delineated by two regions in which UX_2 and UX_3 are different. Is the contact only at those two times (t=.1635 and t=.1685)?

Look at a force plot. Need to figure out which node and element to use:



Try element 5, node 3. And indeed the results look like two "bumps":



Over the whole time we see this:



Looking at a different time period:



We see that the first encounter I this period, at t=0.32, is a "three-bouncer". See the trace for UX_2 and UX_3. See also how the test mass is essentially confined to movements of 0.5mm from the nominal, give or take the location of the structure.

Forces for element 5, node 3 and for element 4, node 5:



Note also how the amplitude for UX_3 is reduced in the encounter (ditto for UX_6 at about t=.365). This just depends on the relative phases when collision occurs. At a later time, the opposite occurs:



So the solution appears to be reasonably robust – at least, it makes sense.

Macro so far in appendix 3.

Try allowing autots but with the suggested iterations at 1000,1000,3000:



Results less credible than before (albeit much quicker to solve!). Revert to autots,off.

9. EFFECT OF LINKING STRUCTURE POINTS

One question is – since there is in fact only one structure, should nodes 2 and 6 be tied together?

Try the effect of linking nodes 2 and 6:

!couple nodes 2 and 6 in UX:
CP, 1, UX, 2, 6
!CP, NSET, Lab, NODE1, NODE2, NODE3, NODE4, NODE5, NODE6, NODE7, NODE8, NODE9, NODE10

Credible results:







On the bounce travelling towards –x, at 0.57 secs, it is node 3 that gets disturbed:



On the bounce travelling towards +x, at 0.6 secs, it is node 5:



The forces are about doubled, presumably because we are now interacting with 24 kg of structure (forces at element 5, node 3):



So the next step is to halve the mass. Effectively what we have done is split the mass of the structure between two nodes. This is legitimate. To keep the same natural frequency in the structure we also halve the structure stiffness:

Replace this: KStruct = 2E6 ! stiffness of structure Mstruct = 12 !dynamic mass of structure

With this: KStruct = 1E6 ! stiffness of structure Mstruct = 6 !dynamic mass of structure

Solution different but still credible:



Forces now sensible (included element 4, node 5)



10. STIFFNESS OF PADS

How much can we reduce the force by changing the stiffness of the pads? They could obviously be made longer, or a spring could be added. Try making them only a tenth as stiff (still much stiffer than the structure)

Change MTM = 40 !Mass of test mass KPad = 1.4E8 !stiffness of polymer pad bPad = 2.4E3 !damping constant of pad KStruct = 1E6 ! stiffness of structure Mstruct = 6 !dynamic mass of structure

To MTM = 40 !Mass of test mass KPad = 1.4E7 !stiffness of polymer pad bPad = 2.4E3 !damping constant of pad KStruct = 1E6 ! stiffness of structure Mstruct = 6 !dynamic mass of structure

Still looks sensible:



The interactions last longer:





And forces are reduced – but not by a factor 10 – maybe a factor 3.



11. INPUT OF COMPLEX TIME HISTORY

Thanks to help from the ANSYS support, we can put in a time-history. Prepare a file with a simple list of times and displacements:

position time 0 0 0.000279625 0.01 0.02 0.000528317 0.03 0.000721407 0.04 0.000845298 0.05 0.000899726 0.06 0.000896978 0.07 0.000858306 0.08 0.00080843

0.090.0007694950.10.0007559220.110.0007713790.120.000808556Etc.



This example was made from this expression

=D3*SIN(E3*A4)+D4*SIN(E4*A4)+D5*SIN(E5*A4)+D6*SIN(E6*A4) In which A4 is the time, and the multipliers were

	D (mm)	E(radians per second)
3	0.0005	10
4	0.0005	15
5	0.0002	30
6	0.0002	50

Prepare a variable and read it in:

!*
*DIM, gdisp,TABLE, 101, 1, 1, TIME, ,
!*DIM, Par, Type, IMAX, JMAX, KMAX, Var1, Var2, Var3, CSYSID
!*
*TREAD,GDISP,'timehist','txt',',1,
!*TREAD, Par, Fname, Ext, --, NSKIP

Then use the variable to give the ground motion: Replace this

NSUBST,1000 Autots,off TIME,.1 D,1,UX,.001 D,7,UX,.001 solve

NSUBST,1000 TIME,0.2 D,1,UX,-.001 D,7,UX,-.001 solve

nsubst,3000 TIME,1 solve FINISH

With this in PREP7 D,1,UX,%GDISP% D,7,UX,%GDISP%

And this for the solver: NSUBST,5000 Autots,off TIME,1 Solve





And forces are much the same as before:



On common axes:



A reminder of the model:



And some detailed results to show at the SUS meeting:







Now with offsets to show "true" position rather than displacement from nominal:

And zoomed in:



APPENDIX 1 ANSYS macro FINISH ! Make sure we are at BEGIN level /CLEAR /PREP7

 $! \mbox{ to make a simple time-history FEA of EQ stops.}$

! ! Parameters

MTM = 40 !Mass of test mass KPad = 1.3E8 !stiffness of polymer pad KStruct = 2E6 ! stiffness of structure Mstruct = 12 !dynamic mass of structure

/triad,lbot !Move co-ord sys triad out of the way

! element types
!
ET,1,COMBIN14 !spring
!*
KEYOPT,1,1,0 !0=linear spring
KEYOPT,1,2,1 !1 = 1D longitudinal in UX
KEYOPT,1,3,0 !2 = 2D rather than 3D; Z=0
throughout
!*

ET,2,CONTAC12 !Gap !* KEYOPT,2,1,0 !Friction type only valkid if mu>0 KEYOPT,2,2,0 10=orientation angle based on theta real const make theta = 0 KEYOPT,2,3,1 !0=no weak SPRing on open dap KEYOPT,2,4,1 !1=use node location for initial gap KEYOPT,2,7,0 !connected with optimised solution time !* ET,3,MASS21 !Point mass 1* KEYOPT,3,1,0 !0=Real consts are mass and inertia KEYOPT,3,2,0 !0=elem coord system parallel to global KEYOPT, 3, 3, 4 !2D mass no rotary intertia **!Real constants** ! For the test mass !R,n,mass R,1,MTM I

! !R,n,theta,kn ,intf,start,ks R, 2,90 ,2e10, ,

!for the gap

!For the polymer pad I !R,n,K,CV1,CV2 R,3,KPad !For the structure ! mass !R.n.mass R,4,MStruct ! spring !R,n,K,CV1,CV2 R,5, kstruct Inodes N,1,-.003 ,2,-.002 ,3,-.0005 ,4,0 ,5,.0005 ,6,.002 ,7,.003 lelements ! Test mass type,3 real,1 E,4 !gaps type,2 real,2 E.5.4 E,4,3 !pads type,1 real,3 E,5,6 E,3,2 !structure type.3 real.4 E.6 E,2 type,1 real,5 E,6,7 E,2,1

! fix everything in Y NSEL,all D,all,UY,0

12. APPENDIX 2 – SECOND MACRO

FINISH ! Make sure we are at BEGIN level /CLEAR /PREP7

! to make a simple time-history FEA of EQ stops.

. ! Parameters ! MTM = 40 !Mass of test mass KPad = 1.4E8 !stiffness of polymer pad KStruct = 2E6 ! stiffness of structure Mstruct = 12 !dynamic mass of structure

/triad,lbot !Move co-ord sys triad out of the way

! element types ! ET,1,COMBIN14 !spring !* KEYOPT,1,1,0 !0=linear spring KEYOPT,1,2,1 !1 = 1D longitudinal in UX KEYOPT,1,3,0 !2 = 2D rather than 3D; Z=0 throughout !*

ET,2,CONTAC12 !Gap !*

KEYOPT,2,1,0 !Friction type only valkid if mu>0 KEYOPT,2,2,0 10=orientation angle based on theta real const make theta = 0KEYOPT,2,3,1 !0=no weak SPRing on open gap KEYOPT,2,4,1 !1=use node location for initial gap KEYOPT,2,7,0 !connected with optimised solution time !* ET,3,MASS21 !Point mass KEYOPT,3,1,0 !0=Real consts are mass and inertia KEYOPT,3,2,0 !0=elem coord system parallel to global KEYOPT,3,3,4 !2D mass no rotary intertia

!Real constants

! ! For the test mass !R,n,mass R,1,MTM !

Ifor the gap

! !R,n,theta,kn ,intf,start,ks R, 2,90 ,2e10, , !For the polymer pad !R,n,K,CV1,CV2 R.3.KPad !For the structure ! mass !R.n.mass R,4,MStruct ! spring !R,n,K,CV1,CV2 R,5, kstruct Inodes N,1,-.003 ,2,-.002 ,3,-.0005 ,4,0 ,5,.0005 ,6,.002 ,7,.003 !elements ! Test mass type.3 real.1 E,4 !gaps type,2 real,2 E,5,4 E,4,3 !pads type,1 real,3 E,5,6 E.3.2

!structure type,3 real,4 E,6 E,2 type,1 real,5

E,6,7

E,2,1

! fix everything in Y

:END

NSEL,all D,all,UY,0 /solve solcontrol,0 !used in VM81, no idea why, if ommitted solution fails. ANTYPE,4 !transient !* TRNOPT,FULL !Full analysis - no shortcuts LUMPM,0 !* OUTRES,ALL,1 kbc,0 !Ramped BC NSUBST,500,0,0 Autots,on TIME,.1 D,1,UX,.001 D,7,UX,.001 solve TIME,0.2 D,1,UX,-.001 D,7,UX,-.001 solve TIME,1 solve FINISH /POST26 !* NSOL,8,1,U,X,UX_1 STORE, MERGE !* NSOL,2,2,U,X,UX_2 STORE, MERGE 1* NSOL,3,3,U,X,UX_3 STORE, MERGE !* NSOL,4,4,U,X,UX_4 STORE, MERGE !* NSOL,5,5,U,X,UX_5 STORE, MERGE !* NSOL,6,6,U,X,UX 6 STORE, MERGE !* NSOL,7,7,U,X,UX 7 STORE, MERGE XVAR,1 PLVAR,8,2,3,4,5,6,7

13. APPENDIX 3. MACRO AT END OF SECTION 8.

FINISH ! Make sure we are at BEGIN level /CLEAR /config,nres,5000 /PREP7

! to make a simple time-history FEA of EQ stops.

! Parameters

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1

MTM = 40 !Mass of test mass KPad = 1.4E8 !stiffness of polymer pad bPad = 2.4E3 !damping constant of pad KStruct = 2E6 ! stiffness of structure Mstruct = 12 !dynamic mass of structure

/triad,lbot !Move co-ord sys triad out of the way

! element types ! ET,1,COMBIN14 !spring !* KEYOPT,1,1,0 !0=linear spring KEYOPT,1,2,1 !1 = 1D longitudinal in UX KEYOPT,1,3,0 !2 = 2D rather than 3D; Z=0 throughout

!*

ET,2,CONTAC12 !Gap

1* KEYOPT,2,1,0 !Friction type only valkid if mu>0 KEYOPT,2,2,0 10=orientation angle based on theta real const make theta = 0 KEYOPT,2,3,1 10=no weak SPRing on open gap KEYOPT,2,4,1 !1=use node location for initial gap KEYOPT,2,7,0 !connected with optimised solution time 1* ET,3,MASS21 !Point mass KEYOPT,3,1,0 !0=Real consts are mass and inertia KEYOPT,3,2,0 !0=elem coord system parallel to global KEYOPT, 3, 3, 4 !2D mass no rotary intertia

ET,4,COMBIN14 !spring for the pads !* KEYOPT,4,1,1 !0=linear spring; 1 required for damping KEYOPT,4,2,1 !1 = 1D longitudinal in UX

KEYOPT,4,3,0 !2 = 2D rather than 3D; Z=0 throughout 1* **!Real constants** ! For the test mass !R,n,mass R,1,MTM L !for the gap I !R,n,theta,kn ,intf,start,ks R, 2,90 ,2e10, , !For the polymer pad !R,n,K,CV1,CV2 R,3,KPad,,bPad !For the structure ! mass !R,n,mass R,4,MStruct L ! spring !R,n,K,CV1,CV2 R.5. kstruct Inodes N.1.-.003 ,2,-.002 ,3,-.0005 ,4,0 ,5,.0005 ,6,.002 ,7,.003 lelements ! Test mass type,3 real,1 E,4 lgaps type,2 real,2 E,5,4 E,4,3 !pads type,4

real.3

E,5,6 E,3,2 !structure type,3 real,4 E,6 E,2 type,1 real,5 E,6,7 E,2,1 ! fix everything in Y NSEL,all D,all,UY,0 /solve solcontrol,0 !used in VM81, no idea why, if ommitted solution fails. ANTYPE,4 !transient !* TRNOPT,FULL !Full analysis - no shortcuts LUMPM,0 !* OUTRES, ALL, 1 kbc,0 !Ramped BC NSUBST,1000 Autots,off TIME,.1 D,1,UX,.001 D,7,UX,.001 solve NSUBST,1000 **TIME**,0.2 D,1,UX,-.001 D,7,UX,-.001 solve nsubst,3000 autots,off TIME,1 solve FINISH /POST26 !* NSOL,8,1,U,X,UX_1 STORE, MERGE !* NSOL,2,2,U,X,UX 2 STORE, MERGE !* NSOL,3,3,U,X,UX_3 STORE, MERGE !* NSOL,4,4,U,X,UX_4

STORE,MERGE !* NSOL,5,5,U,X,UX_5 STORE,MERGE !* NSOL,6,6,U,X,UX_6 STORE,MERGE !* NSOL,7,7,U,X,UX_7 STORE,MERGE XVAR,1 PLVAR,8,2,3,4,5,6,7 :END