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Thermal Lensing from Beam Absorption in a Mirror Substrate

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Abstract

The laser beam is partially absorbed when it passes through the substrate of the input coupler to an optical resonator. The associated temperature profile and the focal length of the resulting graded index lens are estimated. The presence of this lens causes the beam waist to move and to change in size, thus slightly degrading the match between the incident light beam and the resonator. It is shown that, for parameters typical to the first LIGO interferometers, this miss-match is expected to be small, so that it can be removed by slight refocussing of the input beam.

1 Introduction

A small fraction of the incident power is absorbed in the bulk of the input coupler to a resonator, due to the residual absorption found even in the most transparent material. This effect could be particularly significant in the case of monolithic test masses as considered for LIGO, since their thickness is expected to be ~ 15 cm even for the first interferometers. The heat deposited along the beam flows towards the edges of the test mass due to the creation of a temperature profile with nonvanishing gradient. Due to the temperature dependence of the index of refraction, the index takes different values at different distances from the beam axis, which makes the test mass behave like a graded index lens (GRIN). The purpose of this note is to estimate the temperature profile associated with beam absorption (Section 2) and then calculate the focal length of the GRIN and its effect on mode matching (Section 3). A few numerical examples, interesting in connection with LIGO and the 40 m prototype, are given in the Appendix.

2 The Temperature Profile

2.1 Assumptions

1. The test mass has cylindrical shape, and its axis coincides with the axis of the laser beam.

2. End effects are negligible. This is justifiable if the diameter of the beam is much smaller than the length d of the test mass.

3. The amount of heat Q deposited per unit volume is constant across the diameter $2w$ of the laser beam and is vanishing for $r > w$, where r is the distance from the test mass (laser beam) axis.

   Assumption 2 makes the temperature profile look steeper than it really is, while Assumption 3 makes it look shallower. The two approximations thus cancel to some extent and the estimate of the temperature profile is expected to be reasonably accurate.

4. The test mass-laser beam system is considered in a steady state, so that the temperature distribution is stationary.

5. All the heat is disposed of by radiation, as a consequence of the test mass being suspended in vacuum, from very thin wires.

2.2 Calculation of the temperature profile

Assumptions 1,2 define the problem as having cylindrical symmetry, with no dependence on the axial coordinate z. The stationary temperature distribution (Assumption 4) is described by the equation:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{Q}{k} = 0 \tag{1}$$

where k is the heat conductivity of the test mas material. According to Assumption 3, for laser power P at the beam splitter, N recycles and for $r < w$, $Q = \alpha NP/\pi w^2$, α being the absorption coefficient of the test mass material[1].

The boundary conditions are:

- The solution is finite for $r = 0$
- The solution and its derivative are continuous for $r = w$
- The heat flow at the edge of the mass is equal to the radiative heat transfer (Assumption 5), that is:

  $$k\frac{dT(D/2)}{dr} = 4\sigma T_A^3 \left[T_A - T(D/2)\right] \tag{2}$$

  where D is the diameter of the test mass, $\sigma = 5.7 \cdot 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$ is the Stefan-Boltzmann constant and T_A is the ambient temperature.

The solution of (1), that satisfies the above boundary conditions, is:

[1] A power $NP/2$ reaches each resonator input coupler mass, which is traversed twice by the beam

$$T - T_A = \frac{\alpha NP}{2\pi k}\left[\frac{k}{2\sigma T_A^3 D} + \ln\frac{D}{2w} + \frac{1}{2}\left(1 - \frac{r^2}{w^2}\right)\right] \quad ; \; r < w$$
$$T - T_A = \frac{\alpha NP}{2\pi k}\ln\frac{D}{2r} + \frac{\alpha NP}{4\pi\sigma T_A^3 D} \quad ; \quad w \le r \le D/2 \qquad (3)$$

3 Thermal Lensing

3.1 Focal Length of Thermally Induced GRIN

If the index of refraction of a material is n_0 at some given temperature T_0, then, for $T_0 + \Delta T$, the index is, to first order:

$$n = n_0 + \frac{dn}{dT}\Delta T \qquad (4)$$

For $r < w$, where most of the energy carried by the light beam is, Eqs. (3,4) yield:

$$n = n_0 - \frac{dn}{dT} \cdot \frac{\alpha NP}{4\pi k}\left(\frac{r}{w}\right)^2 \qquad (5)$$

For this quadratic index profile, the heated test mass behaves like a lens with focal length[2]:

$$f = \frac{2\pi k w^2}{\alpha NPd(dn/dT)} \qquad (6)$$

3.2 Mode Matching Degradation Due to Thermal Lensing

A beam waist with radius w_0 at a distance x_0 from a lens with focal length f is imaged into a waist with radius w_1 at a distance x_1 from the lens[3], such that:

[2]e. g. J. A. Kueken, Fiberoptics, Tab Books inc., p.149

[3]for light traveling from left to right, $x_0 > 0$ if the object waist is at the left of the lens and $x_1 > 0$ if the image waist is at the right of the lens

$$
\begin{aligned}
\frac{b_1}{f} &= \frac{b_0}{f}\frac{1}{\mathcal{D}} \\
1-\frac{x_1}{f} &= (1-\frac{x_0}{f})\frac{1}{\mathcal{D}} \qquad (7)\\
\mathcal{D} &= (1-x_0/f)^2+(b_0/2f)^2 \quad ; \quad b_i = 2\pi w_i^2/\lambda
\end{aligned}
$$

For the resulting relative change in waist size $\epsilon = (w_1 - w_0)/w_0$ and for the waist displacement $\Delta = x_1 + x_0$ (see Footnote 3 for sign conventions), the fraction of the power shifted to the next higher transverse mode is[4]:

$$
\frac{\delta P}{P} = \epsilon^2 + \left(\frac{\lambda\Delta}{2\pi w_0^2}\right)^2 \qquad (8)
$$

It should be pointed out that the presence of a thermal gradient also leads to stress in the material. This, in turn, causes additional change in the index of refraction and also distorts the mass. Both effects degrade the mode matching and will be addressed separately.

[4] D. Z. Anderson, Appl. Opt. **23**, 2944 (1984)

Appendix: Examples

Assume the test mass is made of fused silica, so that:

(a) $k = 1.3$ W·m^{-1}K^{-1}

(b) $\alpha = 5 \cdot 10^{-3}$m^{-1}

(c) $dn/dT = 1.2 \cdot 10^{-5}$K^{-1}

and also:

(d) $N = 30$ recycles

(e) $P = 1$ W

The extent of mode matching loss due to thermal lensing is given in Table 1 for 3 cases of interest. The power shifted to the next higher mode, shown in the last row of the table, is small, so that a very slight refocussing of the beam will re-establish "perfect" mode matching.

	40 prototype	LIGO: 3 km, 3 km	LIGO: Flat, 5 km
D (cm)	10	20	20
d (cm)	10	15	15
w_0 (cm)	0.215	1.52	1.81
w (cm)	0.215	2.64	1.81
f (m)	210	21,090	9,900
w_1 (cm)	0.213	1.386	1.77
Δ (m)	3.73	102	388
ϵ	0.0093	0.088	0.022
$\delta P/P$	0.0045	0.009	0.01

Table 1: Effect of beam absorption in test mass on mode matching. The figures next to the LIGO entries are the radii of curvature of the mirrors

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