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LIGO-T080257-00-D *LIGO* 03/25/2008

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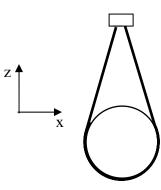
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Notes about fluctuating force acting on the LIGO test mass due to fluctuations of magnetic field

V.P.Mitrofanov (MSU) 25 March, 2008

The LIGO test masses are suspended by a single wire looping around the cylindrical surface, attached at the top to the suspension structure by clamps. The wire loop is electrical short circuit if there are no isolating gaskets in the clamps (?). AC component of the Earth's magnetic field and magnetic field from industrial sources which runs through the loop induces electrical current in the loop. In the DC Earth's magnetic field magnetic force acts on the loop with electrical current and on the test mass correspondingly.



The area of the wire loop S $\approx 0.5 (\pi R_o^2) + R_o H \approx 0.08 \text{ m}^2$, where $R_o = 0.125 \text{ m} - \text{radius of the test mass}$,

H = 0.45 m – the vertical distance between the center of mass and the top clamps (see Fig. 1).

Horizontal AC component of magnetic field \tilde{B}_y induces electromotive force and AC current in the loop:

Fig.1
$$I(\omega) = \frac{-\omega \widetilde{B}_{y}(\omega)S}{R}$$
 (1)

Electrical resistance R of the wire loop: $R = \rho l/A = 0.43$ Ohm,

where the specific resistance $\rho = 1 \times 10^{-7}$ Ohm m (??? for piano steel),

the length of the loop $l \approx \pi R_o + 2H = 1.3$ m,

the cross section area of the wire $A = \pi r^2 = 0.3 \text{ mm}^2 = 3 \text{x} 10^{-7} \text{ m}^2$.

wire diameter 2r = 0.62 mm (??) [G.Gonzalez, Class. Quantum Grav. 17 (2000) 4409].

The inductive impedance of the loop $L \omega$ is much smaller than R for LIGO frequencies.

Magnetic spectra are taken from the Livingston detector log (TO=31/01/2006/ 15:00:37) LO-PEM-EX MAGX

Magnitude
$$\approx 0.7 \times 10^{-6} \text{ (G/rHz)}$$
 (Is $G - \text{Gauss ?}$) = $0.7 \times 10^{-10} \text{ T/rHz}$ at 100 Hz,
Magnitude $\approx 1 \times 10^{-3} \text{ (G/rHz)}$ = $1 \times 10^{-7} \text{ T/rHz}$ at 60 Hz.

Suppose that AC magnetic field inside vacuum chamber is 10 times smaller due to shielding of the chamber walls. Substituting these values in (1) we obtain:

I(
$$\omega$$
) $\approx 8 \times 10^{-10} \text{ A/rHz}$ at 100 Hz,

I(
$$\omega$$
) $\approx 7x \cdot 10^{-7} \text{ A/rHz}$ at 60 Hz.

The loop current interacting with vertical and horizontal DC components of the Earth's magnetic field B_z and B_x creates torques N_x and N_z rotating the loop around a vertical axis Z and horizontal axis X:

$$N_z(\omega) = I(\omega) S B_x$$
, (2)

$$N_x(\omega) = I(\omega) S B_z$$
, (3)

The wire loop is not rigid. Also we must take into account torques created by reaction forces in the clamp. Due to these factors the test mass may move along its axis as well. Accurate calculations of torques and forces are required. We may estimate the amplitude spectral density of the test mass yaw angle $\phi(\omega)$ assuming a rigid wire but taking the torque $N_z(\omega)$ with a factor 0.5:

$$\varphi(\omega) = \frac{I(\omega)SB_x}{2J\omega^2} \quad , \tag{4}$$

where the moment of inertia of the test mass $J = mr^2/4 = 4.2 \times 10^{-2} \text{ kg m}^2$.

Suppose $B_x = 3x10^{-5}$ T (One of the interferometer arms at Livingston observatory is oriented close to the EW direction (?)). We obtain from (4):

 $\phi(\omega) \approx 6x10^{-20} \text{ rad/rHz}$ at 100 Hz,

 $\varphi(\omega) \approx 2 \text{ x} 10^{-16} \text{ rad/rHz}$ at 60 Hz,

One can compare these estimates with upper limit for the angular fluctuations allowed in the suspended test masses (P.Fritschel, N.Mavalvala, D.Shoemaker, D.Sigg, M.Zucker, G.Gonzalez, Appl. Opt. 37 (1998) 6734):

$$\phi \; (f \! > \! 100 \; Hz) \! < \! 7x10^{\text{-}17} \; rad/rHz.$$

To estimate action of the vertical component of the Earth's magnetic field $B_z \approx 5 x 10^{-5} \, T$ consider hypothetic suspension on vertical wires (see Fig.2). Magnetic force acts only on the bottom part of suspension wire and create a torque and a force:

$$F_y(\omega) = 2R_0I(\omega)B_z = 0.25 \text{ m } 7x10^{-7}\text{A/rHz } 5x10^{-5}\text{T} = 9x10^{-14}\text{N/rHz}.$$

Corresponding amplitude spectral density of TM displacement is:



Fig. 2

$$X(\omega) = F_v (\omega)/m \omega^2 \approx 5x10^{-17} m/rHz$$
 at 60 Hz.

Notice that the fluctuating forces acting on the ITM and ETM may be correlated if the fluctuating magnetic fields are correlated. This may compensate their effect in the output signal of the interferometer and needs additional investigation.

The calculated test mass noise associated with fluctuation of magnetic field may give some contribution into 60 Hz line of the present interferometer noise budget. There is an uncertainty in its value associated with the simplified calculations, poor knowledge of magnetic field magnitudes and of some other parameters. Also the magnetic field excess variations (in comparison with the stationary level) are not known. They may be a source of excess nonstationary noise.

But it seems simple to eliminate this noise installing isolating gaskets in the clamps of the test mass wire suspension.