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Study of the Possible Reduction of Parametric Instability Gain Using Apodizing Coatings in Test Masses

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1. Introduction

As part of the study of the possibility of the occurrence of parametric instabilities in advanced gravitational wave interferometers, it was suggested the use of an apodizing coating for the test masses. The main goal is to reduce the parametric gain R by increasing the diffraction losses of the high order optical modes keeping the fundamental mode diffraction loss below 1 ppm.

$$R \approx \frac{2PQ_m}{McL\omega_m^2}\left(\frac{Q_1\Lambda_1}{1+\Delta\omega_1^2/\delta_1^2} - \frac{Q_{1a}\Lambda_{1a}}{1+\Delta\omega_{1a}^2/\delta_{1a}^2}\right) > 1 \qquad (1)$$

Equation (1) shows the relation of the parametric gain R with different cavity parameters. Here P is the total power inside the cavity, M is the mass of the test mass, $Q_{1(a)}$ are the quality factors of the Stokes (anti-Stokes) modes, Q_m is the quality factor of the acoustic mode, $\delta_{1(a)} = \omega_{1(a)}/2Q_{1(a)}$ corresponds to the relaxation rate, L is the cavity length, $\Delta\omega_{1(a)} = \omega_0 - \omega_{1(a)} - \omega_m$ is the possible detuning from the ideal resonance case, and Λ_1 and Λ_{1a} are the overlap factors between optical and acoustic modes as shown in equation (2) [1, 2].

$$\Lambda_{1(a)} = \frac{V\left(\int f_0(\vec{r}_\perp) f_{1(a)}(\vec{r}_\perp) u_z d\vec{r}_\perp\right)^2}{\int |f_0|^2 d\vec{r}_\perp \int |f_{1(a)}|^2 d\vec{r}_\perp \int |\vec{u}|^2 dV}. \qquad (2)$$

Here f_0 and $f_{1(a)}$ describe the optical field distribution over the mirror surface for the fundamental and Stokes (anti-Stokes) modes, respectively, $\vec{u}$ is the spatial displacement vector for the mechanical mode, u_z is the component normal to the mirror surface. The integrals $\int d\vec{r}_\perp$ and $\int dV$ correspond to integration over the mirror surface and mirror volume respectively.

This document concentrates on the effect of an apodizing coating over the diffraction losses for an Advanced LIGO type of cavity. Diffraction losses changes will affect the frequency of the mode, the Q factor and the total losses, also affecting the mode shape and the overlapping parameter as a consequence. Therefore it is not straight forward to determine the effect of diffraction losses changes over the parametric gain, but in general we need higher diffraction losses in order to reduce the parametric instability gain.

The diffraction losses for an Advanced LIGO type of cavity have been previously calculated [3,4]. The Fast Fourier Transform (FFT) method developed at The University of Western Australia allows us to inject any optical mode in to the cavity. Inside the cavity the mode is propagated using a FFT and thus the beam is free to change according to the resonance conditions imposed by the cavity parameters. Previous simulations show that a mode which is not supported by the cavity will morph in to a different mode of the same order but with lower diffraction losses. A change in the mode shape is accompanied by the corresponding frequency shift. In such cases it is not possible to say that the nominal mode resonates inside the cavity. As a result the simulations here presented show the high order modes diffraction losses of the injected mode.

Figure 1 shows a comparison of the diffraction losses obtained using a FFT simulation and the results obtained using the eigenvalues calculations for this cavity using an eigenvector method [5]. For these calculations a test mass of 34 cm in diameter was assumed according to LIGO drawing D-040431-B. A homogeneous coating with 50 ppm losses was also assumed.

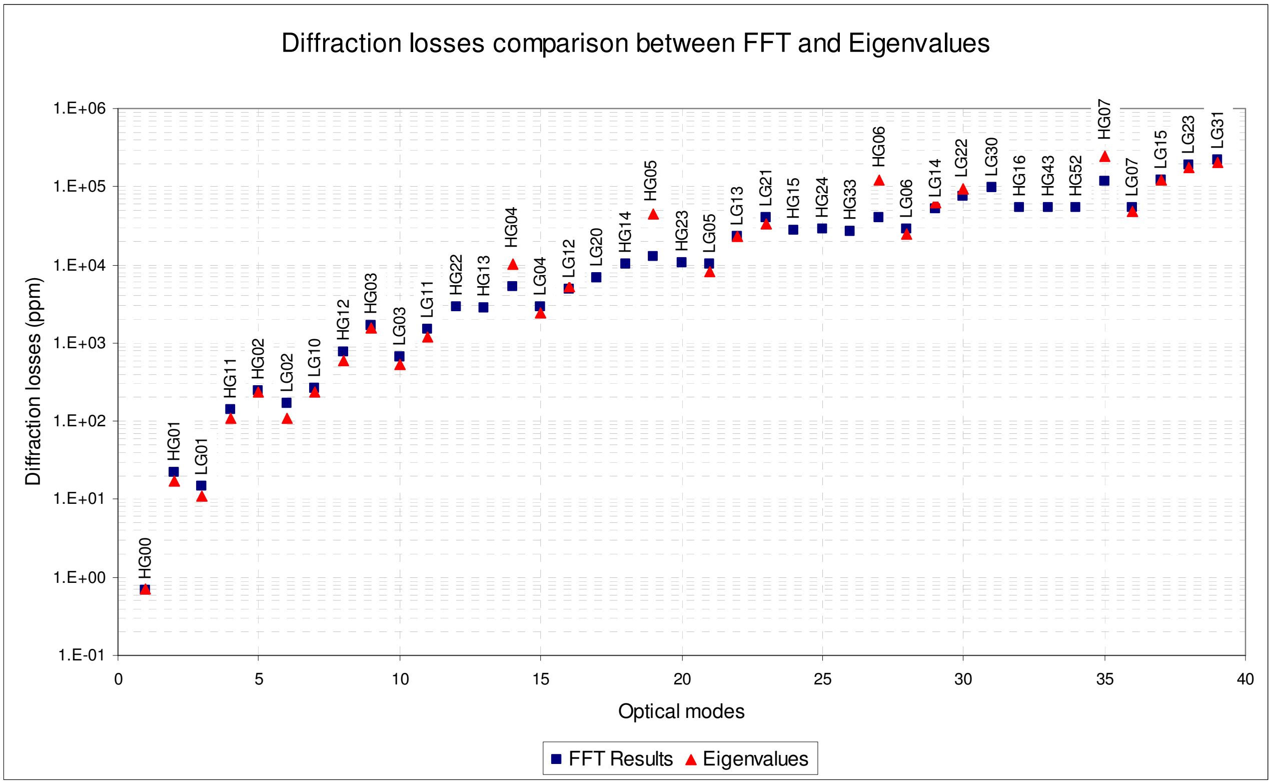

Figure 1: Diffraction losses for an Advanced LIGO type cavity, comparison between FFT and eigenvalues calculations.

2. Apodizing Coating

In order to compare the effect that an apodizing coating will have on the diffraction losses the designs presented in figure 2 were tested using different values for L1, L2 and L3.

Several simulations were done in order to determine the best coating absorption combination by comparing results based on the diffraction losses of the fundamental mode. In all simulations the same substrate was used according to the LIGO document E060001-00, which basically corresponds to a 34 cm diameter test mass including the chamfer and the flat sides for suspension attachment. Let T_i = 5000 ppm and L_i = 15 ppm be respectively the transmission and dielectric losses for the Input Test Mass (ITM) and T_e = 1 ppm and L_e = 15 ppm the corresponding values for the End Test Mass (ETM) [6].In this case it is clear that the major loss contribution comes from the transmission losses of the ITM. The results presented in the next section correspond to the more relevant ones.

The simulations also show that the minimum coating size for the fundamental mode to have diffraction losses of 1 ppm corresponds to a circular coating with a diameter of 33.1 cm. This is assuming that outside the coating all photons will be loss. However by reducing the losses outside the central coating we are able to reduce the size of this central coating proportional to the reduction of the outer ring losses. For these simulations the same coating is assumed for both ITM and ETM.

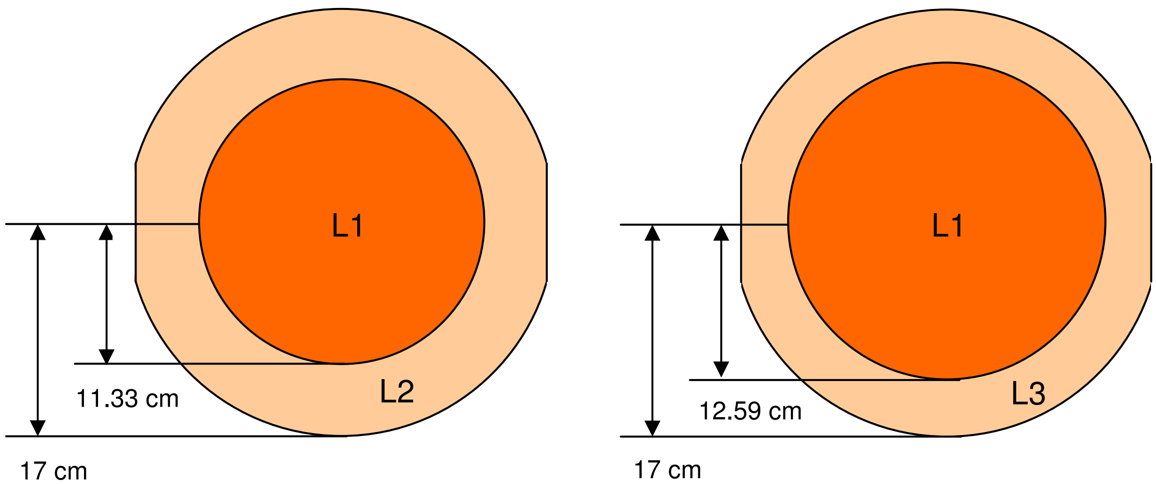

Figure 2: Proposed apodizing coatings design for the ITM and ETM of an Advance LIGO type cavity. The main purpose is to simulate their influence on diffraction losses and ultimately their effect in the parametric gain *R*.

3. Results

Figure 3 shows the diffraction losses for a standard homogeneous coating with L = 50 ppm, also other configurations with homogeneous coatings with losses of 1000 ppm and 10000 ppm have been considered. The figure also include two apodizing coatings, one with losses given by L1 = 50 ppm and L2 = 25000 ppm (coating mean value of 18714 ppm) and one with losses given by L1 = 50 ppm and L3 = 100000 ppm (coating mean value 68961 ppm). However due to the different energy distribution of the higher order modes the absorption losses of the apodizing coatings will be different for each mode. However the simulation results presented in figure 3 show no big difference in terms of diffraction losses when using different coatings.

Normal (L = 50)	Diff.Loss DS (L_1 = 50, L_2 = 25000) mean L = 18714	Diff.Loss (L = 1000)	Diff.Loss (L = 10000)	Diff.Loss DS2 (L_1 = 50 L_3 = 10^5) mean L = 68961
0.6889	0.8388	0.6882	0.6817	0.8745

Table I: Diffraction losses for the fundamental mode HG00 using the different coatings.

Table I show the diffraction losses for different coating losses on the test masses. We can infer from the table that when using a homogeneous coating the higher the losses the lower the diffraction losses. When using an apodizing coating the diffraction losses for the fundamental mode are increased, but always keeping them below 1 ppm.

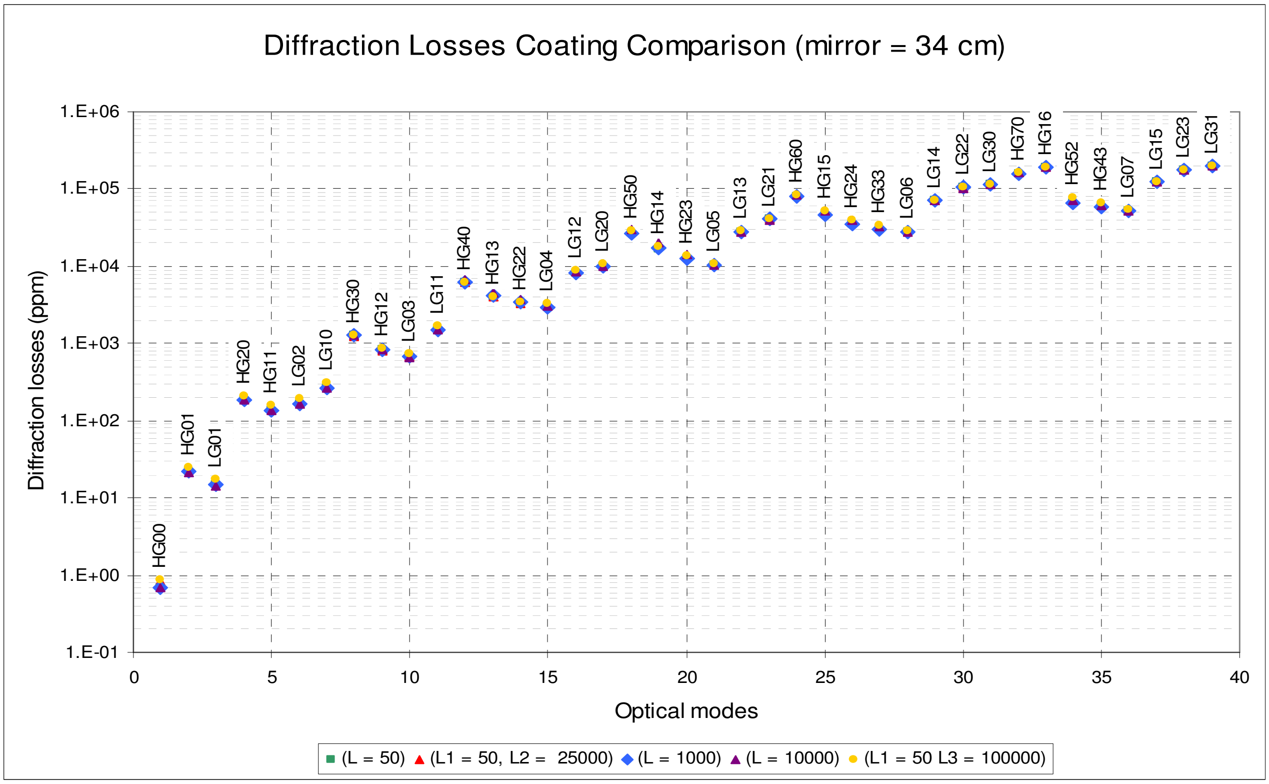

Figure 3: Diffraction losses of higher order modes for an Advanced LIGO type of cavity using different combinations of homogeneous and apodizing coatings.

For higher order modes the diffraction losses comparison between the different coatings will depend of the energy distribution on the beam profile. Therefore modes with higher energy distribution closer to the edge of the test mass will be more affected by the apodizing coating. A similar effect can be seen in the shape of the mode which is also affected by the coating losses, which in consequence will affect the overlapping parameter and therefore the parametric gain *R*. Also due to the higher losses the gain of the cavity is reduced, thus the circulating power inside the cavity also drops.

Figure 4 shows the ratio between the different coatings when compared to the standard coating, which is assumed to have homogeneous losses of 50 ppm. It is interesting to see that the biggest effect of the differential coatings is on the fundamental mode, with an increase of losses of 27%, which is reduced for the higher order modes. Most of the higher order modes have more or less similar diffraction losses except for the more symmetric HG modes.

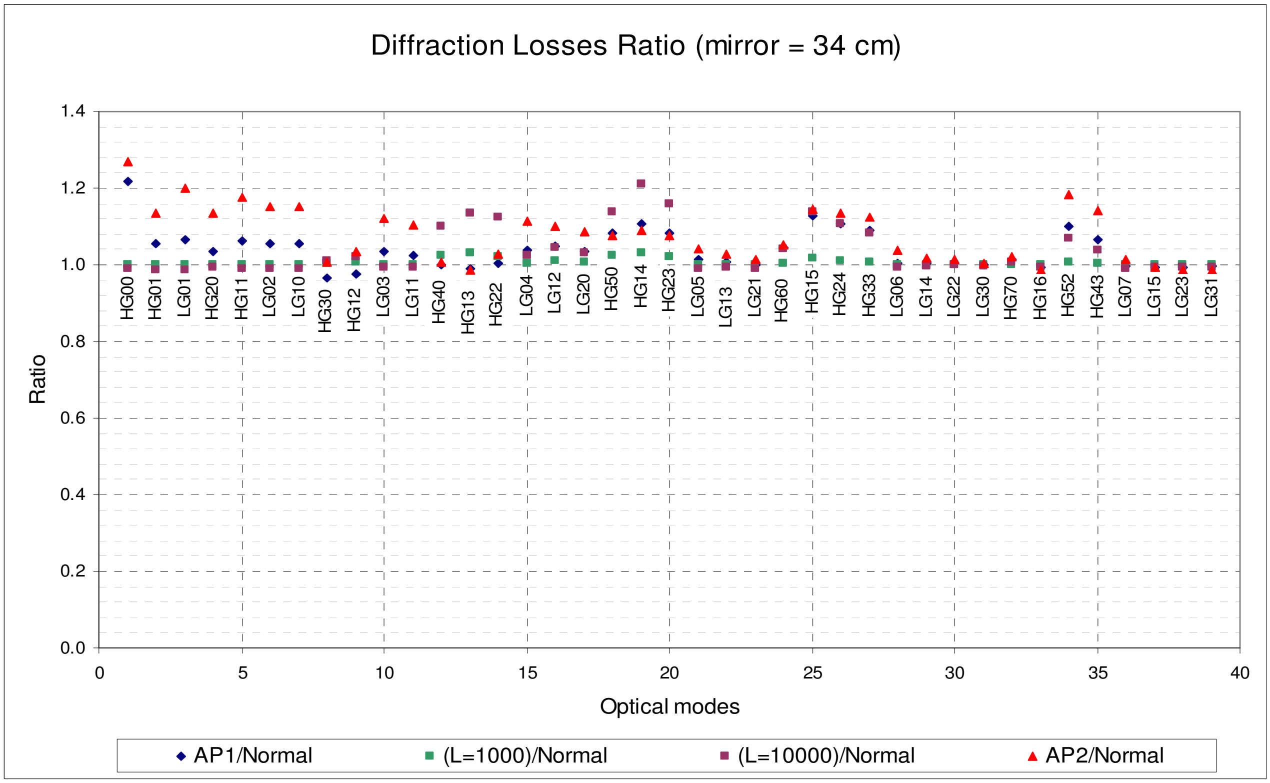

Figure 4: Ratio between the different coatings and a normal coating, assuming that the normal coating has homogeneous losses of 50 ppm.

Figure 5 shows the comparison between the three different coatings, where DiffLoss 50 ppm corresponds to the original calculations with a homogeneous coating with dielectric losses of 50 ppm. DiffLoss AP1 corresponds to the first test with a central area of 50 ppm and an external ring of ~5.7 cm with a loss of 25000 ppm. DiffLoss AP2 corresponds to a central area of 50 ppm as well, but with an external ring of ~4.1 cm with a loss of a 100000 ppm. As we can see from figure 5 there is not much difference between the different coatings, however we can still see that the biggest difference in terms of diffraction losses is for the lower modes, in particular the fundamental mode HG00.

The main difference however is in the coating absorption for each mode, while having a homogeneous coating shows homogeneous absorption it is not the case for the apodizing coatings. In the case of an apodizing coating the absorption will depend on the energy distribution of the mode, therefore it is also important the change in shape of the mode since it will also affect its absorption. This calculation was done by normalizing the circulating power inside the cavity and integrating the field of each mode over the absorption map over the test mass surface.

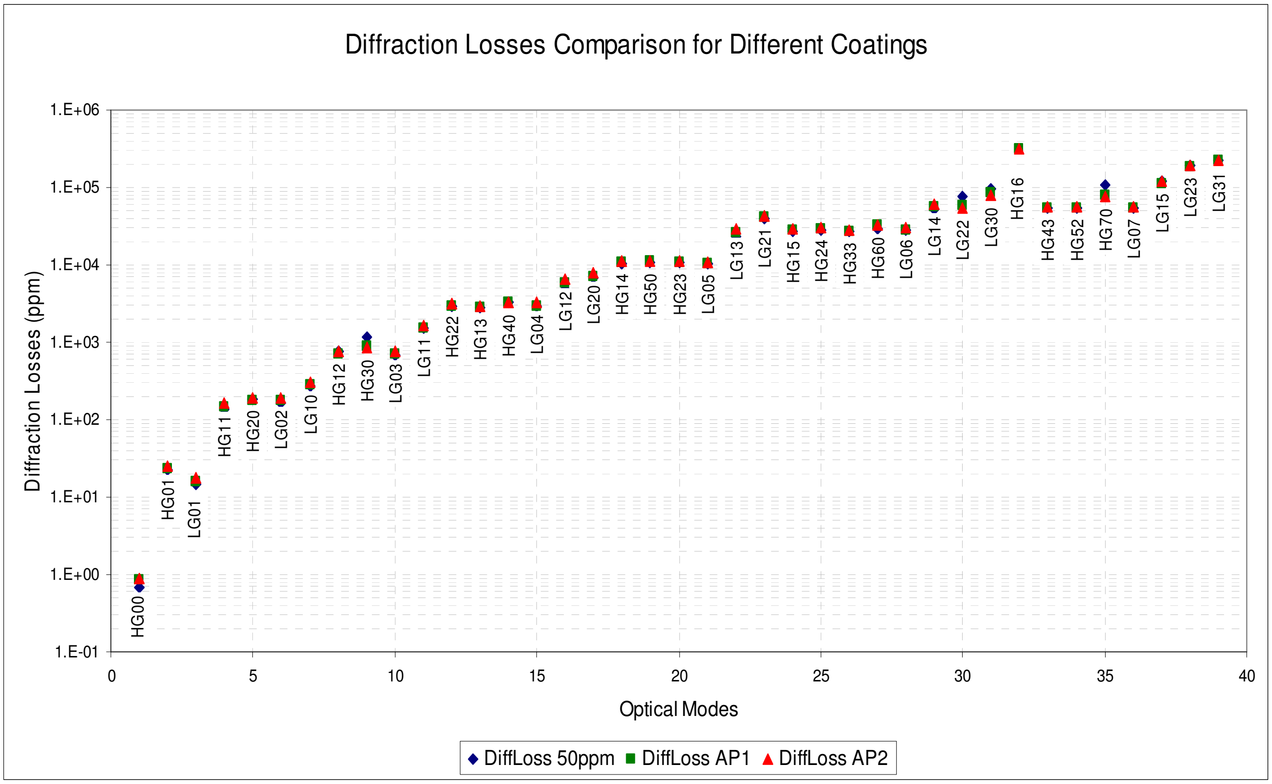

Figure 5: Diffraction losses comparison between the original homogeneous coating and the two apodizing coatings.

Figure 6 shows the results for the two different apodizing coatings. Not surprisingly the coating absorption goes up with the mode order. As in the diffraction losses case this is cause by the mode shape changing inside the cavity. This is caused by the circular symmetry, which favors the resonance of LG modes more than HG modes. We have to remember that in this case no external means of exciting higher order modes have been considered, no mirror tilt, no mechanical resonance and no suspension residual noise for example.

We can also notice in figure 6 that for the lower modes there is not much difference between the two coatings absorptions. But for the higher order modes, starting from order 3, the two curves start to show some difference. This is due to the energy distribution of the higher order modes and the fact that the outer ring of the second coating has higher losses even though it is slightly narrower.

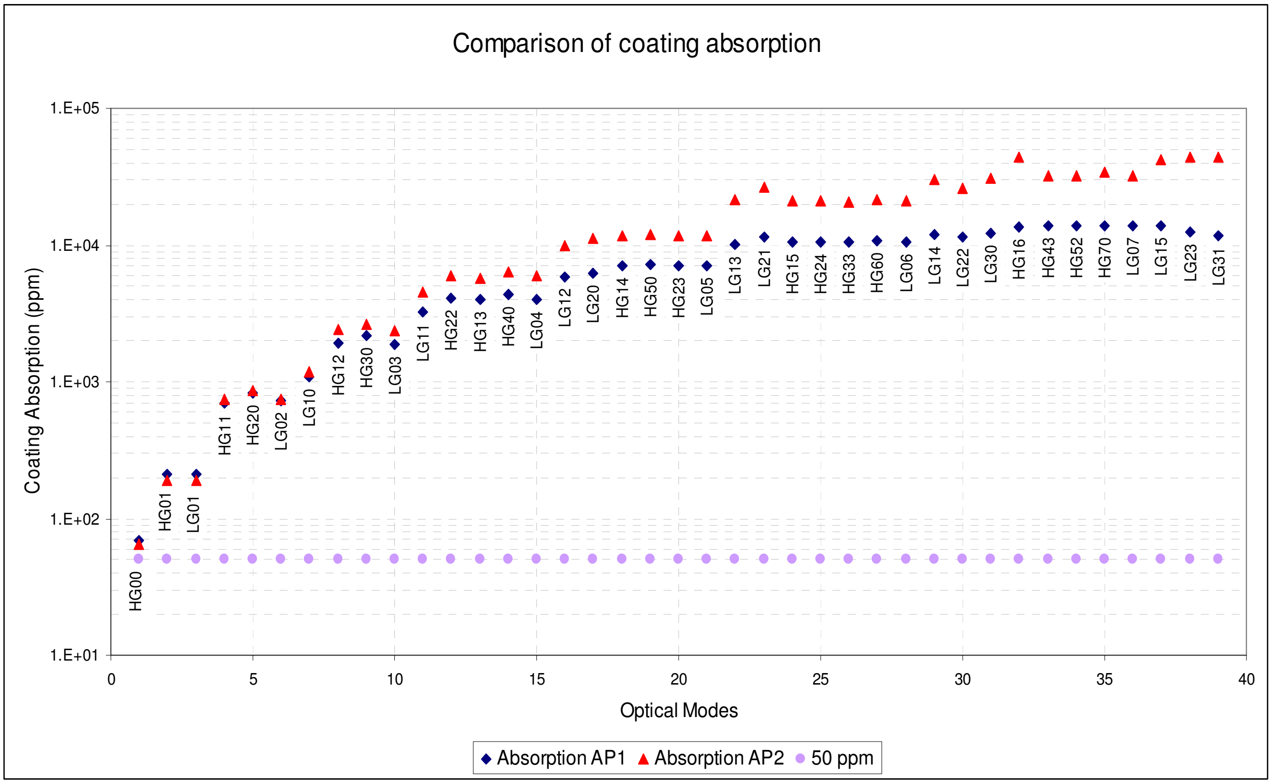

Figure 6: Shows the different dielectric absorption for higher order modes. We notice that there is not much difference between the two coatings when analyzing the lower modes, but there is a clear difference for the higher order ones.

4. Conclusions

Based on these results the use of an apodizing coating will increase the diffraction losses for several modes. Somewhat unanticipated was to see that the effect is bigger in the lower order modes. The diffraction loss change comes with the corresponding frequency shift and a Q factor change for the optical mode. Consequently there is also a change in the mode shape due to the different coating losses. In general these changes are too small and any effect on the overlapping parameter will be negligible. As a result the effect of an apodizing coating on the parametric instabilities gain R for an advanced gravitational wave interferometer it is also expected to be negligible.

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References

[1] V. B. Braginsky et al, *Phys. Lett. A*, **287,** 331, (2001).

[2] V. B. Braginsky et al, *Phys. Lett. A*, **305,** 111, (2002).

[3] B. Bhawal, LIGO technical note, LIGO-T050234-00-E, (2005).

[4] P. Barriga et al, *"Numerical calculations of diffraction losses and their influence in parametric instabilities in advanced Interferometric gravitational wave detectors"*, **submitted**, 2006.

[5] C. Yuanying at al, *Optics Communications*, **234,** 1, (2004).

[6] R. Lawrence, "Active wavefront correction in laser interferometric gravitational wave detectors", *PhD Thesis Department of Physics*, Massachusetts Institute if Technology (2003).