LIGO-T890048-00-R

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#81

List of 5/01/89

Processes that may cause noise from scattered light in the instrumentation chambers and associated vacuum pipes.

Areas of possible concern

1) Transition from the instrumentation building vacuum pipe to the main tubes. Primary mechanism is back scatter to the mirror in the closest test mass chamber.

2) Scattering paths from the beam splitter chamber, where the intensity of light is high, to the antisymmetric output; where the intensity is low, when the interferrometer is locked on a dark fringe. Two cases must be considered: with and without an output mode filter.

3) Scattering in the mode filter tubes analogous to that in the main tubes involving scattering by a mirror, reflection by the wall, and recombination into the principal mode by one of the cavity mirrors.

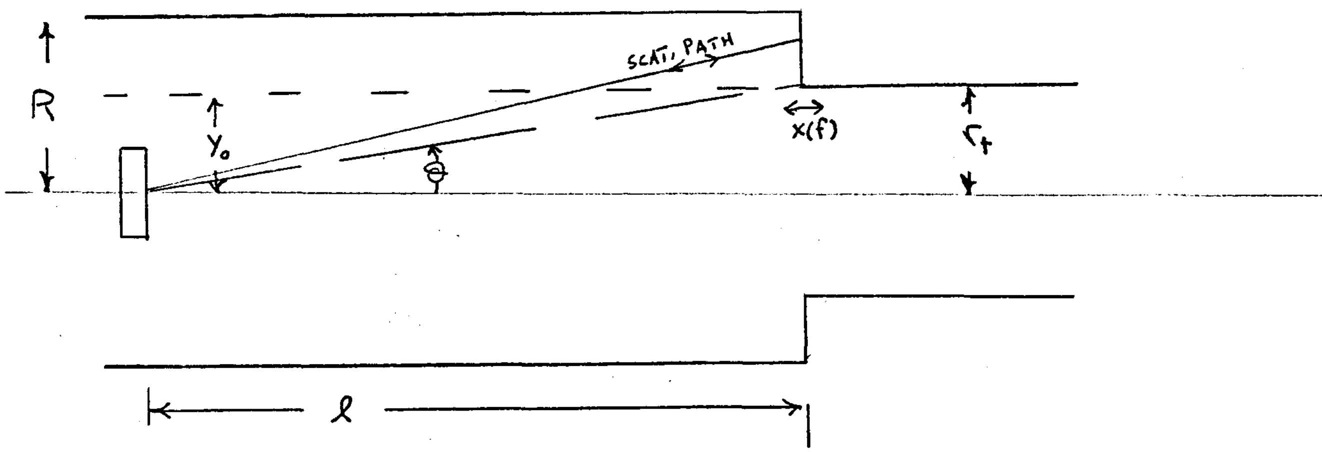
4) Scattering from the many optical components in the input and output train. Again the scattering path being component to wall to component.

5) Additional recombination of ambient scattered light by the optical components into the principal mode.

6) Scattering in the tubes coupling the splitter chambers and the test mass chambers; particularly, the intersection points of input optics beam tubes with the test mass/splitter chamber tubes.

Back scattering from the transition section of pipe in the instrumentation building to the nearest mirror in a test mass chamber.

Geometry



Calculation following KST p40

Assume: Scattering has the following mechanism

1) Scattering by mirror M
2) Diffuse reflection by transition with longitudinal modulation $x(f)$
3) Recombination into main mode at mirror M

Scattered brightness back to mirror

$$\frac{dP_{\text{scat}}}{P dA d\varphi df} = \frac{\alpha}{l^2\theta^2} \underbrace{\left(\frac{dP(\theta)}{P d\Omega}\right)_{\text{surface}}}_{G_{\text{surface}}} \frac{R(R-r_t)}{l^2} \left[4\pi \frac{x(f)}{\lambda}\right]^2$$

$$h(f) = \left[\int_0^{2\pi} P_{\text{recombination}} \frac{dP_{sc}\big/ dA d\varphi df}{P/\lambda L} d\varphi\right]^{1/2} \frac{\lambda f}{c}$$

Integral over the transition section, φ, has a maximum near $\varphi = 0$ and a band of $\Delta\varphi = 2y_0/R$

Two cases: without output mode filter

$$P_{\text{rec}} = \frac{(1-\eta)^2}{2\theta} \left(\frac{\lambda}{L}\right)^{1/2}$$

$$h(f) = \frac{\alpha^{1/2}}{\theta^{3/2}} \frac{(\lambda L)^{1/2}\big(R(R-r_t)\big)^{1/2}}{l^2} G^{1/2} (1-\eta) \left(\frac{\lambda}{L}\right)^{1/4} \left(\frac{\lambda f}{c}\right) 4\pi \frac{x(f)}{\lambda} \left(\frac{2y_0}{R}\right)^{1/2}$$

Use fact that $\theta \sim \frac{y_0}{l}$ nearest point on beam to transition

$$h(f) \;=\; \alpha^{1/2}\,(1-\eta)\,\left(\frac{2(R-r_t)}{l}\right)^{1/2}\,\frac{(\lambda L)^{1/2}}{y_0}\,\left(\frac{\lambda}{L}\right)^{1/4}\,G^{1/2}\,\left(\frac{\lambda f}{c}\right)\,4\pi\,\frac{x(f)}{\lambda}$$

Use

$$\begin{array}{llll} \alpha = 1\times 10^{-6} & L = 4\times 10^{5} cm & R = 3ft = 91cm \\ \eta = .9 & y_0 = 20cm & r_t = 2ft = 61cm \\ \lambda = 5\times 10^{-5} cm & & \\ x(f) = 10^{-5}/f^2 \; cm/Hz^{1/2} \text{ (seismic noise)} & & \end{array}$$

$$h(f) \cong \left(\frac{G}{l}\right)^{1/2} \frac{2.4\times 10^{-21}}{f}$$

So to stay at less than $1/10\ h_{QL}(f) \;=\; \frac{4\times 10^{-24}}{f}$

$$\boxed{\left(\frac{G}{l}\right) < 3\times 10^{-6}}$$

One must be more careful here than for the standard baffle in the 4km tube. Typical values talked about at the end of this section.

With output mode filter

$$P_{\text{rec}} = \frac{2\alpha}{\theta^2}\,\frac{\lambda}{L}$$

Again using $\theta = \frac{y_0}{l}$

$$h(f) \;=\; 2\alpha\left(\frac{\lambda}{L}\right)^{1/2}\,\frac{\left[\lambda L R(R-r_t)\right]^{1/2}}{y_0^2}\,\left(\frac{y_0}{R}\right)^{1/2}\,G^{1/2}\,\left(\frac{4\pi x(f)}{\lambda}\right)\,\left(\frac{\lambda f}{c}\right) \Rightarrow \alpha\,\frac{1}{\theta^2}$$

Result becomes independent of l

$$h(f) \;=\; G^{1/2}\,\frac{2.6\times 10^{-26}}{f}$$

To stay less than $1/10\ h(f) \sim \frac{4\times 10^{-24}}{f}$

No special requirement on the transition section since G is allowed to be larger than 1.

Summary of requirements on transition section

1) Without mode cleaner

$\frac{G}{l} < 3 \times 10^{-6}$

Using black surface like anodized aluminium (Martin-Marietta black)

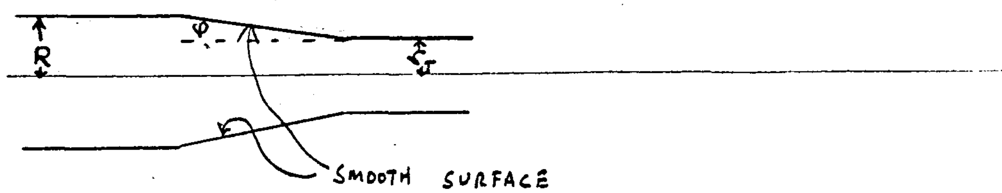
$\theta =$ angle of incidence

$|R| =$ reflectivity $\sim 3 \times 10^{-3}$ $\qquad 0 < \theta < 80$ degrees

$G \sim |R|$ for this material

$l \simeq 1 \times 10^3 cm \sim 10$ meters

Using smooth surface, tapered



Small φ brings scattered rays from mirror closer to small grazing angle,

$g = \left(\frac{2\pi\sigma}{\lambda}\varphi\right)^2 \qquad g < 1$ for smooth surface

$G(\pi - \varphi) \simeq \left(\frac{2\pi\sigma}{\lambda}\right)^2 e^{-\left(\frac{2\pi\sigma}{\lambda}\right)^2} \qquad T/\lambda \gg 1$ diffuse component RW p7

Typical numbers: If $\varphi \sim 0.1$ (transition takes place over a length of 10 feet)

$\sigma/\lambda \sim 1$ still satisfies smooth surface approximation that $g < 1$ and if $T/\lambda \gg 1$

$G(\pi - \theta) \sim 10^{-16}$ plenty good enough

Conclusion: If smooth surface condition is satisfied by tapering the transition, the $G/l < 3 \times 10^{-6}$ is easily satisified.

A possible suggestion is to use an aluminized "kevlar" cone. Only reason for caution would be dirt collecting on the surface. Dust will backscatter.

With mode cleaner no special condition required

Recommend: Tapered section with smooth surface $\frac{2\pi\sigma}{\lambda}\varphi < 1$
and distance greater than 3 meters from last mirror to give tolerance for dust.

3) Scattering in mode filter and coupling tubes

a) Tubes involved with input optics

1) Frequency modulation of the principal mode by scattered rays that have hit moving wall surfaces and are subsequently recombined with the main mode by scattering by an optical component are less important than other scattered beams in the LIGO. The frequency modulation is common to the beams that are ultimately divided by the beam splitter and the effect is the same as frequency noise in the laser.

The effect can be crudely estimated by evaluating the frequency noise from a scattered beam and comparing it to the frequency noise permitted at maximum sensitivity.

If the main beam has a power P and the scattered power into the main mode is P_{scat}, the frequency noise in the recombined beam is

$$\nu(f) = \left(\frac{P_{\text{scat}}}{P_{\text{main}}}\right) \nu_0 \frac{2\pi f}{c} x(f) \qquad \boxed{\begin{array}{c}\textbf{incoherent}\\ \textbf{addition}\end{array}} \qquad (1)$$

$x(f)$ = amplitude spectral density of wall motion $\sim 10^{-5}\big/f^2 \;\; cm\big/Hz^{1/2}$

$f > 10Hz$

ν_0 = light frequency $\sim 6 \times 10^{14} Hz$

The strain sensitivity limited by frequency noise is

$$\underset{\text{freq noise}}{h(f)} = \frac{\nu(f)}{\nu_0}\left[\frac{\Delta F}{F} + \frac{\Delta l}{l}\right]$$

Where $h(f)$ is the limiting sensitivity, $\Delta F/F$ is the finesse balance of the cavities and $\Delta l/l$ the length balance of the cavities.

Assume $\quad h(f) < 3 \times 10^{-25} \text{strain}\big/Hz^{1/2}$ $\quad$ best QL sensitivity

$$\left[\frac{\Delta F}{F} + \frac{\Delta l}{l}\right] \sim 10^{-3}$$

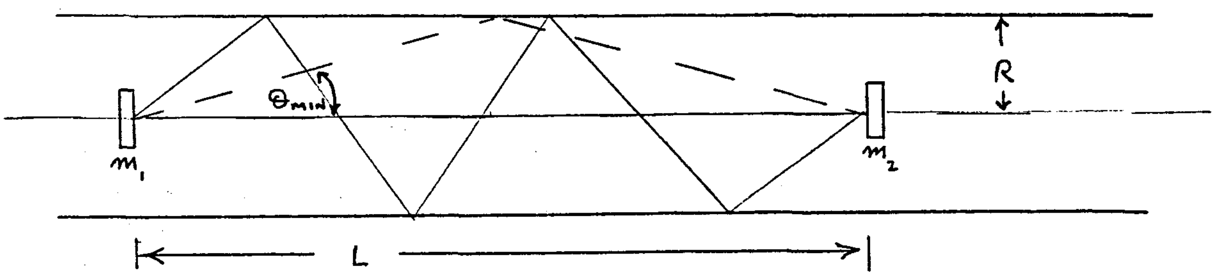
$\nu(f) < 1.8 \times 10^{-7} \quad Hz/Hz^{1/2}$ $\quad$ [without electronic common mode subtraction]

Evaluating the amount of scattered light permitted in the input optics

$$\frac{P_{\text{scat}}}{P_{\text{main}}} < \left(\frac{\nu(f)}{\nu_0}\frac{c}{2\pi f x(f)}\right) \sim 1.4 \times 10^{-7} f \qquad f > 10Hz$$

The attenuation of scattered light that is recombined with the main beam should be 60db at 10Hz.

The scattered - reflected - recombined power in a typical input beam tube



The worst case is that around $\theta_{\text{min}} \sim \frac{2R}{L}$

Scattering by the input mirror M_1

$\frac{dP_{sc}(\theta)}{d\Omega P} = \frac{\alpha}{\theta^2} \qquad \alpha = 10^{-6}$

Assume specular reflection from wall with no loss

$$\frac{P_{\substack{\text{scat}\\ \text{recombined}}}}{P} \cong \int_{\theta_{\text{min}}}^{\pi/2} \frac{\alpha}{\theta^2}\frac{2\alpha}{\theta^2}\frac{\lambda}{L}\,d\theta \simeq \frac{6\alpha^2}{\theta^3_{\text{min}}}\frac{\lambda}{l} = \frac{3}{4}\frac{L^2\lambda}{R^3}\alpha^2$$

For L = 12 meters

R = 9″ ⇒ 23cm $\qquad \frac{P_{\substack{\text{scat}\\ \text{recomb}}}}{P} \sim 4 \times 10^{-15}$

$\alpha = 10^{-6}$

The attenuation of scattered light in the input filter tube is sufficient to allow for coherent superposition in EQ 1.

b) Tubes involved with coupling splitter chambers to test mass chambers

The scattering in these tubes comes after the beam splitter where the interferometer becomes first order sensitive to phase fluctuations induced by the recombination of scattered beams with the main beam. The effect of scattering in this region, before the main cavity, is less sensitive than in the main cavity by the finesse of the cavity. This is seen by noting that, the phase shift is approximately given by

$$\Delta\varphi \;=\; \frac{E_{\substack{\text{scat}\\\text{recombined}}}}{E_{\text{main}}}$$

Which is true both inside and outside of the main cavity. The build up of the field in the cavity increases both numerator and denominator. The important difference is that a phase shift in the beam outside of the cavity is interpreted as a cavity mirror displacement of $\Delta x_M \;=\; \frac{\Delta\varphi_{\text{outside}}}{2\pi}\frac{\lambda}{F}$ while a phase shift of the total beam inside the cavity is interpreted as a mirror motion

$\Delta x_M \;=\; \frac{\Delta\varphi}{2\pi}\lambda$

As a consequence the influence of a scattered field recombined with the main beam in these coupling tubes is less important by a factor $1/F$

F will be 30 (1KHz antenna) $\rightarrow$ 300 (100Hz antenna)

Estimate of $h(f)$ limit due to scattering in the coupling tubes without baffles

Without exit mode filter

$$h(f) \;\sim\; \sqrt{\frac{2}{3}}\,\frac{\alpha^{1/2}(1-\eta)}{F^2}\;\left[\frac{(\lambda L_{\text{main}})^{1/2}}{R}\right]^{3/2}\;\theta_{\min}\,\mu(f) \qquad \text{KST 3.21 modified}$$

$L_{\text{main}} =$ 4km (beam has been expanded)

R = radius of coupling tube = 15" = 3.8cm

$\mu(f) \;=\; 2\times 10^{-9}\big/ f$ $\frac{\text{radians}}{Hz^{1/2}}$ amplitude of tube slope spectrum

F = 30 $l_T \;=\;$ 6 meters $\theta_{\min} \;=\; 2R\big/ l_T \;=\;$ 0.13 radians

$$h(f) = 8 \times 10^{-19} \Big/ f \qquad \textbf{Not Adequate}$$

With exit mode filter

$$h(f) = \frac{2}{\sqrt{3}} \frac{\alpha}{F^2} \left[\frac{(\lambda L_{\text{main}})^{1/2}}{R} \right]^{3/2} \left[\frac{\lambda}{L} \right]^{1/4} \theta_{\text{min}}^{1/2} \, \mu(f) \qquad \begin{array}{c} \text{KST 3.23} \\ \text{modified} \end{array}$$

$$= 1.1 \times 10^{-22} \Big/ f \qquad \textbf{Not Adequate}$$

Conclusion: Coupling tubes need to be baffled or blackened

Not sure that extending KST calculation to this short tube with very different L/R is correct. However - the beam diameter is large in this tube and there are 3 scattering and recombining surfaces in the optical train. $\genfrac{}{}{0pt}{}{\textbf{Blackening}}{\textbf{Baffling}}$ should improve the limits by a factor of 10^5 for both cases: with and without output mode filter.

The baffling should be designed to avoid direct reflection from the wall between the mirrors. In this short tube one can't use the same strategy as in the long beam pipes. The baffles and wall should be black.

I will have to check if corrugating the walls helps here.

c) The output tubes

These tubes are a special case. The light intensity is low when the interferometer is locked which makes the scattered light more critical since the main beam is reduced in intensity by

$$I_{\text{antisym.}} = (1-c)\, I_{\text{in}}$$

c is the contrast of the fringes defined as

$$c = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

where I_{max} is the intensity on a bright fringe I_{min} is the intensity on a "dark" fringe

The hope is to have $1 - c < 0.1$

The phase sensitivity to scattered light is down by a factor 1/F relative to the scattering in the main beam tubes.

The calculation is incomplete but it is clear that the output tubes should also be baffled and blackened.