



*LIGO Laboratory / LIGO Scientific Collaboration*

LIGO-T070120-00-Z

*Enhanced LIGO*

06/04/07

---

**Bullseye Detectors for Mode Matching Measurements  
for e-LIGO**

---

Muzammil A. Arain and Guido Mueller

Distribution of this document:  
LIGO Science Collaboration

This is an internal working note  
of the LIGO Project.

**California Institute of Technology**  
**LIGO Project – MS 18-34**  
**1200 E. California Blvd.**  
**Pasadena, CA 91125**  
Phone (626) 395-2129  
Fax (626) 304-9834  
E-mail: info@ligo.caltech.edu

**Massachusetts Institute of Technology**  
**LIGO Project – NW17-161**  
**175 Albany St**  
**Cambridge, MA 02139**  
Phone (617) 253-4824  
Fax (617) 253-7014  
E-mail: info@ligo.mit.edu

**LIGO Hanford Observatory**  
**P.O. Box 1970**  
**Mail Stop S9-02**  
**Richland WA 99352**  
Phone 509-372-8106  
Fax 509-372-8137

**LIGO Livingston Observatory**  
**P.O. Box 940**  
**Livingston, LA 70754**  
Phone 225-686-3100  
Fax 225-686-7189

<http://www.ligo.caltech.edu/>

**Table of Contents**

<b>1</b>	<b><i>Introduction</i></b>	<b>3</b>
<b>1.1</b>	<b><i>Purpose</i></b>	<b>3</b>
<b>1.2</b>	<b><i>Scope</i></b>	<b>3</b>
<b>1.3</b>	<b><i>Definitions</i></b>	<b>3</b>
<b>1.4</b>	<b><i>Acronyms</i></b>	<b>3</b>
1.4.1	LIGO Documents	3
1.4.2	Non-LIGO Documents	3
<b>2</b>	<b><i>General Description</i></b>	<b>3</b>
<b>3</b>	<b><i>Measurement Description</i></b>	<b>6</b>
<b>4</b>	<b><i>Measurement Preparation</i></b>	<b>8</b>
<b>5</b>	<b><i>Guoy Phase Telescope Design</i></b>	<b>8</b>
<b>6</b>	<b><i>To Probe Further</i></b>	<b>9</b>
<b>7</b>	<b><i>Conclusion</i></b>	<b>9</b>

## 1 Introduction

### 1.1 Purpose

This document proposes the use of Bullseye sensors for determining the mode mismatch in Initial LIGO. Particularly this applies to LLO where the current mode matching is not perfect. This operation may be performed after S5 run before decommissioning. The main task is to determine the repositioning of MMT<sub>2</sub>. This is the easiest way to change the mode matching. This will be especially helpful in the operation of Enhanced LIGO.

### 1.2 Scope

This document is prepared for the purpose of describing Bullseye sensors to determine mode mismatch at LLO. Typical readers of this include people involved in designing core optics, input/output system, and thermal compensation system.

### 1.3 Definitions

### 1.4 Acronyms

IMC: Input Mode Cleaner

MMT: Mode Matching Telescope

NRSB: Non-Resonant Side Band

TCS: Thermal Compensation System

#### 1.4.1 LIGO Documents

1. R. Adhikari et. al., "Input Optics Final Design," LIGO-T980009-01-D.

#### 1.4.2 Non-LIGO Documents

2. G. Mueller, Q. -z. Shu, R. Adhikari, D. B. Tanner, D. Reitze, D. Sigg, N. Mavalvala, and J. Camp, "Determination and optimization of mode matching into optical cavities by heterodyne detection," *Opt. Lett.* **25**, 266-268 (2000).
3. G. Mueller, "Modematching measurements February 2007".

## 2 General Description

The modematching between the Input Mode Cleaner (IMC) and the main interferometer should be improved during the break between S5 and eLIGO. The reflected field at LLO shows a Bullseye structure and it was estimated that about 5% of the light could be recovered by improving the modematching. One obstacle to measure the modematching is the recycling cavity and the TCS system. The recycling cavity in LIGO is a degenerate cavity and TCS keeps it barely stable. Virtually all Hermite Gaussian eigenmodes can resonate in this cavity. Subsequently, the carrier TEM<sub>00</sub>-mode is defined by the arm cavity eigenmode and we only need to improve the mode matching into the arm cavity. TCS operates on the philosophy of maintaining good overlap between the carrier and the sideband in the recycling cavity or may be optimizing the signal to

noise ratio. To our understanding TCS does not take into account mode matching of the carrier light into the arm cavity directly. However, in e-LIGO, the situation may change because of the higher TCS power requirement due to more absorbed power in the arms. Therefore, it would be prudent to perform some measurements to analyze the mode structure of both arm cavities and the recycling cavity.

Bullseye sensors are an effective tool in determining the mode miss-match due to beam waist size and beam waist location miss-match. A complete description of Bullseye sensor is present in Ref. 1 and 2. If the input mode is slightly miss-matched from the arm cavity mode, higher order cylindrical modes are induced in the cavity. Here we describe the mode mismatch as a mismatch between beam waist locations and beam sizes. Most significant are the two lowest order Laguerre-Gauss Modes. The zeroth order mode is the normal TEM<sub>00</sub> mode while the first higher order mode has a bullseye pattern. The amplitude of the bullseye mode provides information about the mode miss-match due to beam waist size and beam waist location deviation from the arm cavity mode. The amplitude  $\epsilon$  of the bullseye mode can be expressed as:

$$\epsilon = -\left(\frac{w'_0}{w_0} - 1\right) - i\frac{b}{2Z_R} = \epsilon' + i\epsilon'',$$

where  $w'_0$  and  $w_0$  are waist sizes of the input beam and the cavity mode waist size, respectively;  $b$  is the displacement of the input beam waist position relative to the cavity mode waist position along the cavity axis;  $Z_R$  is the Rayleigh length of the cavity mode. If we measure the coupling coefficient  $\epsilon$ , the mismatches of beam parameters can be readily obtained.

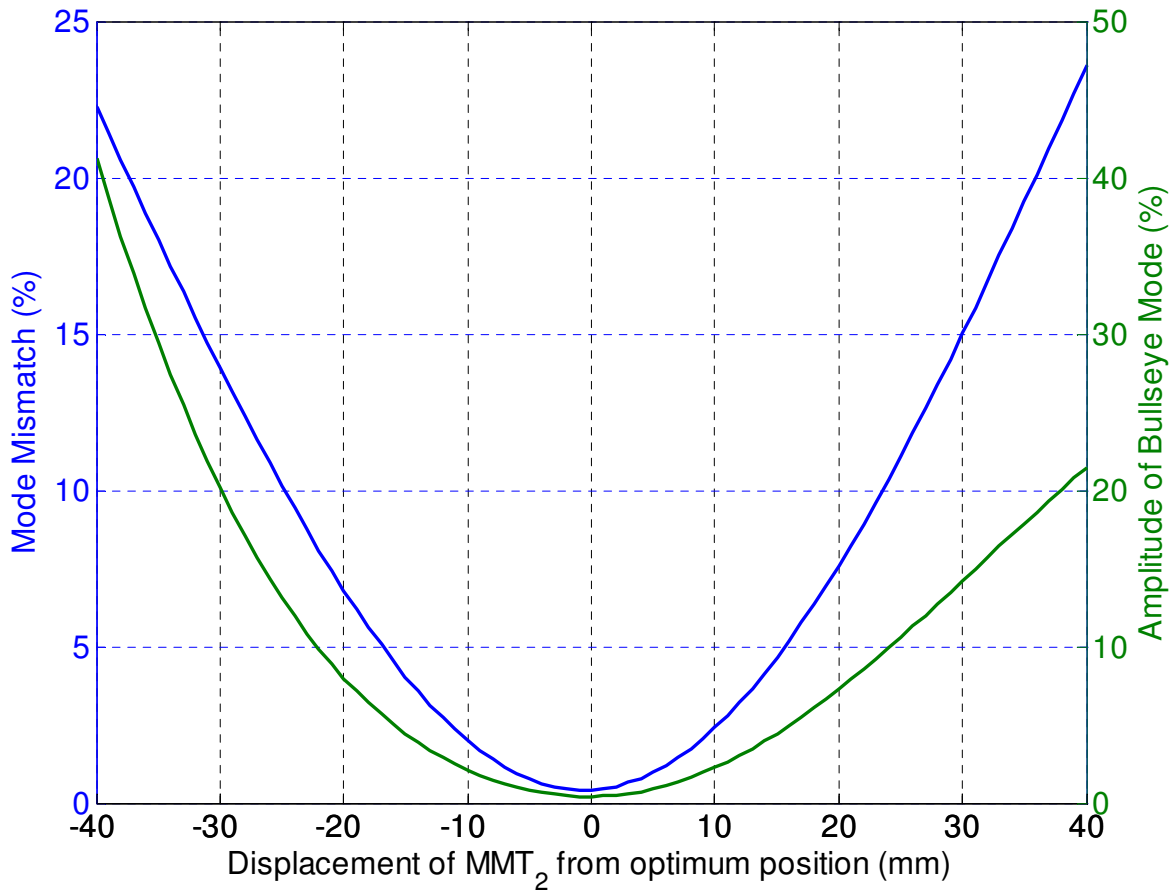
Since the mode miss-match due to beam waist size miss-match and beam waist location mismatch occurs in quadrature, the two types of errors can be determined independently by using two telescopes with different Guoy phase telescopes. In principle, the two sensors can be designed in such a way that  $S_1 \propto \epsilon'$  and  $S_2 \propto \epsilon''$  where  $S_{1,2}$  are the signals from the two Bullseye photodetectors. Thus we can estimate the two mismatches independently. However, in practice, due to positioning errors, the signal  $S_{1(2)}$  may have some dependence on  $\epsilon'(\epsilon'')$ . More importantly, the mode matching improvement is being planned by repositioning of MMT<sub>2</sub> that induces both beam waist size and location mismatches. Hence, it is not specifically required to differentiate between the two types of mismatches. Therefore, we can define a new signal as  $S(S_1, S_2)$  such that the measurement sensitivity is optimum at a particular point. At a particular point, any of the two signals, a linear combination of two, or quadratic combination of two can provide the best sensitivity. However, if we do measurements in such a way that we have an independent measurement of the two signals, we can form any combination for maximum sensitivity.

As mentioned earlier, the purpose of these measurements is to improve the mode matching by repositioning MMT<sub>2</sub>. The effect of moving MMT<sub>2</sub> by a small amount (using HEPI) on mode matching can be determined by measuring the slope of the Bullseye signal at a specific port. For small variations in MMT<sub>2</sub> position, the signal can be described as:

$$S(x_1 + \Delta x) = \frac{\partial S}{\partial x}(x_1 + \Delta x)$$

where  $x_1$  is the offset of  $MMT_2$  from a reference position while  $\Delta x$  is the small increment. Applying a linear fit on the measured data will allow us to estimate the slope  $\frac{\partial S}{\partial x}$ . This information can then be extrapolated to determine the right amount of  $MMT_2$  repositioning.

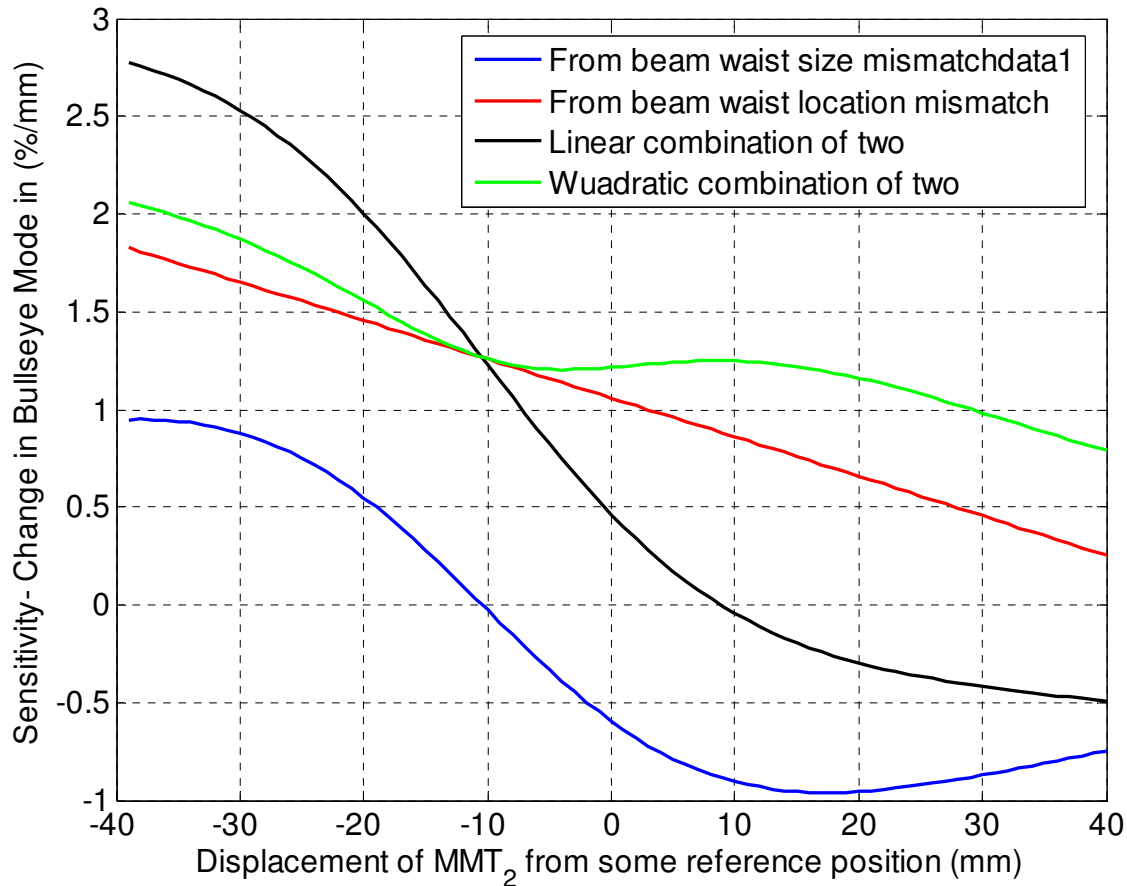
One important issue with the Bullseye measurement is the sensitivity of the measurements. The sensitivity in this scenario can be defined as the change in Bullseye signal with an incremental change in the position of  $MMT_2$ . The amplitude of the Bullseye signal is measured by calculating the beam waist and location inside the cavity for every position of  $MMT_2$ . Note that the Bullseye signal is sensitive to the amplitude of the Bullseye mode while the mode mismatch depends upon the intensity of the  $TEM_{00}$  mode. Therefore, there could be significant Bullseye amplitude even for low amount of mode mismatches as shown in Fig. 1. Typical values for Initial LIGO have been assumed in this case.



**Fig. 1: Mode mismatch into the arm cavity as a function of the  $MMT_2$  mirror position plotted on left y-axis. The amplitude of the Bullseye signal is plotted on the right y-axis.**

Based upon data in Fig. 1, the sensitivity can be calculated as shown in Fig. 2. This shows that a variety of sensitivity can be achieved for various combinations of the two signals. The sensitivity of the Bullseye measurement at UF years ago was about 1.5% in terms of amplitude of the 10 mode.

In the UF case, this was around 0, at LLO it will be around an offset. However, given the improvements in the demod boards and the superior stability of everything, measuring a slope with a 4mm change should be possible. Here 4 mm change in the position of  $MMT_2$  can be realized using HEPI.



**Fig. 2: Sensitivity of the Bullseye signal for the case of Fig. 1.**

### 3 Measurement Description

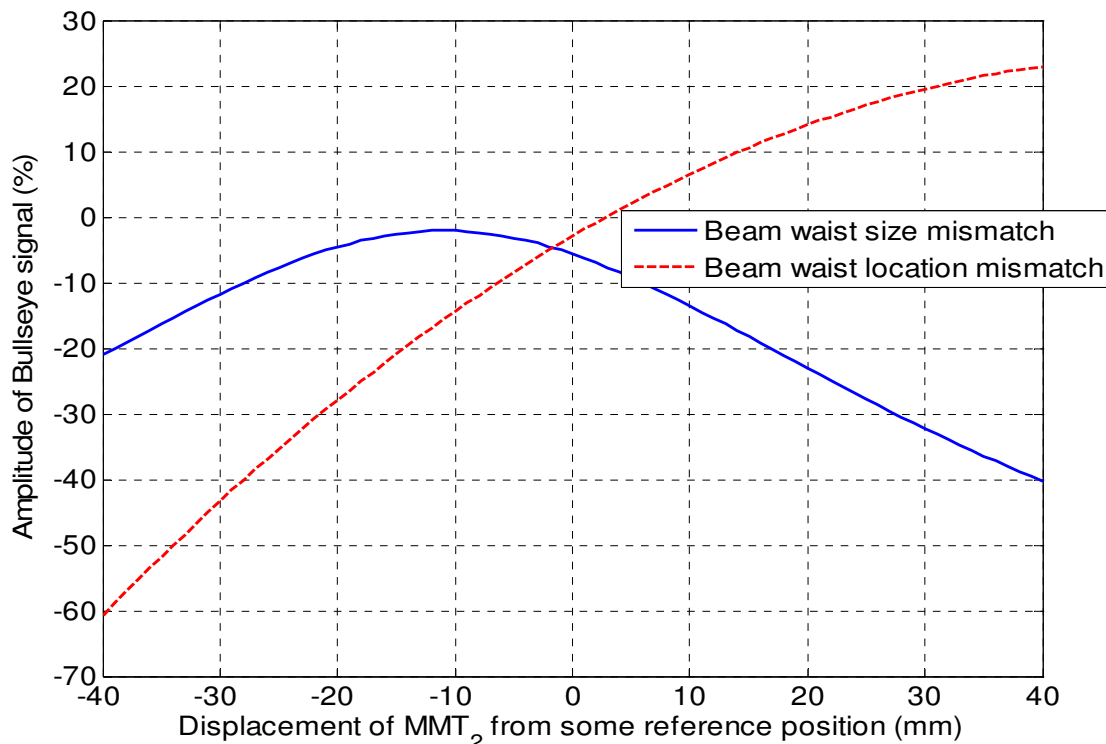
One way to measure the mode matching into each arm is to misalign the recycling cavity and the second arm cavity, and by placing a Bullseye detector at the reflected port, and measure the Bullseye signal for several distances between  $MMT_2$  and  $MMT_3$ .

For these measurements we first should increase the distance between  $MMT_2$  and  $MMT_3$  by as much as possible using HEPI. Then we should misalign the PR mirror and one of the ITMs and lock the second arm cavity on the carrier using the non-resonant sideband. Non-resonant side band (NRSB) is elected because TCS does not alter the NRSB. The Bullseye detectors placed inside the reflected port measures the beat between the 00-mode of the reflected carrier and the 10-mode of

the sideband used to lock the cavity. This measurement should then be repeated for the other cavity. Then the distance between  $MMT_2$  and  $MMT_3$  should be reduced in steps of 1 mm and both measurements should be repeated for each distance. If both offsets are identical within a certain margin, the average value is the value by how much we have to change the distance between the MMTs.

This measurement assumes that the recycling cavity has no impact on the mode matching (except that the ROC of the PR changes the ROC of the wavefront). This is something we need to confirm. For that purpose we should repeat the measurement for the fully locked interferometer (incl. TCS) using again the non-resonant sideband and the Bullseye in the refl port. If the offset obtained using this measurement is same as the previous one, this means that the arm cavity mode is not significantly changed by the TCS operation at full load.

An important point to consider is the fact that by changing the position of  $MMT_2$ , we have only one degree of freedom. This is clearly shown in Fig. 3 where the blue curve (Bullseye signal due to beam waist size mismatch) and green curve (Bullseye signal due to beam waist location mismatch) do not pass through the zero at the same time. This effect may not enable us to completely recover the mode matching but we can get within 1% of the mod mismatch.



**Fig. 3: Bullseye amplitude due to beam waist size and location mismatch as a function of  $MMT_2$  mirror displacement.**

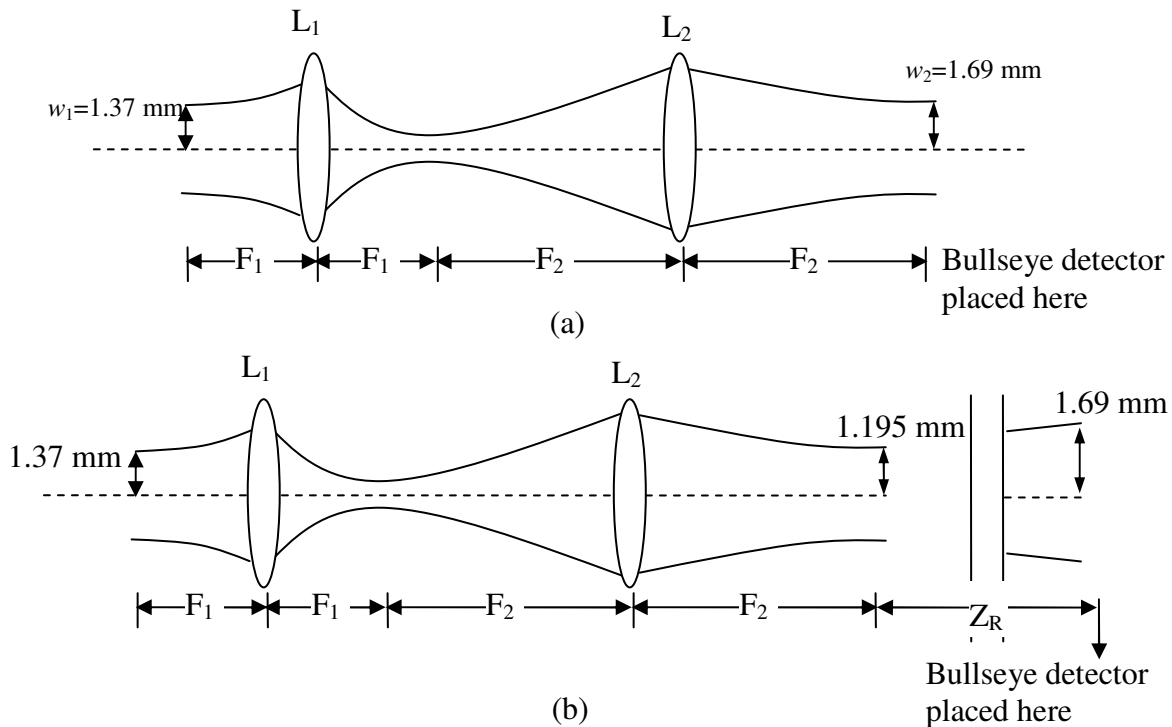
### 4 Measurement Preparation

In preparation for the Bullseye measurements, it would be very useful to measure the beam size of the reflected field again. Valera and Andrii did measure this in 2003 but a new measurement to confirm their findings would be very useful. However, we can use their data to design the Guoy phase telescopes on the optics table for the Bullseye detectors.

A photodetector board at 63 MHz and a demodulator board will be required for these measurements with two Guoy phase telescopes. The telescope design will be described in the next section. There are spare demodulator boards available at LLO/LHO while one spare photodetector at NRSB is available at LLO (**required**).

### 5 Guoy Phase Telescope Design

Two Guoy phase telescopes are required Bullseye sensor measurements with a relative Guoy phase difference of  $45^0$  that will make the signals from the two photodetectors in quadrature to each other. The schematic diagram for the two Guoy phase telescopes is shown in Fig. 3.



**Fig. 3: Schematic diagram of Guoy phase telescopes.**

Here we start with a 1.37 mm beam waist at the reflected port. Fig. 3 (a) shows one Guoy phase telescope that has a Guoy phase of about 180 degrees. The proposed two lens afocal system images the beam to the right beam size at the photodetector. For maximum common mode rejection, the



beam size should be 1.69 mm so that the central and outer portion receives same amount of TEM<sub>00</sub> light. The second telescope should have an additional 45 degree Guoy phase. This is ensured by going an extra Rayleigh range away from the beam waist. Since the beam waist at the photodetector has to be 1.69 mm so the minimum beam waist should be a factor of  $\sqrt{2}$  smaller than 1.69, i.e., 1.195 mm. This also gives the value of Rayleigh range as 4.2412. Now all that is left is to select the focal lengths of the lenses. Forming an afocal system also fixes the distances between the two lenses as the sum of the focal lengths of the two lenses. This system is very tolerant to small positioning errors. For this afocal system, the focal lengths should be selected such that  $\frac{F_2}{F_1} = \frac{w_2}{w_1}$ .

The optical layout details are provided in Table 1.

**Table 5: Parameters for Fig. 1.27**

<i>Definition</i>	<i>Unit</i>	<i>Value Telescope 1</i>	<i>Value Telescope 2</i>
$w_1 =$ Waist Size at the Refl Port	mm	1.37	1.37
$d_1 =$ Distance b/w waist and L <sub>1</sub>	mm	149	183.3
$F_1 =$ Focal length of L <sub>1</sub>	mm	149	183.3
$F_2 =$ Focal length of L <sub>2</sub>	mm	183.3	160.4
$d_2 =$ Distance b/w L <sub>1</sub> and L <sub>2</sub> = (F <sub>1</sub> +F <sub>2</sub> )	mm	332.3	343.7
$d_3 =$ Distance b/w L <sub>2</sub> and Bullseye PD	mm	183.3	4.4073
$w_1 =$ Waist Size at the Bullseye PD	mm	1.685	1.695

Here the lenses used are commercially available CVI lenses.

## 6 To Probe Further

If the first two results, i.e., Bullseye signal slope and offset for locked x-arm and locked y-arm respectively lead to different offsets, then the two cavities are either different or the beam-splitter has a significant curvature which impacts the modematching. If this happens, then we should be prepared to use more diagnostics tools and analyses to determine the optimal target common mode. If the first two results agree within reasonable limits but the third is off, then the power recycling cavity is probably not what we expect it to be and the modematching between the recycling cavity and the arms has to improve first. In that case a fourth Bullseye measurement using the resonant sideband could help to explain the situation and can provide some insight to the TCS requirements.

## 7 Conclusion

Bullseye measurements are described to improve the mode matching into the arm cavity. These measurements will provide valuable information about the arm cavity modes and recycling cavity mode changes. At the end of analysis, optimal position for the MMT<sub>2</sub> mirror will be determined to improve the mode matching. The availability of Bullseye detectors and demodulator boards should be ensured and a new beam waist measurement size at the reflected port will be helpful.