



Date:	4 January 2008
LIGO Number	T080003-01
Subject:	Thermal Noise Increase due to a Gold Coated Barrel
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Distribution:	aligo_coc, aligo_aos, aligo_sys

Phil Willems has proposed the addition of a gold coating on the barrel of the test masses and the compensation plates in order to reduce the emissivity and reduce the radial temperature gradients due to radiation from the barrel<sup>1</sup>. Since gold is a lossy material it may also help to passively damp higher order acoustic modes which might otherwise be excited into oscillation by higher order optical cavity modes. However, there is concern that the lossy gold coating might lead to an unacceptably large increase in Brownian thermal noise in the optical readout. The purpose of this technical note is to report on a finite element analysis used to calculate the increase in thermal noise (using Levin's approach<sup>2</sup>) due to the addition of a gold coating.

An axisymmetric finite element model of the Advanced LIGO test mass was developed, as depicted in Figure 1. For simplicity (and to enable an axisymmetric model) a perfect right circular cylinder was assumed; The following geometric features were not included: wedge angle, bevels, flats and bonded "ears" (used to weld the fused silica suspension ribbons). In addition, the gold layer will need a protective SiO<sub>2</sub> overcoat layer and may need an interstitial layer to bond it to the substrate (e.g. Ni). Neither of these layers are included. The Si O<sub>2</sub> protective layer should be low loss and similar to the substrate. The interstitial layer may be exceedingly thin and inconsequential.

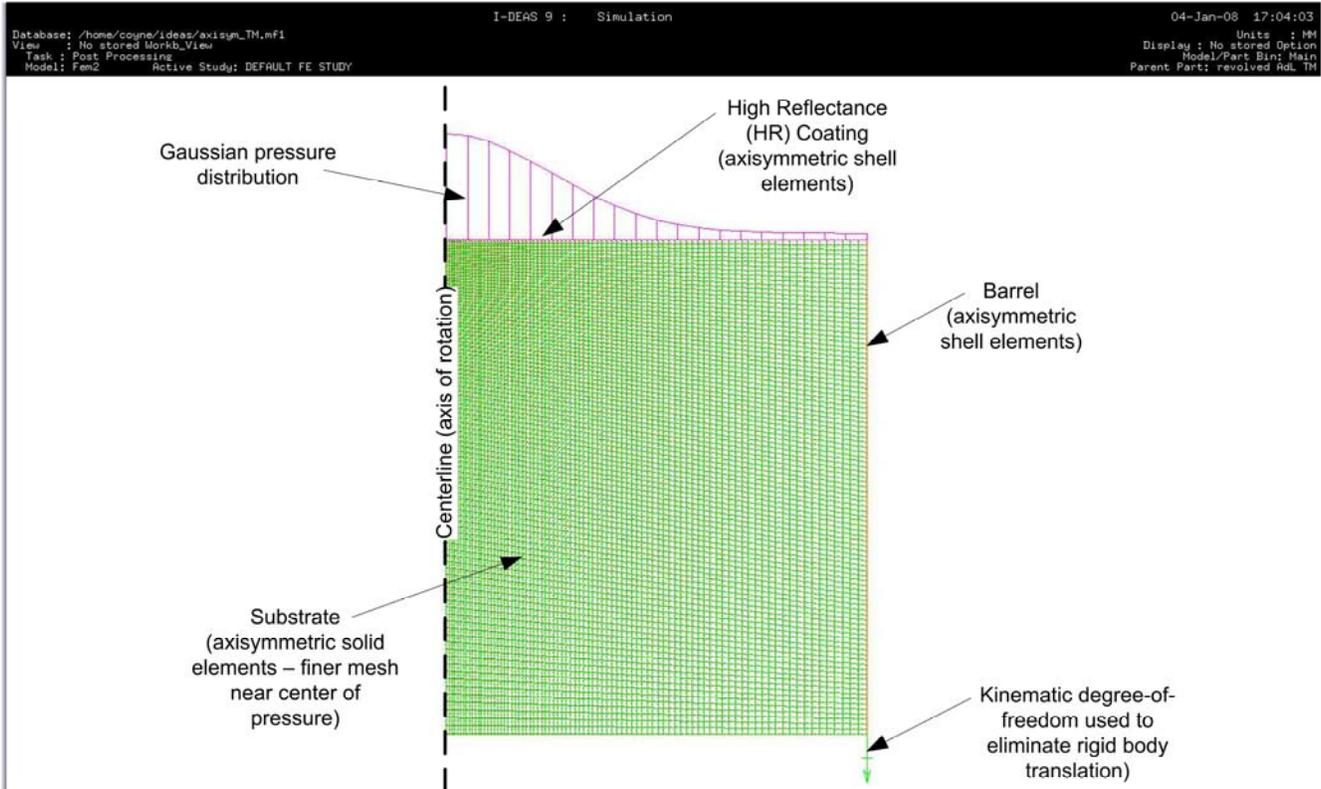
The elements approximating the fused silica material of the test mass substrate are comprised of parabolic quadrilateral axisymmetric solid elements. The High Reflectance (HR) and barrel coatings are comprised of axisymmetric shell elements of appropriate thickness. The mesh was refined to insure convergence. The parameters and property values used in the analysis are listed in Table 1.

The variation in strain energy is depicted in the contours of Figures 2 and 3. The variations of strain energy across the HR and barrel coatings are depicted in Figures 4 and 5. The strain integrals (summations of total element strain) for the substrate, HR coating and barrel coating domains are listed in Table 2. The displacement noise (and strain), at 100 Hz, due to the Brownian motion are also compared in Table 2.

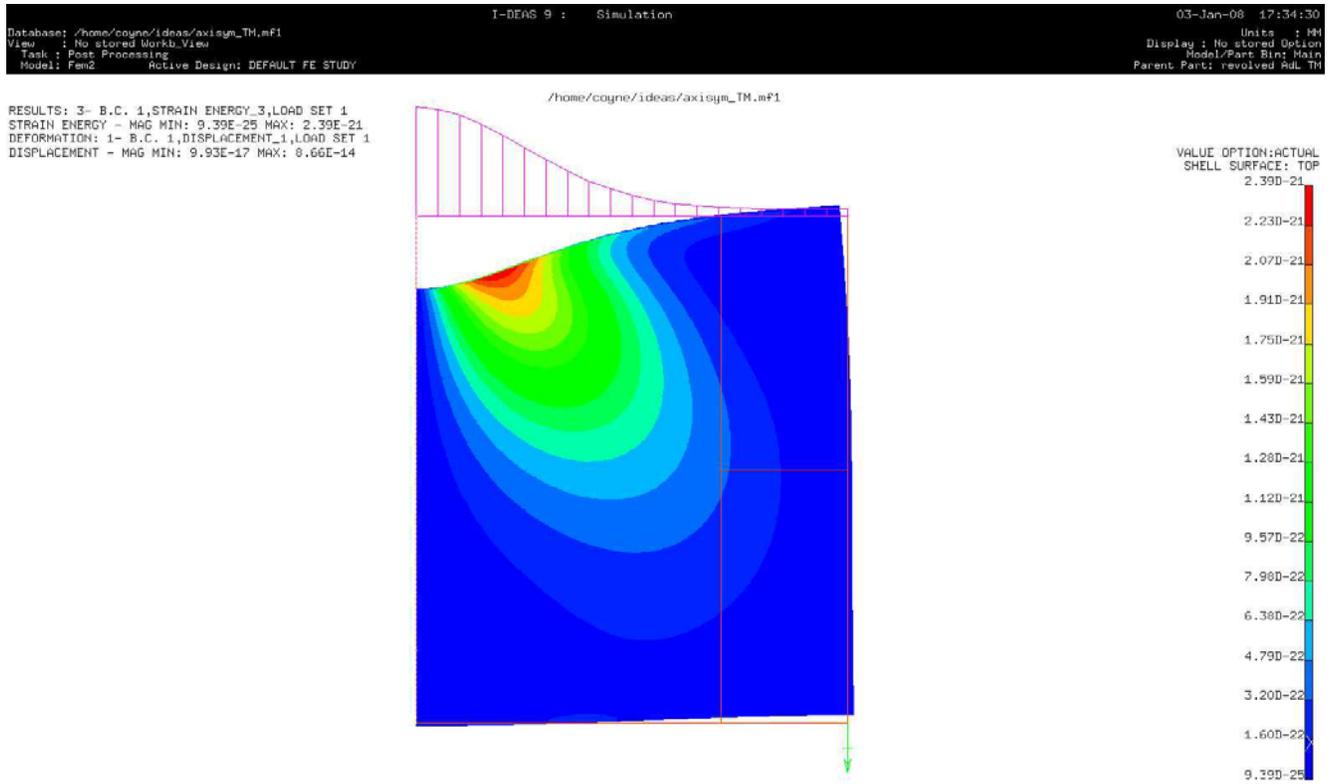
<sup>1</sup> P. Willems, "Heating of the ITM by the Compensation Plate in Advanced LIGO", LIGO-T070123-02, 3Dec2007.

<sup>2</sup> Y. Levin, "Internal thermal noise in the LIGO test masses: A direct approach", Phy Rev D, v57, n2, 15Jan98, p659-663.

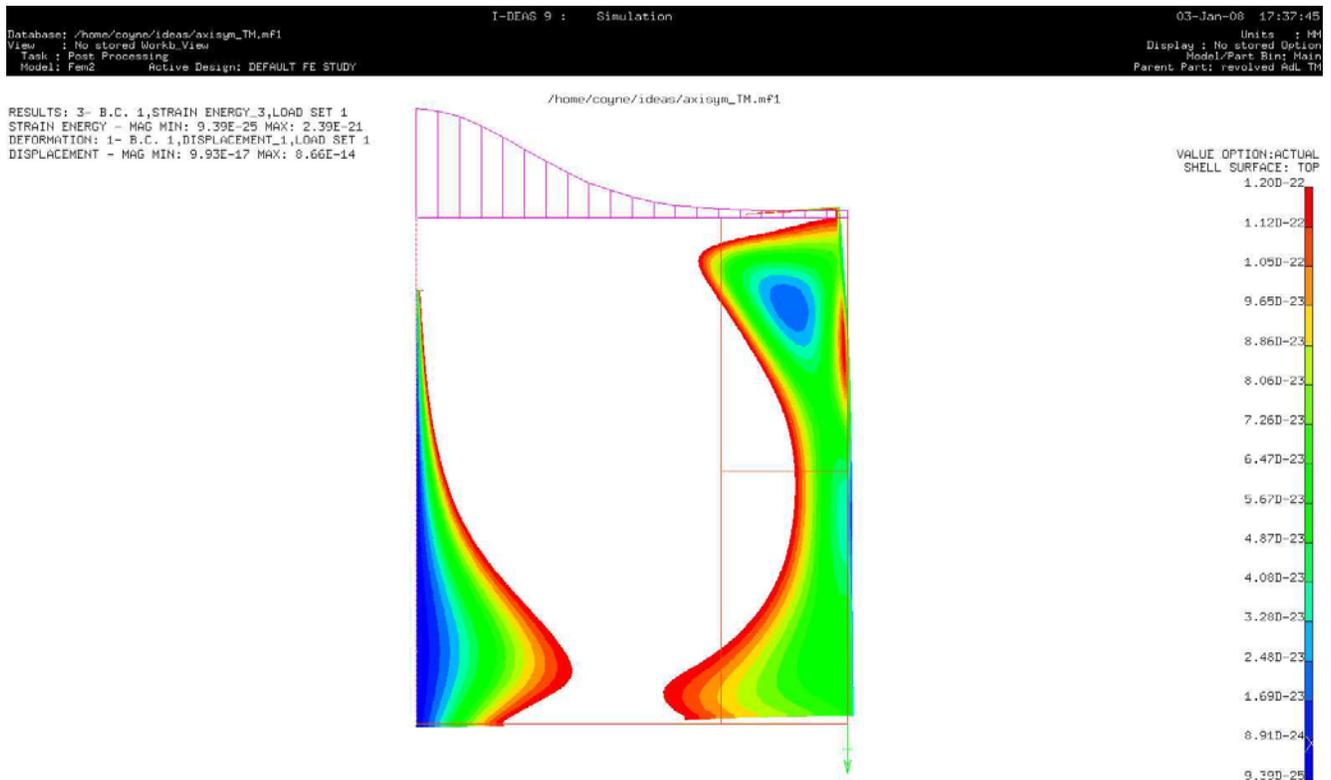
The total strain noise (at 100 Hz) compares well with the Bench 6.2 calculation ( $3e-24$   $1/\sqrt{\text{Hz}}$ ). The gold barrel coating only increases the strain noise ASD by 1%. This is due to the fact that the Brownian thermal noise is dominated by the HR coating ( $3e-24$  HR,  $0.3e-24$  substrate,  $0.5e-24$  gold). Even if new HR coating formulations result in significantly lower loss factor, the HR coating is still likely to dominate over the gold barrel coating contribution.



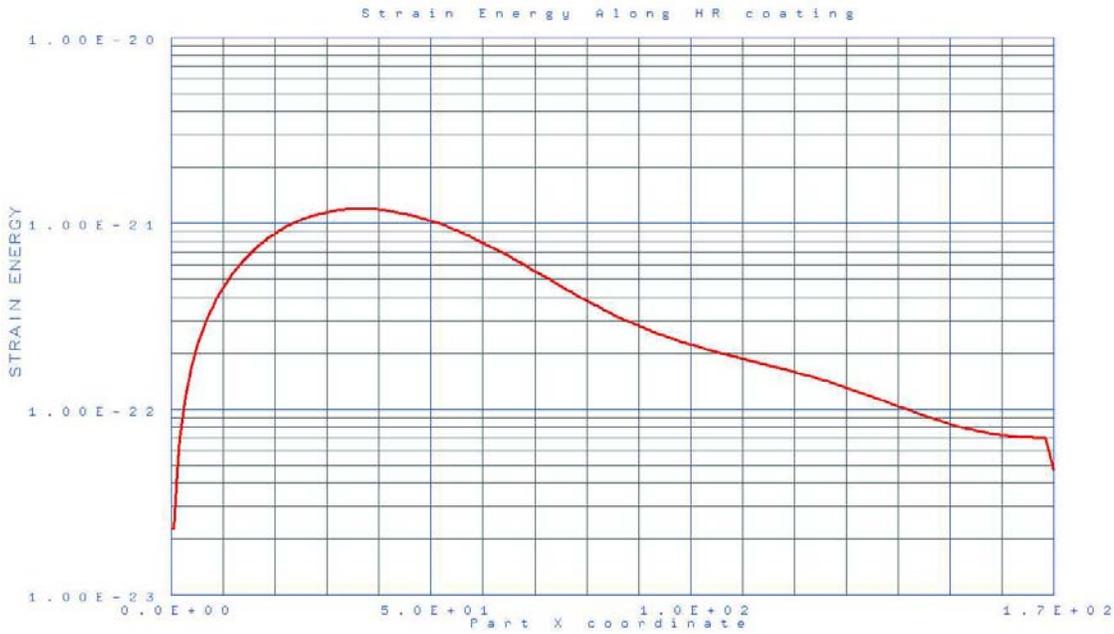
**Figure 1: finite element model**



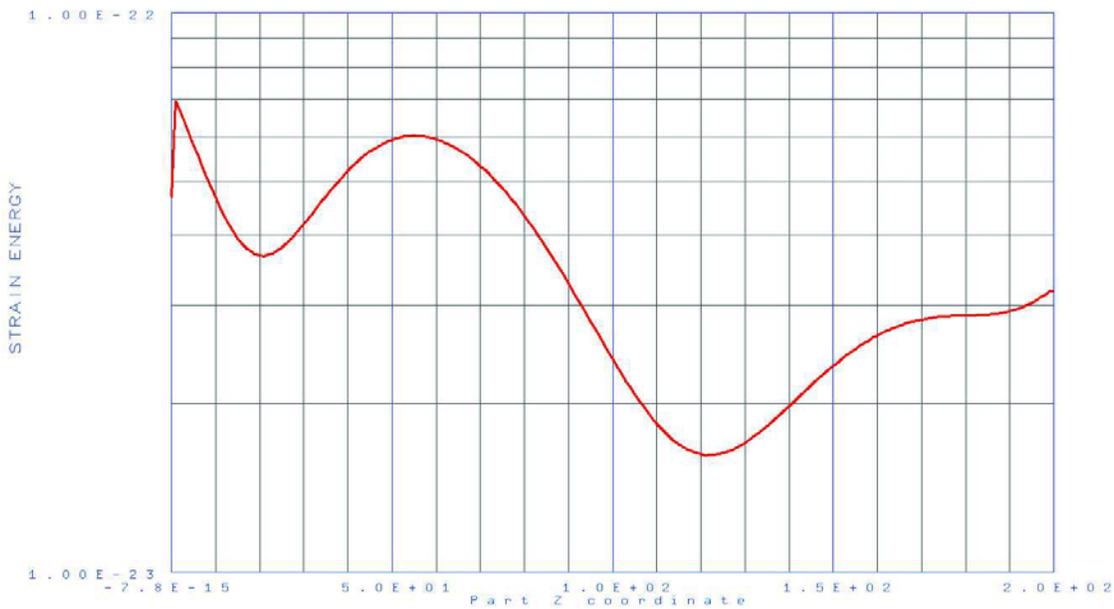
**Figure 2: Strain Energy Contours for a 1 microN Gaussian force (linear contour scale, 0 to 100%)**



**Figure 3: Strain Energy Contours for a 1 microN Gaussian force (linear contour scale, 0 to 5%)**



**Figure 4: HR Coating Strain Energy (mm-mN/rad) at each nodal point for a 1 microN Gaussian force**



**Figure 5: Barrel Coating Strain Energy (mm-mN/rad) at each nodal point for a 1 microN Gaussian force**

**Table 1: Parameters**

Symbol	Parameter	Value	Units	Source
a	TM radius	0.170	m	M050397-02
H	TM thickness	0.200	m	Ibid
$r_0$	Gaussian beam radius	0.060	m	Bench 6.2
$E_{FS}$	TM elastic modulus	71.8	GPa	Fused Silica; Bench 6.2 uses 72.7GPa
$\nu_{FS}$	TM Poisson's ratio	0.16	-	Bench 6.2 uses 0.167
$\phi_{FS}$	TM bulk loss factor	$2.6 \times 10^{-10}$ at 100 Hz	-	Fused Silica: from Bench 6.2: $\phi = c_2 f^n$ , $c_2 = 7.6e-12$ $n = 0.77$ , $f = \text{frequency (Hz)}$
$t_{HR}$	High Reflectance dielectric coating thickness	10	$\mu\text{m}$	~40 layers @ $\lambda/4$
$E_{HR}$	HR coating elastic modulus	106.5	GPa	Ta <sub>2</sub> O <sub>5</sub> /SiO <sub>2</sub> : T970176-00 Bench 6.2 uses ~94 GPa
$\nu_{HR}$	HR coating Poisson's ratio	0.21	-	Ta <sub>2</sub> O <sub>5</sub> /SiO <sub>2</sub> : T970176-00 Bench 6.2 uses 0.2038
$\phi_{HR}$	HR coating loss factor	$3.38 \times 10^{-4}$	-	Eqn[23], G. Harry et. al., CQG, 19(2002), with effective coating elastic modulus, $Y'=94$ GPa, substrate modulus, $Y=72.7$ GPa, $\phi_{\perp} = 1.36e-4$ and $\phi_{\parallel} = 1.79e-4$
$t_{Au}$	gold barrel coating thickness	100	nm	Phil Willems estimate 11-dec-2007
$E_{Au}$	gold barrel coating elastic modulus	80	GPa	H. Anderson (ed.), AIP Physics Vade Mecum, 1981
$\nu_{Au}$	gold barrel coating Poisson's ratio	0.42	-	Ibid
$\phi_{Au}$	gold barrel coating loss factor	$9 \times 10^{-3}$	-	recent unpublished measurement by Andri Gretarsson

**Table 2: Strain Energy and Displacement Noise Results**

<b>Symbol</b>	<b>Parameter</b>	<b>Value</b>	<b>Units</b>
$U_{FS}$	Total substrate strain energy normalized by the force amplitude squared	2.48e-11	m <sup>2</sup> /J
$U_{HR}$	Total substrate strain energy normalized by the force amplitude squared	2.27e-15	m <sup>2</sup> /J
$U_{Au}$	Total substrate strain energy normalized by the force amplitude squared	1.80e-18	m <sup>2</sup> /J
$S_{x,FS}(f=100\text{Hz})$	Displacement noise PSD contribution from the substrate	3.33e-43	m <sup>2</sup> /Hz
$S_{x,HR}(f=100\text{Hz})$	Displacement noise PSD contribution from the HR coating	3.91e-41	m <sup>2</sup> /Hz
$S_{x,Au}(f=100\text{Hz})$	Displacement noise PSD contribution from the barrel coating	8.27e-43	m <sup>2</sup> /Hz
$\sqrt{S_x}(f=100\text{Hz})$	Displacement noise ASD: Total	6.34e-21	m/ $\sqrt{\text{Hz}}$
	Displacement noise ASD: without barrel	6.28e-21	m/ $\sqrt{\text{Hz}}$
	Displacement noise ASD: substrate only	5.77e-22	m/ $\sqrt{\text{Hz}}$
	Displacement noise ASD: without HR	1.08e-21	m/ $\sqrt{\text{Hz}}$
$h(f=100\text{Hz})$	Strain noise ASD: Total	3.17e-24	1/ $\sqrt{\text{Hz}}$
	Strain noise ASD: without barrel	3.14e-24	1/ $\sqrt{\text{Hz}}$
	Strain noise ASD: substrate only	2.89e-25	1/ $\sqrt{\text{Hz}}$
	Strain noise ASD: without HR	5.39e-25	1/ $\sqrt{\text{Hz}}$
	Strain noise ASD: HR coating only (compare to 2.93e-24 from Bench 6.2)	3.12e-24	1/ $\sqrt{\text{Hz}}$