Earthquake stop analysis part 3

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July/August 2006

1 Introduction

This note continues the work in T060098.

The conclusions from that work were that the model was basically working, but a few issues remained to be addressed:

- Firm up on details of model (stiffness of beam etc)
- Give more realistic numbers for spring constant and damping in rubber
- Decide how to handle the stops between masses
- Decide how to get at "ground" motion for the Seismic platform.

2 Spring constant and damping in rubber.

2.1 Experiment

We have carried out a simple experiment with two steel balls which are suspended in the manner of a "Newton's cradle". The experiment consists in lifting one ball to a measured location, allowing it to swing to hit the second ball, and then measuring the maximum motion of the second ball as a result of the impact. The balls weigh several kg and the cradle wires are ~3m long, to the forces are representative and the motions are slow enough and large enough to allow the essential results to be captured with simple measurement techniques.

In the first test, the balls impacted directly. In the second test, a piece of the proposed earthquake stop material was fixed to one mass. For each test, several data points were taken with varying sizes of swing. For each data point many trials were made as follows. Define the "input" ball as the one which is raised by hand to start the test, and the "output" ball as the one whose motion after impact is to be measured. A retort stand was set up at the given distance behind the input ball, so that the input ball could be conveniently pulled back by hand by a known amount. The input ball was then allowed to swing and impact the output ball, the swing of the output ball being judged by eye. A second retort stand was placed behind the output ball. The location of the second stand was adjusted iteratively until the output ball was seen to just touch the output ball at the peak of the ball's swing. The position of both stands was then measured.

The stiffness of the piece of EQ stop material was also measured statically. The idea is to model this with FEA and adjust the material damping properties to give the observed results.

2.2 Experiment results

These are reported in an email reproduced below:
The "input" ball weighed 13.88 kg
The "output" ball weighed 13.82 kg

The pendulums were 2470+68mm long (vertical distance)
The balls were attached 68mm above their CG

The steel-steel contacts gave the following results (input motion-output motion)

#1 297-265
#2 184-178
#3 303-275
#4 116-115

All dimensions are in mm and are have +/- 1mm error bars

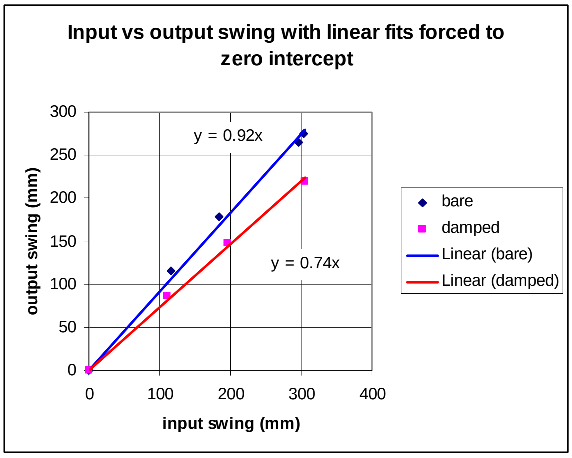
The steel flourel contacts gave the following results (input motion-output motion)

#5 110-87
#6 305-220
#7 197-148

All dimensions are in mm and are have +/- 1mm error bars. The flourel stop dimensions for the above results were approximately a 10mm diameter cylinder 6mm long. This was attached with superglue.

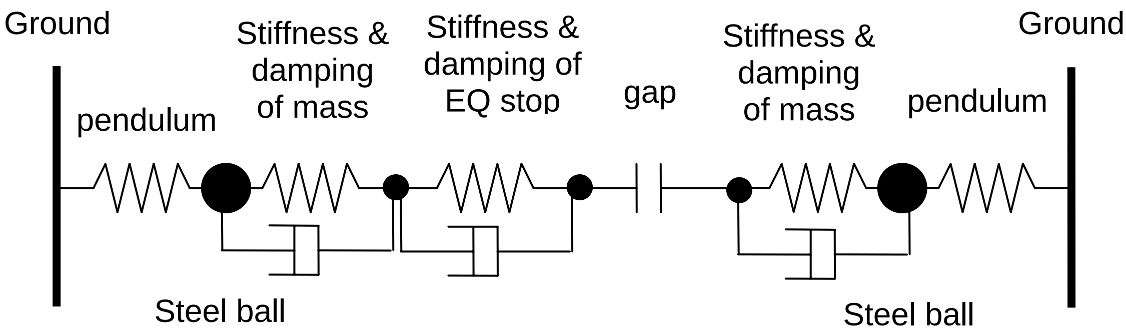
The static deformation of the stop deforms 1.5mm (+/- 0.2) under 13.88kg between appropriately radiused surfaces.

The radius of the ball was measured at 150mm +/- 1mm.
A graph of the results shows reasonable linearity:



2.3 FEA model

The FEA model has point masses for the balls, springs to represent their stiffness and damping, springs to represent the effect of gravity (in a pendulum sense), and a spring to represent the stiffness and damping of the EQ stop material.



2.3.1 Spring properties

2.3.1.1 The pendulum springs

For a pendulum the frequency is given by $f = \sqrt{g/L}$, and for a spring-mass it is $f = \sqrt{k/m}$.

To find a spring that gives the same effect as the pendulum, equate the two expressions whence

$$k = \frac{mg}{L}$$

In our case

$$k = \frac{13.8*9.81}{2.538} = 53.3 \text{ N/m}$$

2.3.1.2 The springs that represent the stiffness of the masses

These are not critical values, they are there only to allow the case without EQ stops to be represented as a baseline. Assume the same stiffness as a cylinder whose length is equal to the radius of the ball and whose diameter is a fifth that of the ball. The stiffness is then

$$k = \frac{Ea}{l} = \frac{\pi r^2 E}{l}$$

In our case $r = \frac{r_{ball}}{5}$ and $l = r_{ball}$

$$k = \frac{\pi r_{ball}{}^2 E}{25 * r_{ball}}$$

$$k = \frac{\pi \times 0.15 \times 207 \times 10^9}{25} = 3.9E9 \text{ N/m}$$

2.3.1.3 The spring that represents the Flourel pad

The stiffness is found from the measurements reported above as

$$k_{pad} = \frac{F}{\delta} = \frac{9.81 \times 13.88}{.0015} = 90.8E3 \text{ N/m.}$$

2.3.1.4 Damping constants - steel

These will be refined in the FEA, but in the first instance we need a number to start with. Taking the approach of T060053 section 5,

$$b = \frac{\sqrt{km}}{Q}$$

Assume for the steel Q = 10,000. Then

$$b_{ball} = \frac{\sqrt{km}}{Q} = \frac{\sqrt{3.9E9 * 13.8}}{10000} = 23 \text{ N/m/s}$$

2.3.1.5 Damping constants - flourel

Dennis has kindly supplied graphs for the damping properties of viton, which is similar to Flourel (both can be tailored in properties with various additives) – appendix 2. He also supplied an analysis of how to generate the relevant damping coefficient, appendix 3, relevant part reproduced here.

The interaction time of the contact is about 0.03 seconds (based on an initial calculation with essentially no damping in the impact). So the effective frequency (associated with the strain rate during impact) is ~1/0.03 = 33 Hz. From the nomograph for viton, the real part of the elastic modulus at low frequency (static) is

Estatic = 1100 psi = 7.6 MPa

and the modulus at 33 Hz (and 75F) is

Edynamic = 1700 psi = 12 Mpa = ~1.58 x Estatic

If the full diameter of the 10mm diameter by 6 mm long viton cylinder is involved in the impact, then the stiffness should be about

kstatic = E A / L = 7.6E6 * pi * 0.005^2/0.006 = 99E3 N/m (similar to the measured 91E3)

kdynamic = 1.58 x kstatic = 143.5E3 N/m

The loss factor at 33 Hz and 75F is 0.4, so the Q = 1/loss = 2.5. The associated damping coefficient is

bpad = Sqrt(k m)/Q = Sqrt(143.5E3 * 13.8)/2.5 = 563 N/m/s

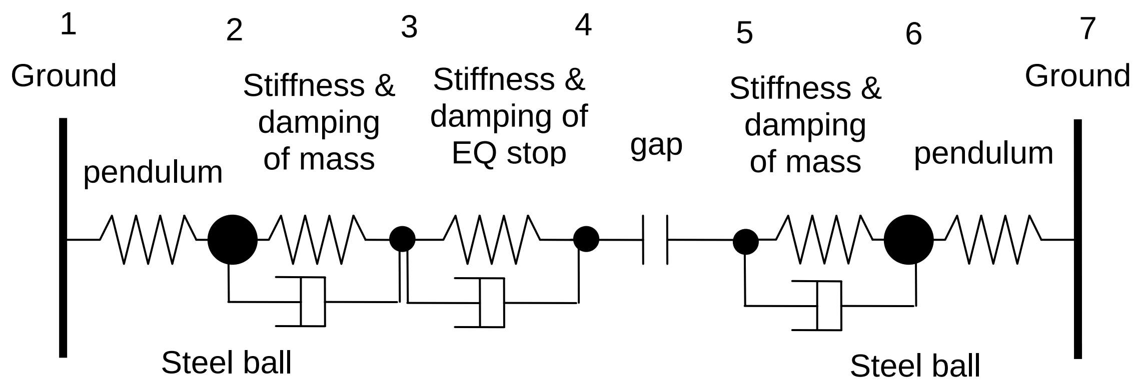
And, importantly for future work, b varies with the square of the stiffness.

2.4 FEA results

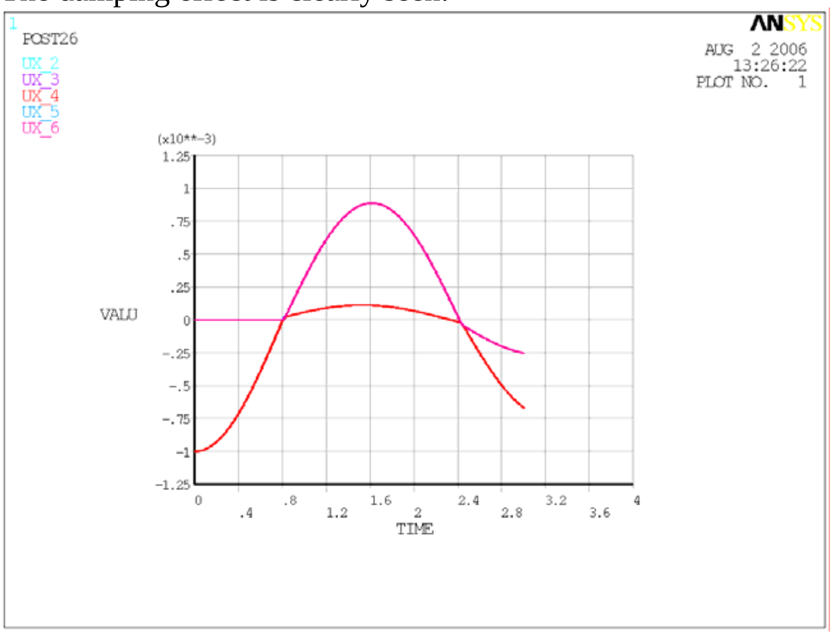
2.4.1 Initial results

After much messing with ANSYS, and a vital mistake pointed out by Dennis Coyne (see appendix 3 for detail), here is the first successful run of a model (macro, as modified by Dennis, in appendix 1):

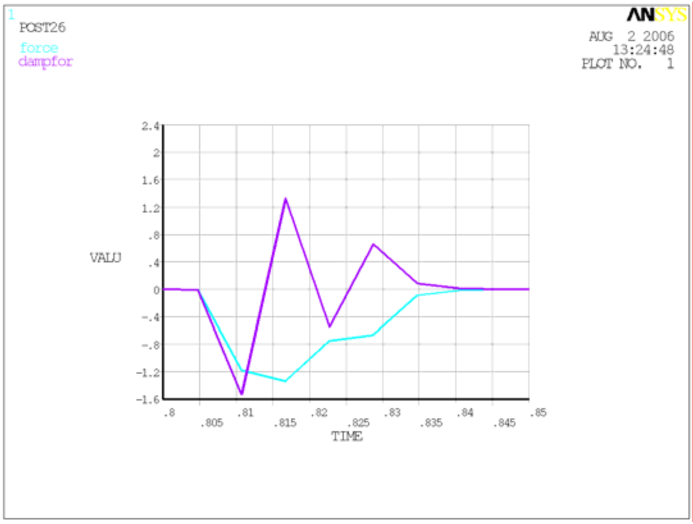
The nodes are numbered thus:



The damping effect is clearly seen:



However on closer study of the impact, there is clearly room for more refined time stepping. This graph shows the force and damping force during the impact: (terms SMISC,1 and NMISC,3 for COMBIN14).



To refine this, split the job up and make substeps as follows:

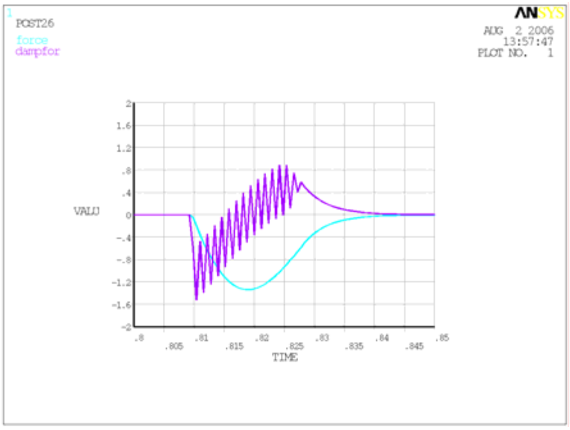
Step	purpose	time	Substeps
1	Initial conditions	0 to 0.001	2
2	swing	.001 to 0.79	50
3	impact	0.79 to 0.85	100
4	rebound	0.85 to 1.65	50

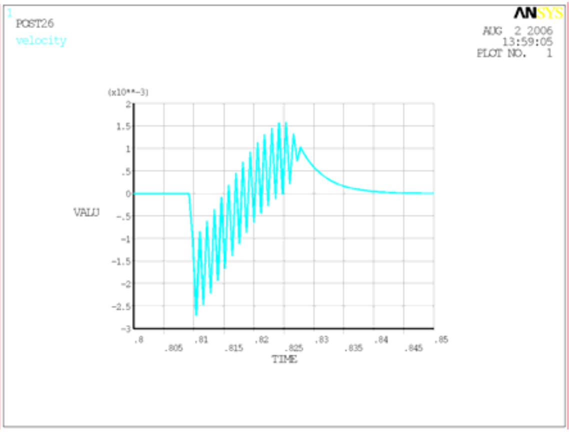
```
/solve
!*
ANTYPE,4      !transient
TRNOPT,FULL   !Full analysis - no shortcuts

solcontrol,0    !used in VM81, no idea why, if ommitted solution fails.

neqit,200

timint,off
d,2,UX,initdisp
d,3,UX,initdisp
d,4,UX,initdisp
nsubst,2
kbc,1  !stepped BC
OUTRES,ALL,ALL
time,0.001
solve

timint,on
ddele,2,ux
ddele,3,ux
ddele,4,ux
NSUBST,50
time,0.79
solve

NSUBST,100
time,0.85
solve

NSUBST,50
time,1.65
```

solve

FINISH

This looks better, but still some way to go. I have also plotted the velocity (NMISC,2) – which takes the same form as the damping.

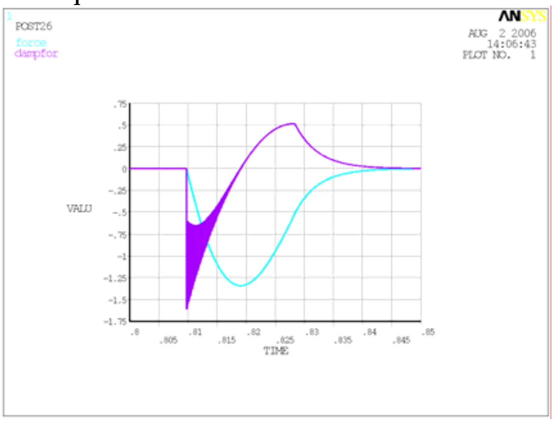




Split the substeps thus

Step	purpose	time	Substeps
1	Initial conditions	0 to 0.001	2
2	swing	.001 to 0.8	50
3	Initial impact	0.79 to 0.83	500
4	Full impact	0.83 to 0.85	100
5	rebound	0.85 to 1.65	50

Not quite there:



After some playing with step sizes,
This:

deltim,.01
time,0.8
solve

deltim,.0001
time,0.806
solve

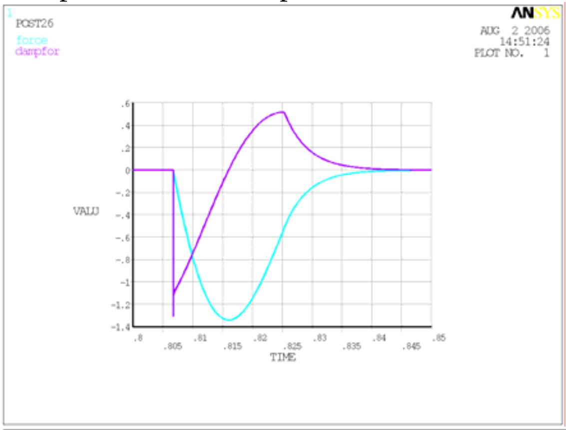
deltim,.000001
time,0.808

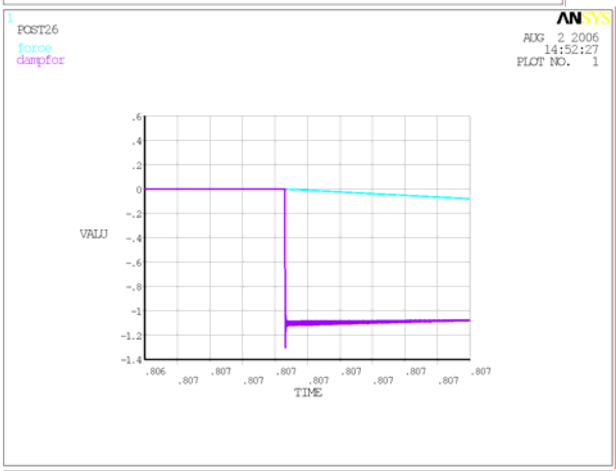
solve

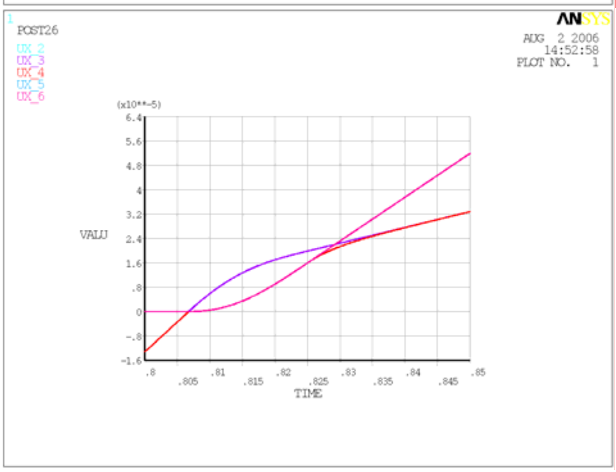
deltim,.0002
time,0.85
solve

deltim,.01
time,1.65
solve

gives the following in which the curves look reasonably smooth and continuous with the exception of a small spike whose area must be very low:







The way I interpret these results is as follows:

- At about 0.807 secs nodes 4 and 5 come into contact. The pad, element 3, starts to be compressed giving a negative velocity of one end relative to the other and a negative (compressive) damping force.
- At about 0.817 secs the pad is fully compressed and starts to unload as the output ball's speed exceeds that of the input ball. Relative velocity, and damping force, are zero. The spring force in the pad has peaked.
- At about 0.825 seconds the damping and spring forces in the pad are equal and opposite – there is no net closing force on the gap, which starts to open. Node 4, which had been following node 6, no longer does so.
- At about 0.84 seconds the pad has resumed its natural length, node 4 is now following node 3.

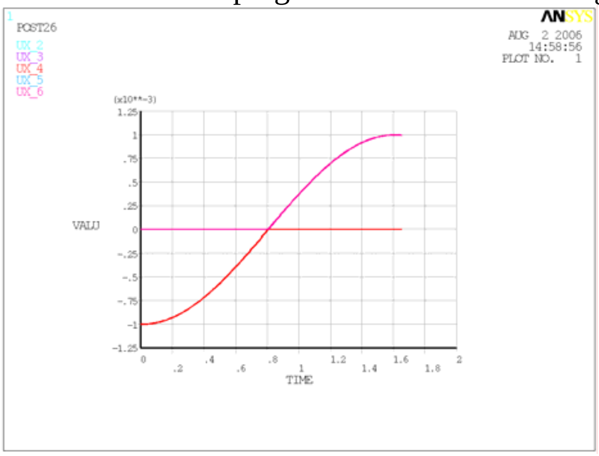
The all seems to make sense, so now proceed with matching the experiment.

2.4.2 Match FEA to steel-only result.

For this, make the pad very stiff and remove all damping:

kball = 3.9E9 !(N/m) stiffness of ball
bball = 23 !(N/m/s) damping in steel
mball = 13.85 !(kg) mass of ball
kpad = 143.5E9 !(N/m) stiffness of flourel EQ stop.
bpad = 1 !(N/m/s) damping constant of pad

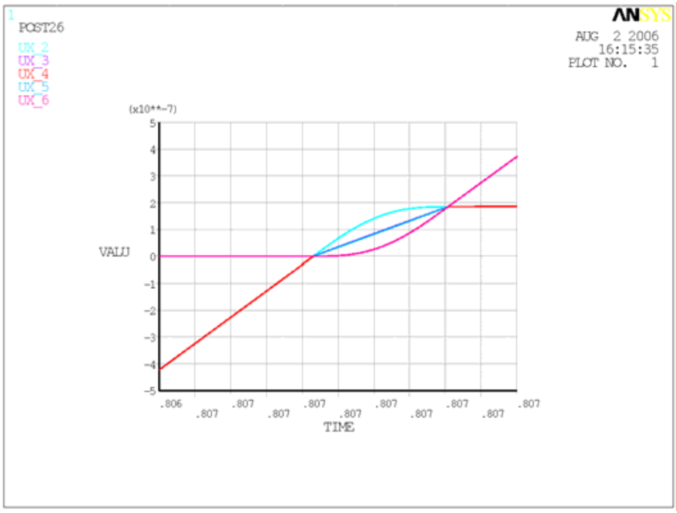
Hmmm – the damping in the steel is not having much effect:



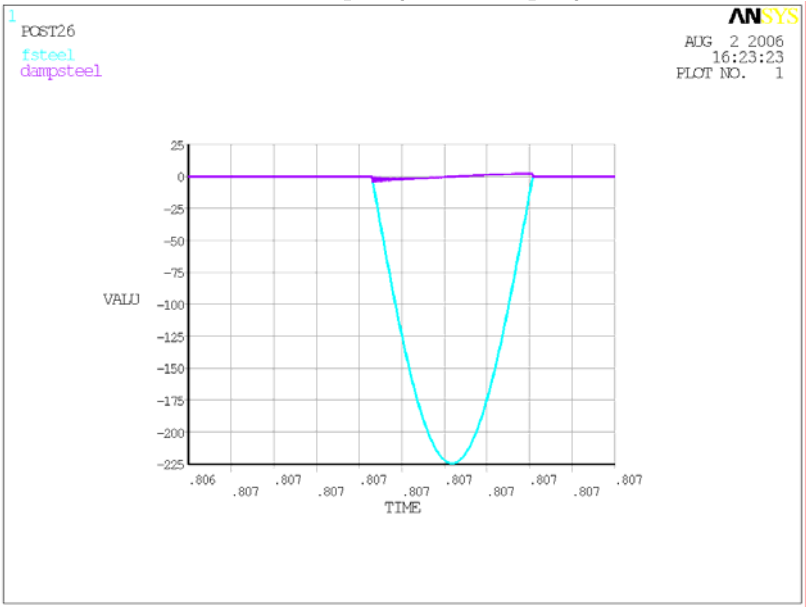
So increase it:
kpend = 53.3 !(N/m) stiffness of artificial spring to represent pendulum
kball = 3.9E9 !(N/m) stiffness of ball
bball = 2300 !(N/m/s) damping in steel
mball = 13.85 !(kg) mass of ball
kpad = 143.5E9 !(N/m) stiffness of flourel EQ stop.
bpad = 1 !(N/m/s) damping constant of pad

Overall plot looks just the same.

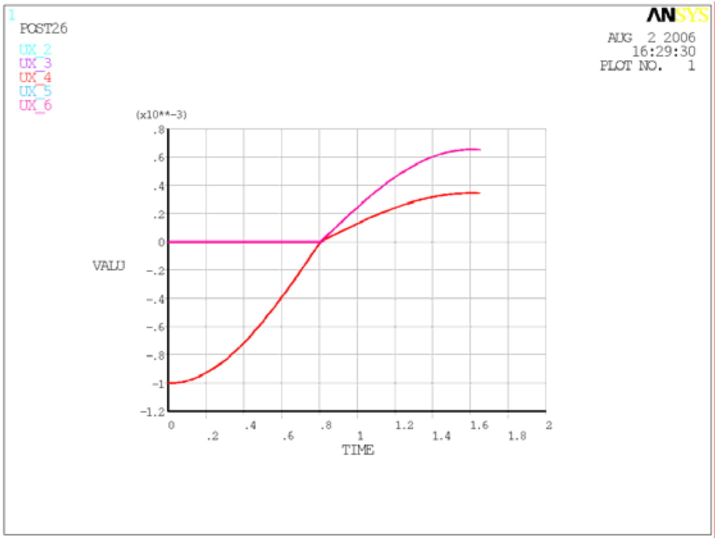
Look in detail at the contact (time range from .8065 to .8070)



We see now that point 5 (and, probably, points 3 and 4) follow a path midway between 2 and 6. This makes sense. Spring and damping force in one of the masses:



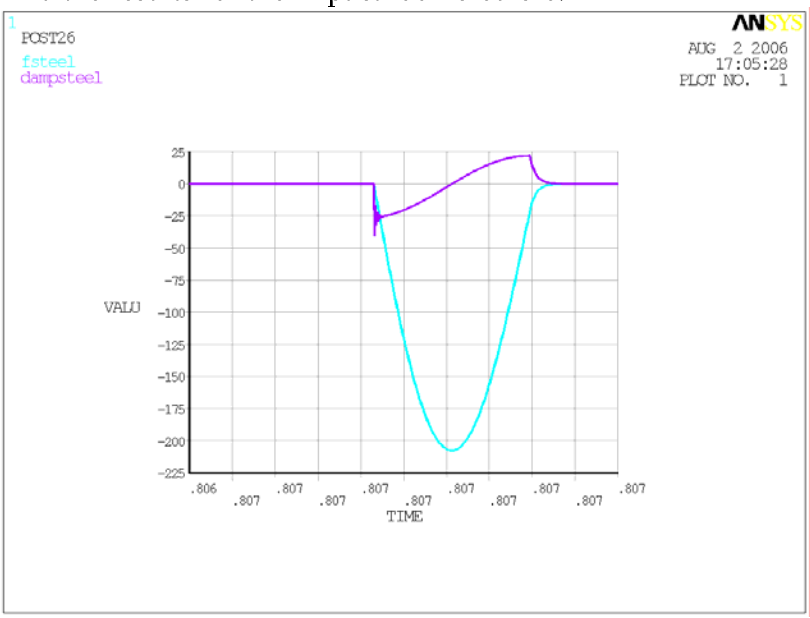
And of course, with a low relative velocity between the ends of the spring that represents the mass, there is very little damping. However, we are stuck with the fact that there is energy loss even without the polymer – from the results of the experiment. Try increasing the damping by a further factor 100. And, with this unfeasibly large damping, it works:



Time to “experiment”

Damping	Response (target = 0.92) Obtain using **extrem,6** command
23	1
2300	1
230000	0.6548E-03
23000	0.9312E-03
40000	0.8888E-03
27000	0.9207E-03 (close enough)

And the results for the impact look credible:



We are left with the question of where this damping is coming from. It is not air or suspension friction. I did a simple test swinging one ball on it’s own. After 6 swings, a ~100mm swing had decayed less than 2mm. It may be that the energy loss is associated with the cast surfaces rubbing during impact. This rubbing would not occur with the pad present, so we may be underestimating the loss caused by the pad (which is conservative in this case).

Another useful suggestion from Dennis is that if the balls are cast then there might be quite a lot of internal damping associated with the high contact stresses. See appendix 5.

Q=b^2/(k*m) = (3.9E9*13.8)/ 27000^2 = 73.

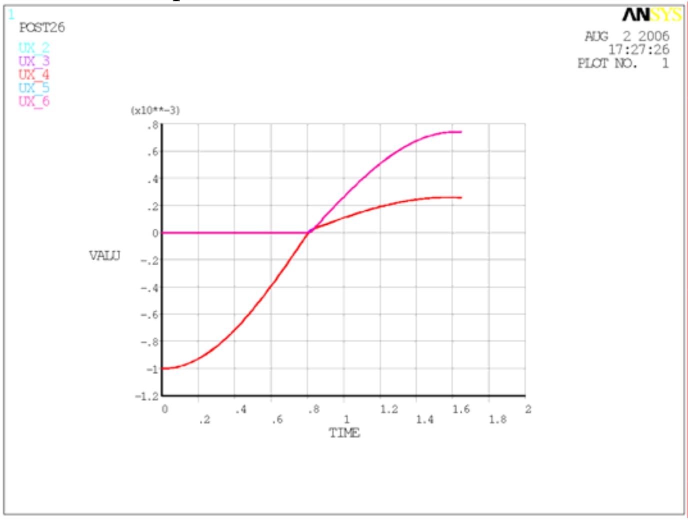
2.4.3 Match FEA to result with polymer pad.

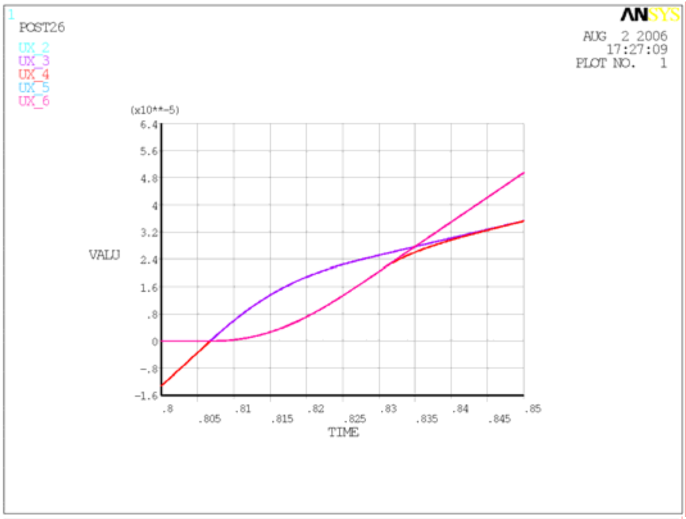
Reintroduce the polymer with its real stiffness and iterate to get the right damping.

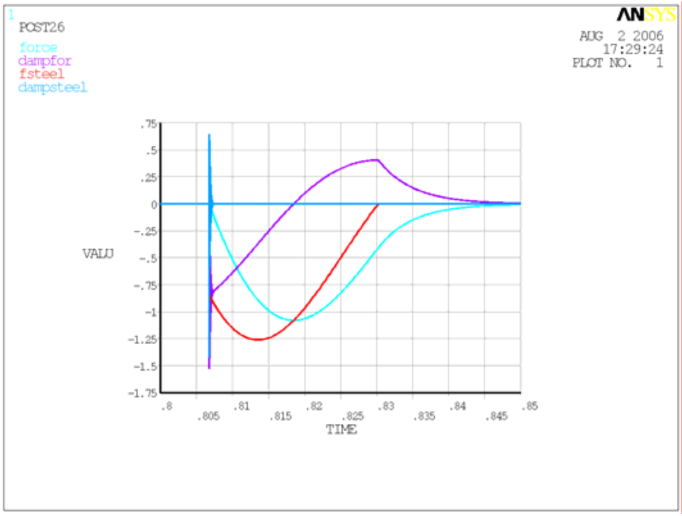
kpend = 53.3 !(N/m) stiffness of artificial spring to represent pendulum
kball = 3.9E9 !(N/m) stiffness of ball
bball = 27000   !(N/m/s) damping in steel
mball = 13.85 !(kg) mass of ball
kpad = 90.8E3 !(N/m) stiffness of flourel EQ stop.
bpad = 563   !(N/m/s) damping constant of pad

bpad	Response (target 0.74)
563	0.6995E-03
500	0.7176E-03
430	0.7405E-03 (close enough)

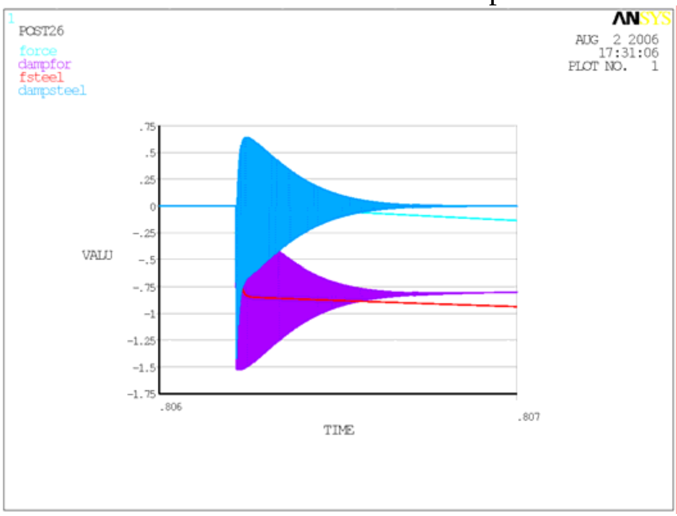
And, for completeness, several results:







note fsteel is the sum of force and dampfor



time from 0.8065 to 0.8075

Given the uncertainties about the damping in the steel (and at least two of the proposed mechanisms would not be present when the polymer pad is in place). Try running again with no damping in the steel:

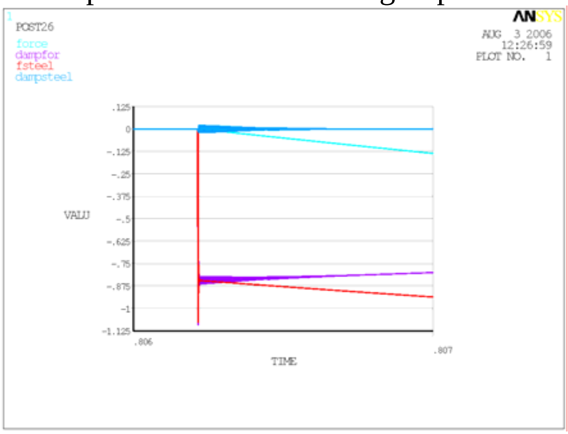
kpend = 53.3 !(N/m) stiffness of artificial spring to represent pendulum
kball = 3.9E9 !(N/m) stiffness of ball
bball = 27 !(N/m/s) damping in steel

mball = 13.85 !(kg) mass of ball
kpad = 90.8E3 !(N/m) stiffness of flourel EQ stop.
bpad = 430   !(N/m/s) damping constant of pad

The output motion is now `0.7405E-03,` in other words, no change! And just to prove that I didn't run the previous job again here is the parameter listing:

```
NAME                              VALUE                        TYPE  DIMENSIONS
 BBALL                             27.0000000                     SCALAR
 BPAD                              430.000000                     SCALAR
 INITDISP                        -1.000000000E-03                 SCALAR
 KBALL                            3.900000000E+09                 SCALAR
 KPAD                              90800.0000                     SCALAR
 KPEND                             53.3000000                     SCALAR
 MBALL                             13.8500000                     SCALAR
```

And a plot of the forces during impact:



time from 0.8065 to 0.8075

2.5 Check for non-linear scale effects

To check that the input distance does not have a big effect, set initdisp to 10 and 100mm.

Input	Output as %
1mm	74.05 (`0.7405E-03`)
10mm	74.05 (`0.7405E-02`)
100mm	74.05 (`0.7405E-01`)

So the behaviour is in fact completely linear with input distance.

2.6 Conclusion from the Newton's cradle experiment

A piece of Flourel of the composition used for the spring seatings in the SEI system, measuring approximately 10mm diameter by 6mm long, has a spring constant of 91E3 N/m. It may be modelled in an impact situation similar to that of the earthquake stops in LIGO by a spring/damper having that spring constant and a damping constant of 430 N/m/s. These numbers are both consistent with published data on similar materials.
For completeness , the full macro in it's final form is given in appendix 5.

3 Details of model

Return to the beam model used in T060098-00-K. The macro starts by taking the stiffness of the structure as an input and working out the properties of an equivalent beam:

length=1.7 !m
youngs=70E9 !Pa
stiffness=2E6 !N/m
beamI=stiffness*length3/(3*youngs)**
beamY=(beamI*12)0.25**
beamA=beamY2**
mass=50 !check with Tim
density=mass/(length*beamA)

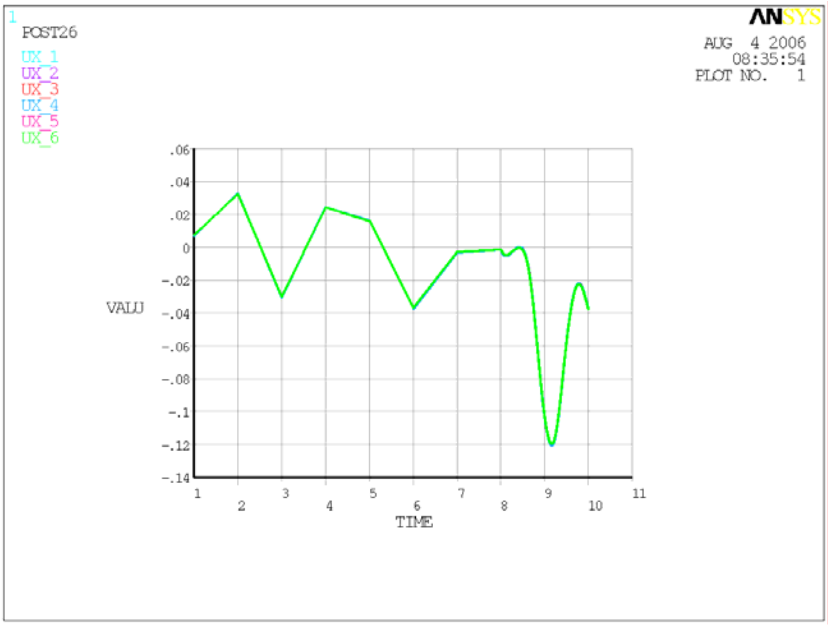
MTM = 40 !Mass of test mass
KPad = 1.4E7 !stiffness of polymer pad
bPad = 2.4E3 !damping constant of pad

The mass and stiffness numbers need updating for the current structure – although we already know that the mass of the controls structure was about 80kg, rather than the 50 used in the macro.

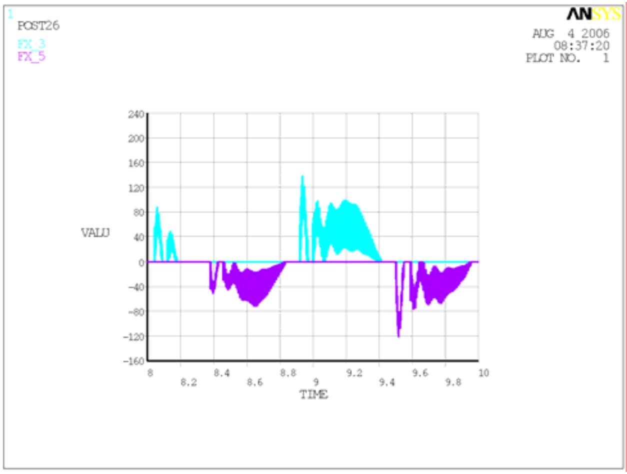
Much more significant is the stiffness I used before for the pad - I had 1.4E7 whereas we now know that a piece of Flourel of the right sort of size has a stiffness of about 90E3 N/m – a lot less.

While I wait for the updated stiffness of the structure (which in truth won't affect the answers much) I will run the macro with the reduced pad stiffness. For three pads the stiffness is 270E3 N/m and the damping is 3*430 = 1300 N/m/s. (I also fixed the error in the macro so I am now using CV1 not CV2 for the damping.)

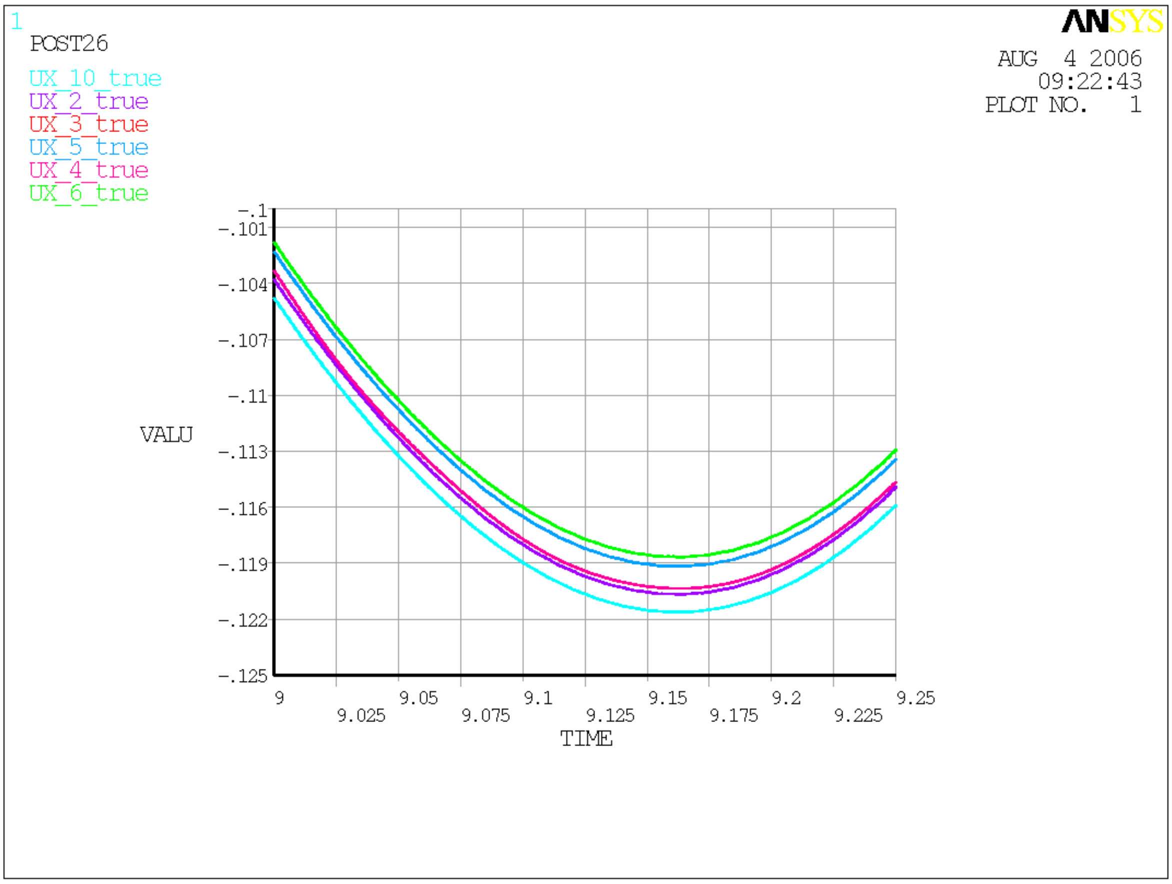
Macro in appendix 6 for reference.
Initial results look OK:



And the forces are much lower than before:

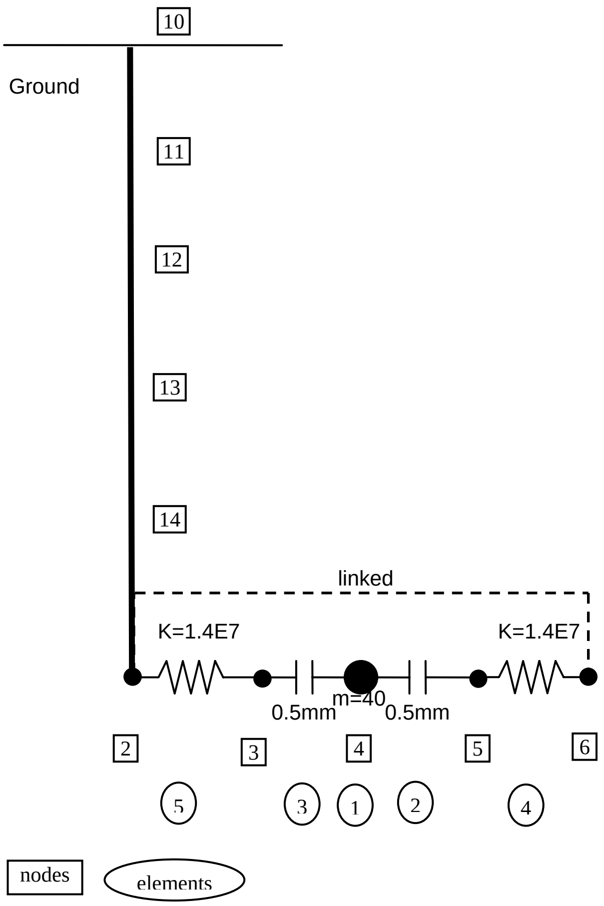


The contact occurring around t=9 is much more subtle than before:



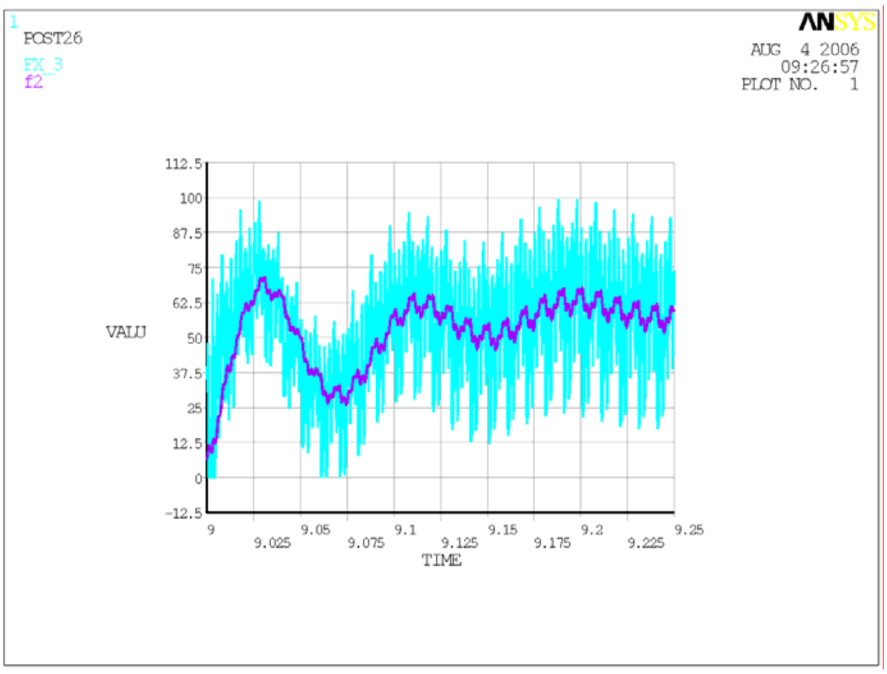
Checking with ANSYS plots shows that the trace for point 3 is hidden by that for point 4 – in other words, the gap is closed throughout this plot.

A reminder of the model:



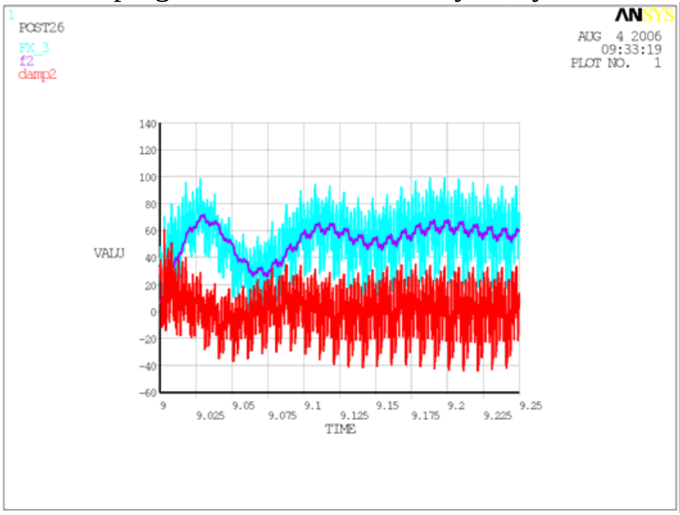
Interestingly the force extracted by two different routes is a bit different:

ESOL,9,5,3 ,F,X,FX_3
ESOL,23,5, ,SMIS,1,f2
ESOL,24,5, ,NMIS,3,damp2

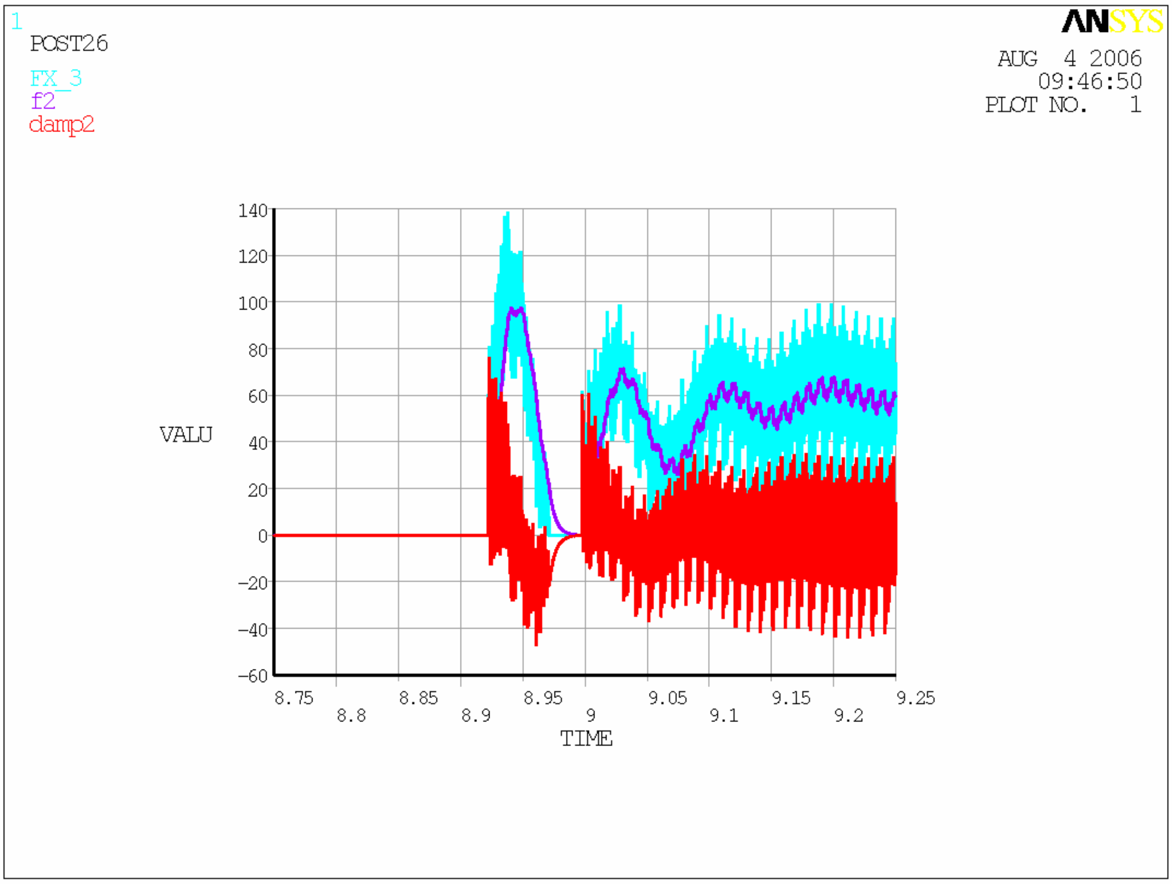


But the form suggests a steady contact rather than the series of bumps seen previously.

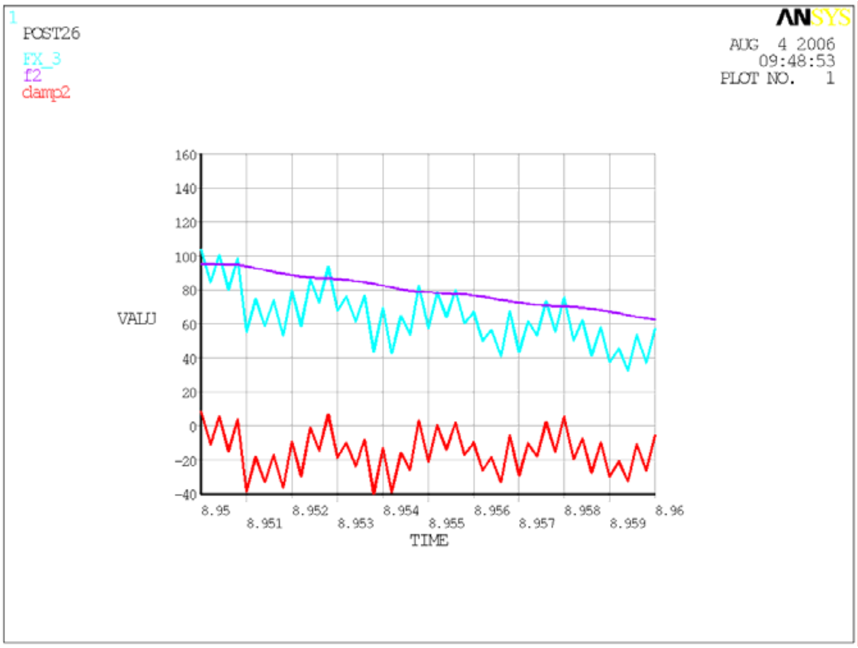
The damping force is small and very noisy, like one of the force plots:



Contact starts a bit earlier, and we can see an initial bounce before steady contact is established:



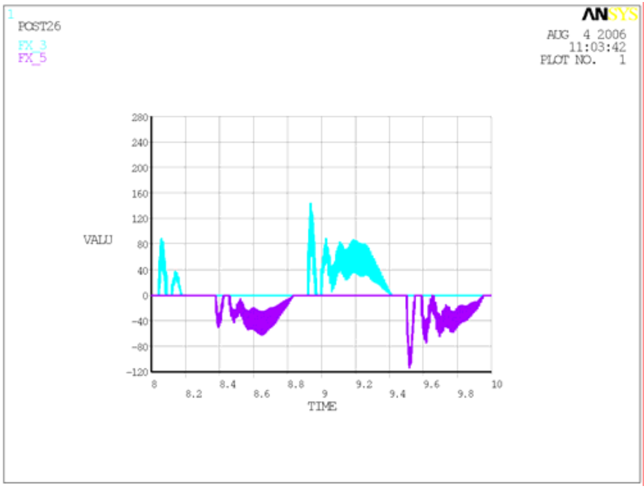
Look at the “noise” in more detail:

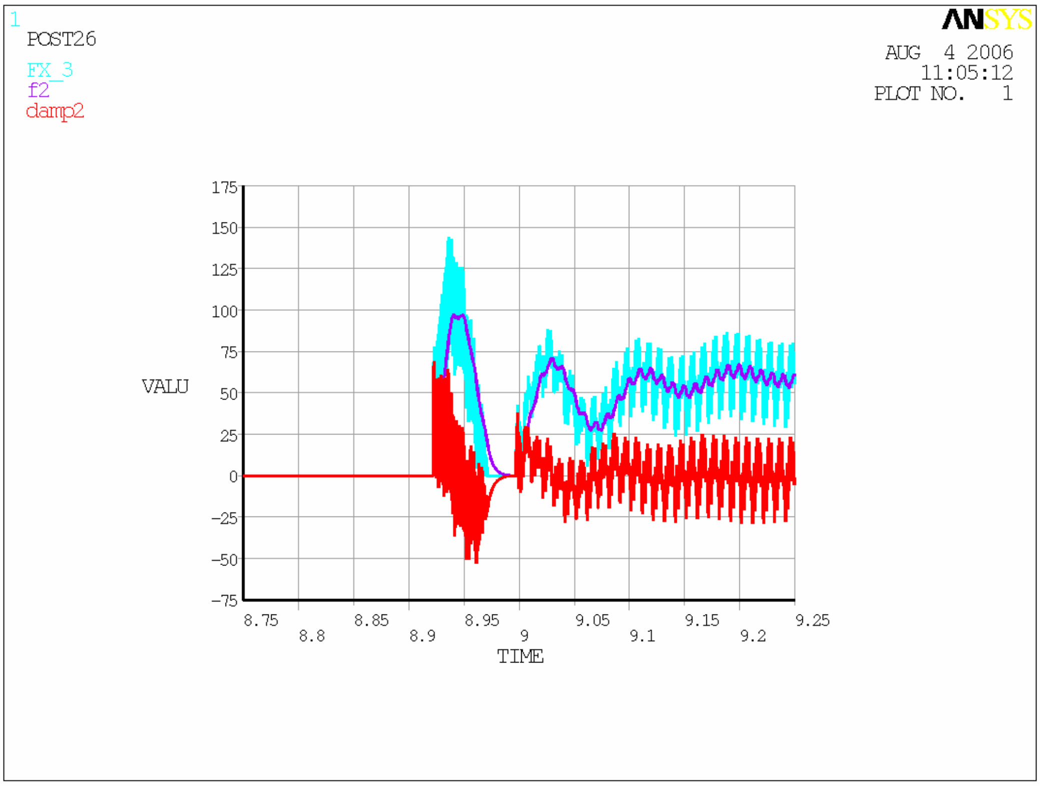


There are 5000 substeps per second in the analysis, giving a substep length of 0.0002 secs. This corresponds to the period of the noise spikes.

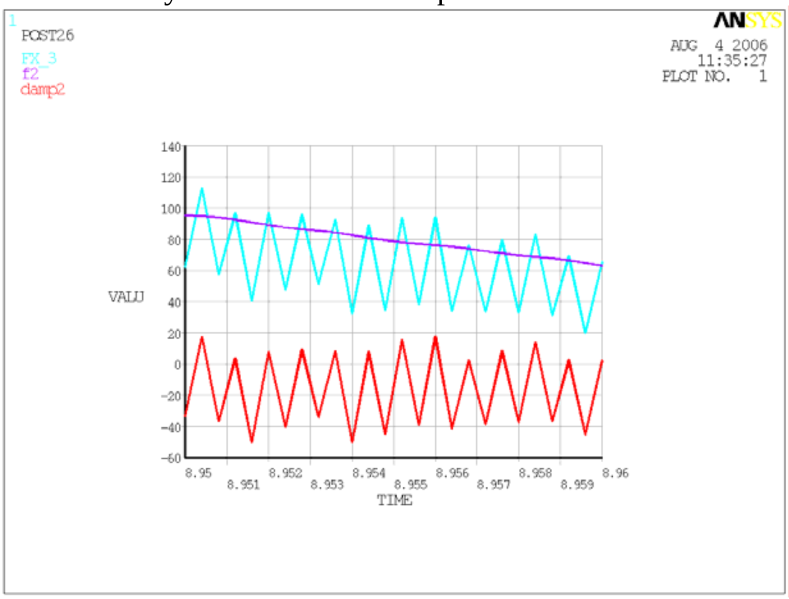
To check solution robustness, try with half the number of substeps (twice as many would take a long time!)

Forces look much the same:





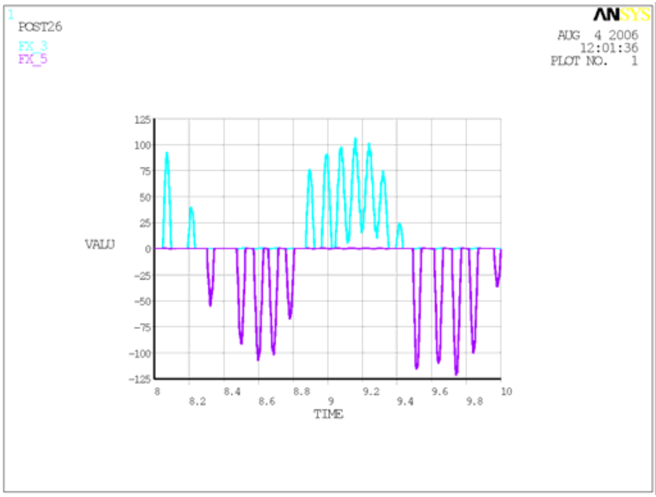
And the noisy force has twice the period but similar values:

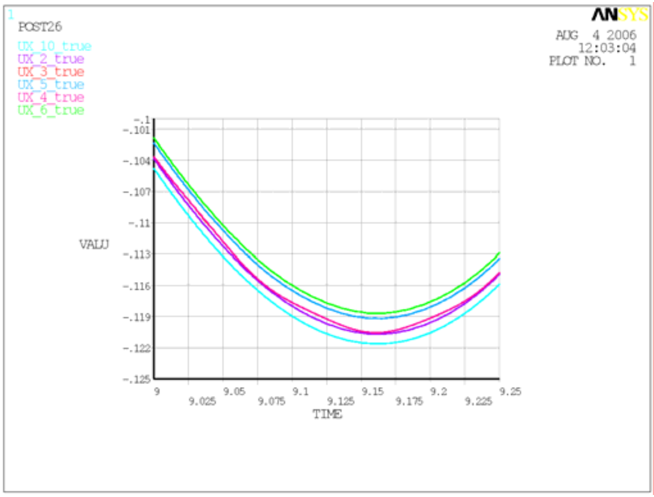


Out of interest, what happens if the Flourel damping is reduced?

Try
KPad = 270E3 !stiffness of polymer pad
bPad = 130 !damping constant of pad

Answer: the "bumping" is back, as one might have expected. However, the peak forces look much the same:





So it looks as though the damping does have an effect but is not particularly important for reducing peak forces.

From all of this I conclude that a small flourel stop with an active volume ~10mm diameter by ~6mm thick would work fine.

APPENDIX 1 – macro used in section 2.4.1

```
FINISH  ! Make sure we are at BEGIN level
/CLEAR
*abbr,doit,doit
*abbr,jreplo,/replot
/config,nres,51000
/PREP7

! to make a simple model of Newton's cradle.
! See LIGO-T-060176
! See Structural Guide | Chapter 5. Transient Dynamic Analysis
!                      | 5.3.2. Establish Initial Conditions
! for how to set up initial conditions
! Parameters
!
kpend = 53.3  !(N/m) stiffness of artifial spring to represent pendulum
kball = 3.9E9 !(N/m) stiffness of ball
bball = 23    !(N/m/s) damping in steel
mball = 13.85  !(kg) mass of ball
kpad = 143.5E3 !(N/m) stiffness of flourel EQ stop.
bpad = 563     !(N/m/s) damping constant of pad
initdisp = -0.001  !(m) initial displacement of input ball


/triad,lbot  !Move co-ord sys triad out of the way

! element types

ET,2,CONTAC12   !Gap
!*
KEYOPT,2,1,0  !Friction type only valid if mu>0
KEYOPT,2,2,0  !0=orientation angle based on theta real const make theta = 0
KEYOPT,2,3,0  !0=no weak spring on open gap
KEYOPT,2,4,1  !1=use node location for initial gap
KEYOPT,2,7,0  !connected with optimised solution time
!*

ET,3,MASS21    !Point mass
!*
KEYOPT,3,1,0  !0=Real consts are mass and inertia
KEYOPT,3,2,0  !0=elem coord system parallel to global
KEYOPT,3,3,4  !2D mass no rotary intertia

ET,4,COMBIN14   !spring for the pads and balls
!*
KEYOPT,4,1,0  !0=linear; 1 nonlinear (if CV2 is nonzero)
KEYOPT,4,2,1  !1 = 1D longitudinal in UX
KEYOPT,4,3,0  !2 = 2D rather than 3D; Z=0 throughout
!*

ET,5,COMBIN14   !spring for the pendulum effect
!*
KEYOPT,5,1,0  !0=linear spring; 1 required for damping
KEYOPT,5,2,1  !1 = 1D longitudinal in UX
KEYOPT,5,3,0  !2 = 2D rather than 3D; Z=0 throughout


!Real constants
!                                             !R,n,mass
! For the balls                               R,1,mball
```

```
!

!for the gap
!
!R,n,theta,kn  ,intf,start,ks
R, 2,90   ,2e12,    ,

!For the polymer pad
!
!R,n,K,CV1,CV2
R,3,kpad,bpad

!For the k of the balls
!
!R,n,K,CV1,CV2
R,4,kball,bball

!For the pendulum effect
!
!R,n,K,CV1,CV2
R,5,kpend



!nodes
n,1,-.075
,2,0
,3,.075
,4,.081
,5,.081
,6,.156
,7,.231


!elements

! Balls
type,3
real,1
E,2
E,6

!pads
type,4
real,3
E,3,4

!k of balls
type,4
real,4
E,2,3
E,5,6

!k of pendulum
type,5
real,5
E,1,2
E,6,7

!gaps
type,2
real,2
E,5,4

!fix all nodes in y
nsel,all
D,all,UY,0

!fix nodes 1 and 7 to gives pendulum effect
D,1,UX,0
D,7,UX,0

FINISH


/solve
!*
ANTYPE,4      !transient
TRNOPT,FULL   !Full analysis - no shortcuts

solcontrol,0    !used in VM81, no idea why, if
ommitted solution fails.

timint,off
d,2,UX,initdisp
d,3,UX,initdisp
d,4,UX,initdisp
nsubst,2
kbc,1  !stepped BC
OUTRES,ALL,ALL
time,0.001


solve


timint,on
ddele,2,ux
ddele,3,ux
ddele,4,ux

NSUBST,500
time,3
solve


FINISH
/POST26
numvar,200


!*
NSOL,2,2,U,X,UX_2
STORE,MERGE
!*
NSOL,3,3,U,X,UX_3
STORE,MERGE
!*
NSOL,4,4,U,X,UX_4
STORE,MERGE
```

```
!*
NSOL,5,5,U,X,UX_5
STORE,MERGE
!*
NSOL,6,6,U,X,UX_6
STORE,MERGE
XVAR,1
PLVAR,8,2,3,4,5,6
```

4 APPENDIX 2 – damping properties of viton.

Ahid Nashif, David Jones, John Henderson, Vibration Damping, John Wiley & Sons, cr 1985.

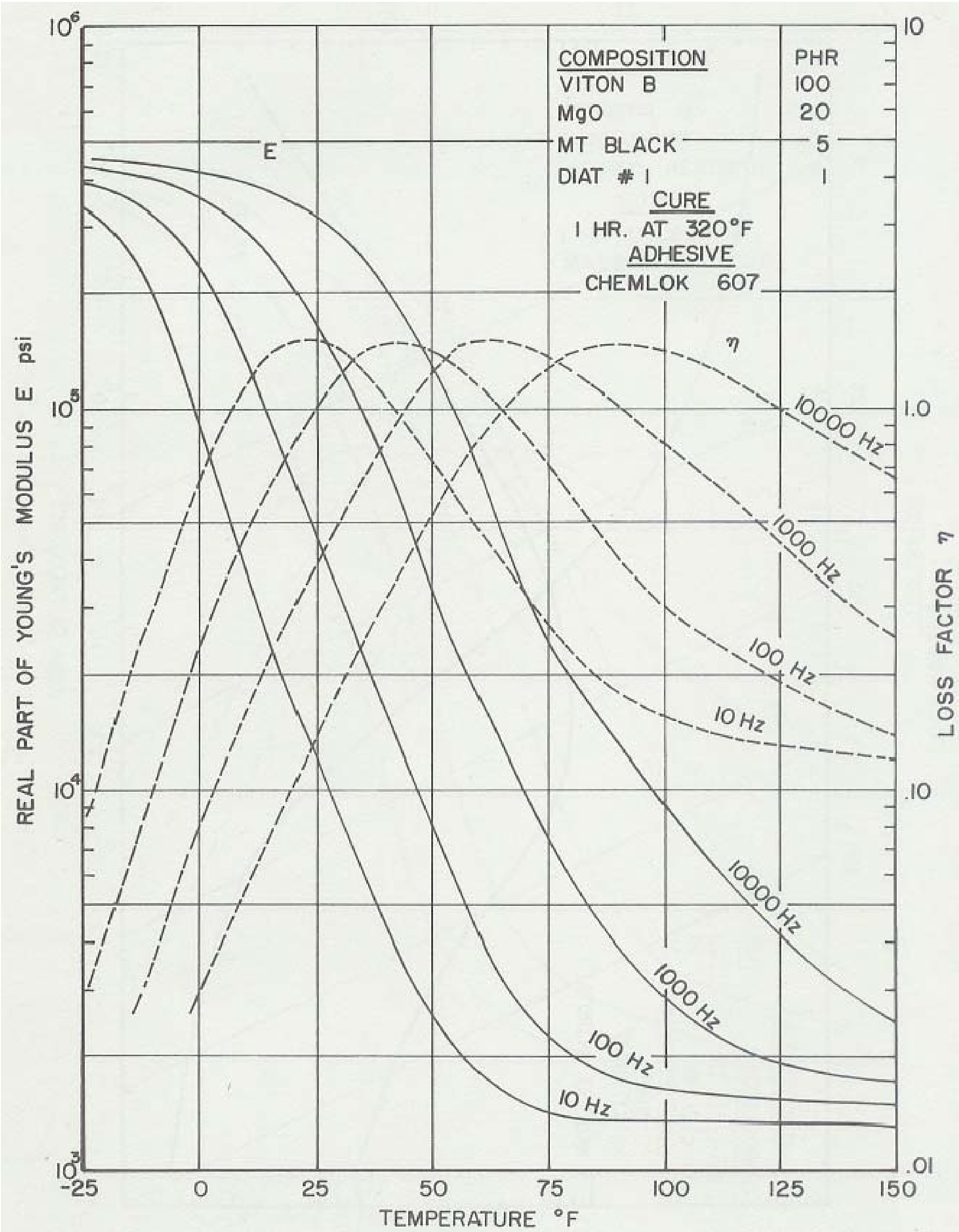


Figure 1: Complex modulus properties of Viton-B: Damping properties with temperature

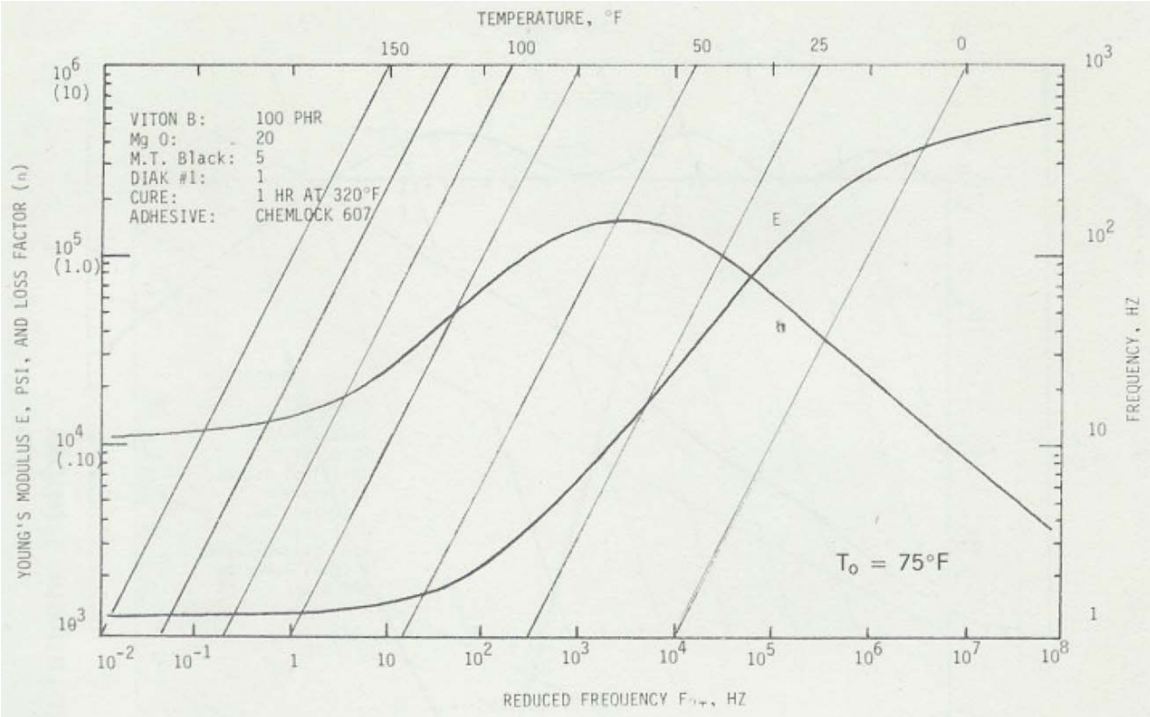


Figure 2: Complex modulus properties of Viton-B: Nomogram

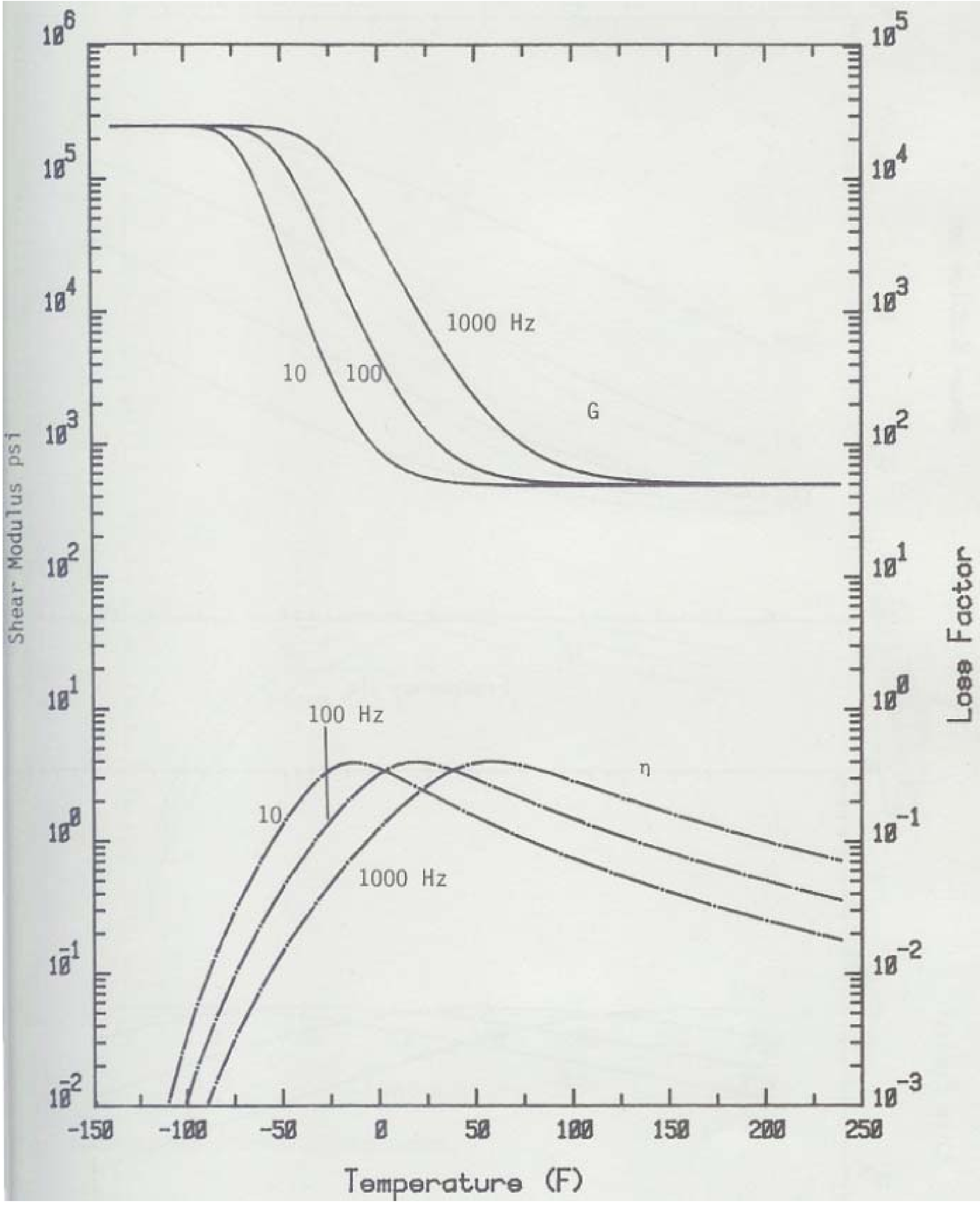


Figure 3: Damping properties of Fel-Pro Nitrile Rubber: Damping properties with temperature

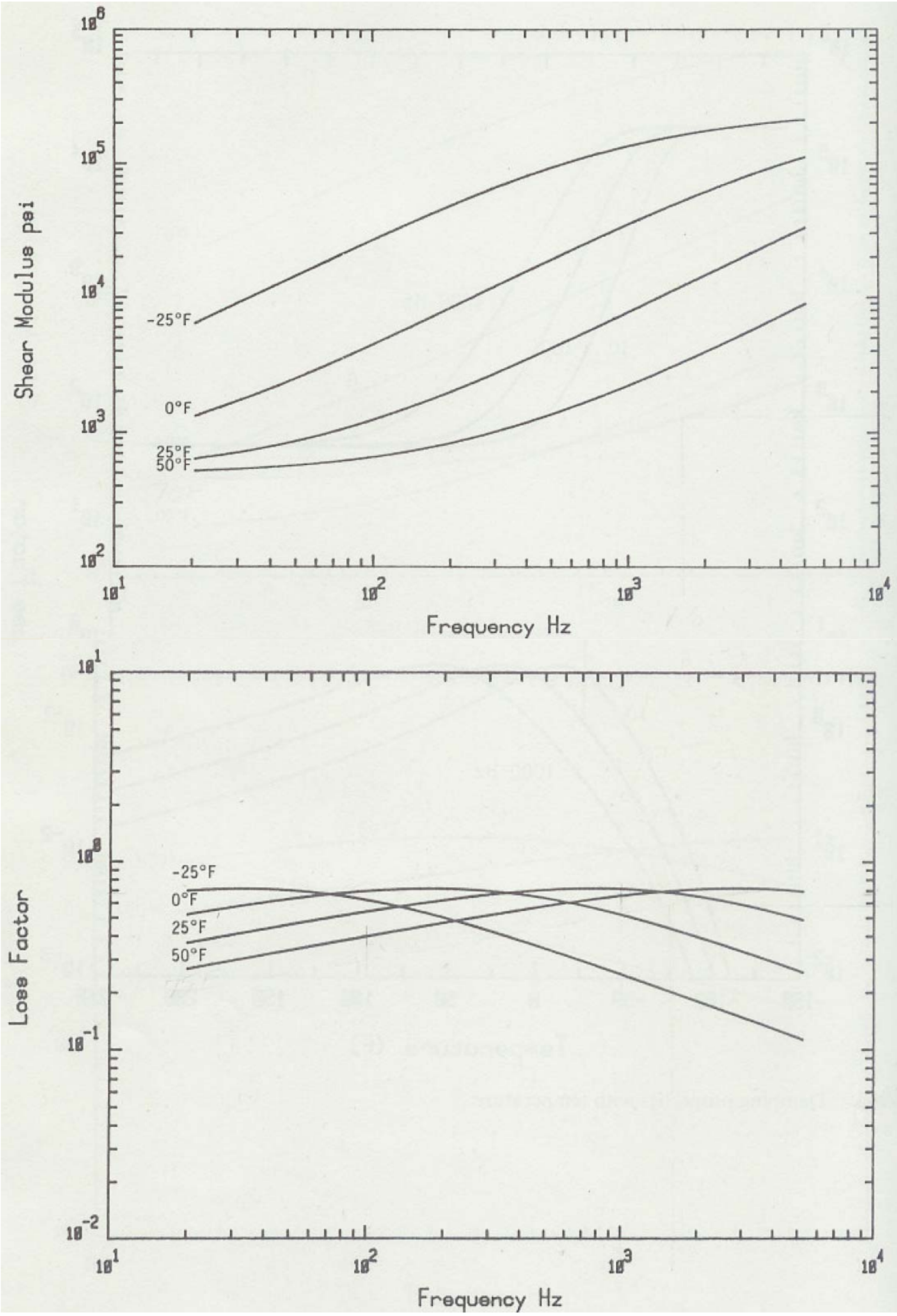


Figure 4: Damping properties of Fel-Pro Nitrile Rubber: Damping properties with frequency

5 APPENDIX 3 – email from Dennis Coyne re use of ANSYS and damping in flourel.

From: Dennis Coyne [mailto:coyne@ligo.caltech.edu]
Sent: 02 August 2006 02:52
To: Greenhalgh, RJS (Justin)
Cc: Dennis Coyne
Subject: RE: viton & nitrile damping properties

Hi Justin,
I found your error in the Newton's cradle, ANSYS macro. You were setting the nonlinear (velocity squared) coefficient, CV2, instead of the linear damping coefficient, CV1. If you make the following changes in the macro:

```
KEYOPT,4,1,0  !0=linear, 1 = nonlinear (if CV2 is nonzero)
R,3,kpad,bpad
R,4,kball,bball
```

then you get some effect of the damping (less than 1 mm rebound of the impacted ball). If you make the bpad damping coefficient 10 times higher (370 N/m/s), then you get a considerable effect.

As for how to relate the viscoelastic damping data to the single, viscous damping coefficient that you are using in your analysis, we must first decide on an effective frequency and temperature for the property data. (BTW in case you didn't know, viton is a Dupont fluoroelastomer similar to 3M's flourel. Both can be tuned for different properties by additives and differing ratios of the components.) The interaction time of the contact is about 0.03 seconds (based on your initial calculation with essentially no damping in the impact). So the effective frequency (associated with the strain rate during impact) is ~1/0.03 = 33 Hz. From the nomograph for viton, the real part of the elastic modulus at low frequency (static) is

Estatic = 1100 psi = 7.6 MPa

and the modulus at 33 Hz (and 75F) is

Edynamic = 1700 psi = 12 Mpa = ~1.58 x Estatic

If the full diameter of the 10mm diameter by 6 mm long viton cylinder is involved in the impact, then the stiffness should be about

kstatic = E A / L = 7.6E6 * pi * 0.005^2/0.006 = 99E3 N/m (similar to the measured 91E3)

kdynamic = 1.58 x kstatic = 143.5E3 N/m

The loss factor at 33 Hz and 75F is 0.4, so the Q = 1/loss = 2.5. The associated damping coefficient is

bpad = Sqrt(k m)/Q = Sqrt(143.5E3 * 13.8)/2.5 = 563 N/m/s

The rebound amplitude of the impacted ball with the dynamic kpad and bpad values above is about 0.9 times the initial ball amplitude. If the bpad value is increased to 750 N/m/s, then the rebound amplitude is ~0.75, consistent with your experiment (see the attached time response plot). This implies a loss factor of (750/563)*0.4 = 0.53. Since the impact duration for this revised calculation is about 0.022 sec, the loss should be higher; I'd estimate about 0.45 from the log plot. Given the approximations involved and the likely considerable possible variation in specific material properties from one viton or flourel to another, I'd say that this is reasonably good agreement.

Better still, one could use the viscoelastic material formulation in ANSYS for the link180 element instead of the COMBIN14 spring-damper element.

Dennis

6 Appendix 4 email from Dennis Coyne re damping in cast ferrous metals

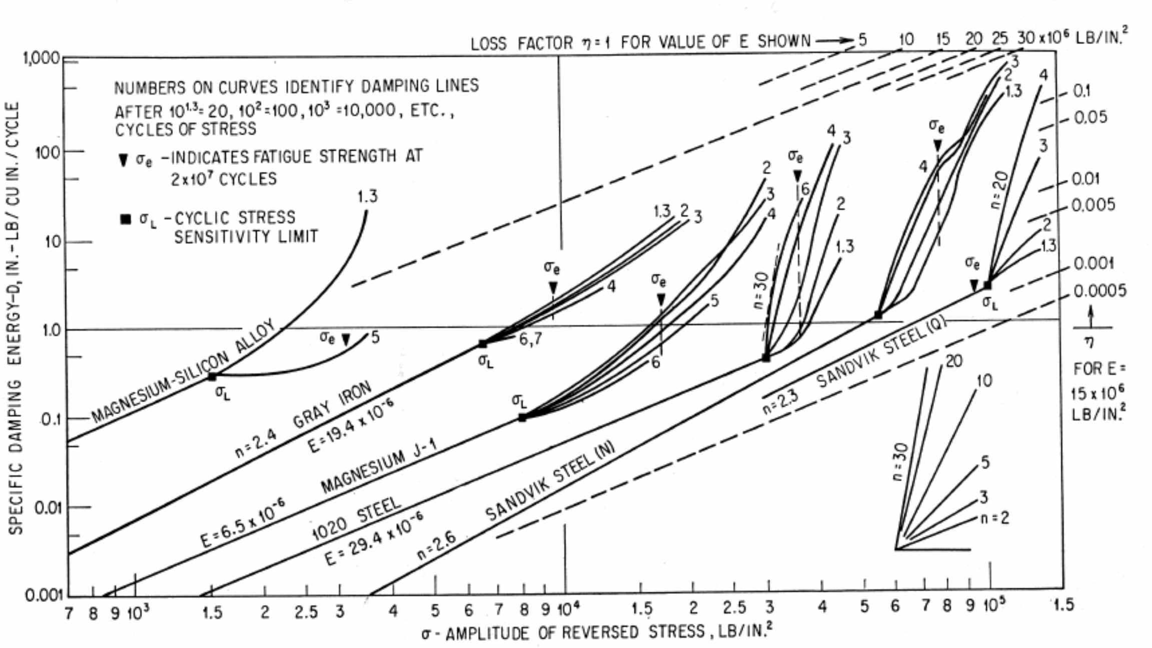


FIG. 36.15. Specific damping energy of various materials as a function of amplitude of reversed stress and number of fatigue cycles. Number of cycles is 10 to power indicated on curve. For example, a curve marked "3" is for 10^3 or 1,000 cycles.

C. Harris (ed.) and C. Crede (ed.), Shock and Vibration Handbook, McGraw Hill, cr 1961, Vol. 2, Chapter 36: B. Lazan, L. Goodman, "Material and Interface Damping"

As you noted it could be a interface damping effect, but it is quite a high damping value. Is it possible that the balls are "gray iron" and not steel? According to C. Harris & C. Crede (ed.), Shock and Vibration Handbook, Vol. 2, Chapter 36, pg. 36-31 and 36-34, if the amplitude of reversed stress is close to the endurance limit for cast gray iron (7E3 psi = 48 MPa) then the loss could be ~0.05 or Q ~ 20. (See the attached excerpted figure from Harris & Crede which included curves for gray iron and steel).

Contact compressive stress for sphere on sphere (W. Young, Roark's Formulas for Stress & Strain, 6th ed., Table 33, case 1b:

P = 207 N (from your fsteel plot)
E = 19.4E6 psi = 134 GPa (cast gray iron)
nu = 0.29
Ce = 2(1-nu^2)/E = 1.37E-11 m^2/N
Kd = D/2 = 0.15 m
a = 0.721*(P*Kd*Ce)^0.33 = 5.42E-4 m
StressComp = 1.5*P/(pi*a^2) = 336 MPa (49 ksi)

So the maximum compressive stress on impact is much larger than the endurance limit for cast gray steel. The damping increases dramatically when the stress exceeds the endurance limit.

If the balls are indeed cast steel, then

E = 29.4E6 psi = 203 GPa
Ce = 9.068e-12 m^2/N
a = 4.73e-4 m
StressComp = 442 Mpa (64 ksi)

which is well in excess of the endurance limit for 1020 steel (don't know is this is cast or not). At these stress levels high damping would be expected.

The above argument assumes that the damping is stress dependent and increases with higher stress. The four data points for the steel-steel contact roughly confirm this impact stress dependence:

point	input (mm)	output (mm)	ratio
#4	116	115	0.99
#2	184	178	0.97
#1	297	265	0.89
#3	303	275	0.91

In the subsequent tests with a flourel pad, the stress in the steel should be far less than the endurance limit. It would probably be best to set the ball damping properties to be far lower than you currently have in order to match the flourel damping properties.

Dennis

7 Appendix 5. Macro as used in section

```
FINISH  ! Make sure we are at BEGIN level
/CLEAR
*abbr,doit,doit
*abbr,jreplo,/replot
/config,nres,51000
/PREP7

! to make a simple model of Newton's cradle.
! See LIGO-T-060176
! See Structural Guide | Chapter 5. Transient Dynamic Analysis
!                      | 5.3.2. Establish Initial Conditions
! for how to set up initial conditions
! Parameters
!
kpend = 53.3  !(N/m) stiffness of artificial spring to represent pendulum
kball = 3.9E9 !(N/m) stiffness of ball
bball = 27    !(N/m/s) damping in steel
mball = 13.85  !(kg) mass of ball
kpad = 90.8E3 !(N/m) stiffness of flourel EQ stop.
bpad = 430    !(N/m/s) damping constant of pad
initdisp = -0.100  !(m) initial displacement of input ball


/triad,lbot  !Move co-ord sys triad out of the way

! element types

ET,2,CONTAC12   !Gap
!*
KEYOPT,2,1,0  !Friction type only valid if mu>0
KEYOPT,2,2,0  !0=orientation angle based on theta real const make theta = 0
KEYOPT,2,3,0  !0=no weak spring on open gap
KEYOPT,2,4,1  !1=use node location for initial gap
KEYOPT,2,7,0  !connected with optimised solution time
!*

ET,3,MASS21    !Point mass
!*
KEYOPT,3,1,0  !0=Real consts are mass and inertia
KEYOPT,3,2,0  !0=elem coord system parallel to global
KEYOPT,3,3,4  !2D mass no rotary intertia

ET,4,COMBIN14   !spring for the pads and balls
!*
KEYOPT,4,1,0  !0=linear; 1 nonlinear (if CV2 is nonzero)
KEYOPT,4,2,1  !1 = 1D longitudinal in UX
KEYOPT,4,3,0  !2 = 2D rather than 3D; Z=0 throughout
!*

ET,5,COMBIN14   !spring for the pendulum effect
!*
KEYOPT,5,1,0  !0=linear spring; 1 required for damping
KEYOPT,5,2,1  !1 = 1D longitudinal in UX
KEYOPT,5,3,0  !2 = 2D rather than 3D; Z=0 throughout


!Real constants
```

```
!
! For the balls
!R,n,mass
R,1,mball
!

!for the gap
!
!R,n,theta,kn  ,intf,start,ks
R, 2,90   ,2e12,    ,

!For the polymer pad
!
!R,n,K,CV1,CV2
R,3,kpad,bpad

!For the k of the balls
!
!R,n,K,CV1,CV2
R,4,kball,bball

!For the pendulum effect
!
!R,n,K,CV1,CV2
R,5,kpend



!nodes
n,1,-.075
,2,0
,3,.075
,4,.081
,5,.081
,6,.156
,7,.231


!elements

! Balls
type,3
real,1
E,2
E,6

!pads
type,4
real,3
E,3,4

!k of balls
type,4
real,4
E,2,3
E,5,6

!k of pendulum
type,5
real,5
E,1,2
E,6,7

!gaps
type,2
real,2
E,5,4

!fix all nodes in y
nsel,all
D,all,UY,0

!fix nodes 1 and 7 to gives pendulum effect
D,1,UX,0
D,7,UX,0

FINISH


/solve
!*
ANTYPE,4      !transient
TRNOPT,FULL   !Full analysis - no shortcuts

solcontrol,0    !used in VM81, no idea why, if ommitted solution fails.

neqit,200

timint,off
d,2,UX,initdisp
d,3,UX,initdisp
d,4,UX,initdisp
nsubst,2
kbc,1  !stepped BC
OUTRES,ALL,ALL
time,0.001
solve

timint,on
ddele,2,ux
ddele,3,ux
ddele,4,ux
deltim,.01
time,0.8
solve

deltim,.0001
time,0.806
solve

deltim,.000001
time,0.808
solve

deltim,.0002
time,0.85
solve
```

```
deltim,.01
time,1.65
solve

FINISH

/POST26
numvar,200

!*
NSOL,2,2,U,X,UX_2
STORE,MERGE
!*
NSOL,3,3,U,X,UX_3
STORE,MERGE
!*
NSOL,4,4,U,X,UX_4
STORE,MERGE
!*
NSOL,5,5,U,X,UX_5
STORE,MERGE
!*
NSOL,6,6,U,X,UX_6
STORE,MERGE
XVAR,1
PLVAR,2,3,4,5,6

ESOL,7,3, ,SMIS,1,force
STORE,MERGE
ESOL,8,3, ,NMIS,3,dampfor
STORE,MERGE
ESOL,9,3, ,NMIS,2,velocity
Store, merge

ESOL,10,4, ,SMIS,1,fsteel
STORE,MERGE
ESOL,11,4, ,NMIS,3,dampsteel
STORE,MERGE
ESOL,12,4, ,NMIS,2,vsteel
Store, merge

extrem,6

/eof
!a few useful commands for cut and paste:
/XRANGE,0.8,0.9
/YRANGE,-0.12e-1,0.2e-1,1
/XRANGE,1.55,1.65
/YRANGE,.09,.105,1
```

8 Appendix 6

```
FINISH  ! Make sure we are at BEGIN level
/CLEAR
*abbr,doit,doit
*abbr,jreplo,/replot
/config,nres,51000
/PREP7

! to make a simple beam model with given stiffness and masS.
!
! Parameters
!
length=1.7 !m
youngs=70E9 !Pa
stiffness=2E6 !N/m
beamI=stiffness*length**3/(3*youngs)
beamY=(beamI*12)**0.25
beamA=beamY**2
mass=50 !check with Tim
density=mass/(length*beamA)

MTM = 40 !Mass of test mass
KPad = 270E3 !stiffness of polymer pad
bPad = 1300  !damping constant of pad

/triad,lbot  !Move co-ord sys triad out of the way

! element types

ET,2,CONTAC12   !Gap
!*
KEYOPT,2,1,0  !Friction type only valkid if mu>0
KEYOPT,2,2,0  !0=orientation angle based on theta real const make theta = 0
KEYOPT,2,3,1  !0=no weak SPRing on open gap
KEYOPT,2,4,1  !1=use node location for initial gap
KEYOPT,2,7,0  !connected with optimised solution time
!*

ET,3,MASS21    !Point mass
!*
KEYOPT,3,1,0  !0=Real consts are mass and inertia
KEYOPT,3,2,0  !0=elem coord system parallel to global
KEYOPT,3,3,4  !2D mass no rotary intertia

ET,4,COMBIN14   !spring for the pads
!*
KEYOPT,4,1,0  !0=linear spring; 1 for damping nonzero CV2
KEYOPT,4,2,1  !1 = 1D longitudinal in UX
KEYOPT,4,3,0  !2 = 2D rather than 3D; Z=0 throughout
!*

ET,5,BEAM3
!*
KEYOPT,5,6,0
KEYOPT,5,9,0
KEYOPT,5,10,0
!*
```

```
!Real constants
!
! For the test mass
!R,n,mass
R,1,MTM
!

!for the gap
!
!R,n,theta,kn  ,intf,start,ks
R, 2,90   ,2e10,    ,

!For the polymer pad
!
!R,n,K,CV1,CV2
R,3,KPad,bPad

!For the beam
!
R,10,beamA,beamI,beamY


!material for the beam
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,youngs
MPDATA,PRXY,1,,0.3
MPDATA,DENS,1,,density

!nodes
N,10,-0.002,length
N,14,-.002,length/5
N,13,-.002,2*length/5
N,12,-.002,3*length/5
N,11,-.002,4*length/5
,2,-.002
,3,-.0005
,4,0
,5,.0005
,6,.002


!elements

! Test mass
type,3
real,1
E,4

!gaps
type,2
real,2
E,5,4
E,4,3

!pads
type,4
real,3
E,5,6
E,3,2
```

```
!beam
type,5
real,10
mat,1
e,10,11
e,11,12
e,12,13
e,13,14
e,14,2

!fix all nodes in y
nsel,all
D,all,UY,0
!fix node 10 in rotation
d,10,rotz,0

!couple nodes 2 and 6 in UX:
CP,  1,    UX,  2,     6
!CP, NSET, Lab, NODE1, NODE2, NODE3, NODE4, NODE5, NODE6, NODE7, NODE8, NODE9,
NODE10

!*
*DIM,  gdisp,TABLE, 1001,  1,   1,    TIME, ,
!*DIM, Par,  Type, IMAX, JMAX, KMAX, Var1, Var2, Var3, CSYSID
!*
*TREAD,GDISP,'montana1001','txt' ,' ', 1,
!*TREAD, Par, Fname,     Ext  , --, NSKIP
D,10,UX,%GDISP%

FINISH
!/eof

/solve
solcontrol,0    !used in VM81, no idea why, if ommitted solution fails.
ANTYPE,4  !transient
!*
TRNOPT,FULL   !Full analysis - no shortcuts
LUMPM,0
!*

OUTRES,ALL,LAST
kbc,0  !Ramped BC

NSUBST,5000
Autots,off
*do,jtime,1,8
TIME,jtime
solve
*enddo

OUTRES,ALL,1

NSUBST,5000
Autots,off
*do,jtime,9,10
TIME,jtime
solve
*enddo
```

```
FINISH
/POST26
numvar,200

!*
NSOL,8,10,U,X,UX_1
STORE,MERGE
!*
NSOL,2,2,U,X,UX_2
STORE,MERGE
!*
NSOL,3,3,U,X,UX_3
STORE,MERGE
!*
NSOL,4,4,U,X,UX_4
STORE,MERGE
!*
NSOL,5,5,U,X,UX_5
STORE,MERGE
!*
NSOL,6,6,U,X,UX_6
STORE,MERGE


FORCE,TOTAL
ESOL,9,5,3 ,F,X,FX_3
STORE,MERGE
FORCE,TOTAL
!*
!*
ESOL,10,4,5 ,F,X,FX_5
STORE,MERGE



XVAR,1
PLVAR,8,2,3,4,5,6

!
! Name: UX_10_true
! ID:  13
! Function: nsol(10 ,U,X)-.002
NSOL,200,10,U,X
FILLDATA,198,,,,-.002,0
REALVAR,198,198
PROD,197,198,194
ADD,13,200,197,,UX_10_true
!
!
! Name: UX_2_true
! ID:  14
! Function: nsol(2 ,U,X)-.001
NSOL,200,2,U,X
FILLDATA,198,,,,-.001,0
REALVAR,198,198
PROD,197,198,194
ADD,14,200,197,,UX_2_true
!
! Name: UX_3_true
```

```
! ID:  15
! Function: nsol(3 ,U,X)-.0005
NSOL,200,3,U,X
FILLDATA,198,,,,-.0005,0
REALVAR,198,198
PROD,197,198,194
ADD,15,200,197,,UX_3_true
!
! Name: UX_4_true
! ID:  17
! Function: nsol(4 ,U,X)
NSOL,17,4,U,X,UX_4_true
!
! Name: UX_5_true
! ID:  16
! Function: nsol(5 ,U,X)+.0005
NSOL,200,5,U,X
FILLDATA,198,,,,.0005,0
REALVAR,198,198
ADD,16,200,198,,UX_5_true
!
! Name: UX_6_true
! ID:  18
! Function: nsol(6 ,U,X)+.001
NSOL,200,6,U,X
FILLDATA,198,,,,.001,0
REALVAR,198,198
ADD,18,200,198,,UX_6_true
!


ESOL,20,3, ,SMIS,1,force
STORE,MERGE
ESOL,21,3, ,NMIS,3,dampfor
STORE,MERGE
ESOL,22,3, ,NMIS,2,velocity
Store, merge

ESOL,23,4, ,SMIS,1,fsteel
STORE,MERGE
ESOL,24,4, ,NMIS,3,dampsteel
STORE,MERGE
ESOL,25,4, ,NMIS,2,vsteel
Store, merge


:END
```

9 Appendix 7