New Folder Name Scaling of Pressure Spec

LIGO-T900036-00-R

Scaling of Pressure Spec with Chemical Composition

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Given the pressure spec for hydrogen in the LIGO beam tubes, p_{H_2}, the spec for another gas is[1]:

$$p_G = C p_{H_2} \quad ; \quad C = \frac{(n_{0H_2} - 1)^2 M_{H_2}^{\frac{1}{2}}}{(n_{0G} - 1)^2 M_G^{\frac{1}{2}}} \tag{1}$$

While the indexes of refraction are known for hydrogen, oxygen and for other gasses with simple molecules, it is hard to come by the measured indexes for more complicated molecules, which may be in a liquid-vapor equilibrium at room temperature and atmospheric pressure. In that case, a value of C can be calculated by using the approximate additivity of the molar refractivity A, which for a gas is[2]:

$$A = \frac{RT}{p}\frac{n^2 - 1}{3} \tag{2}$$

If a molecule contains $\mathcal{N}_i$ atoms of species i, then:

$$A = \sum_i \mathcal{N}_i A_i \tag{3}$$

[1] A. Abramovici, Pressure Specification for LIGO, February 1989

[2] see e. g. M. Born, E. Wolf, Principles of Optics, Pergamon Press, 1975, pp. 88-90

Table 1: Atomic refractivities of several elements, for sodium D light

Element	A
Hydrogen	1.028
Carbon	2.591
O=	2.122
-O-	1.643
OH	2.553
S=	7.921
-S-	7.729
F	0.81
Cl	5.844
Br	8.741
I	13.954
N (primary aliphatic)	2.376
N (secondary aliphatic)	2.582
N (aromatic)	3.55
N≡C	5.459
Ethylenic bond	1.575
Acetylenic bond	1.977

where A_i are called atomic refractivities. Some atomic refractivities are shown in Table 1.

Since for a gas $n^2 - 1 \simeq 2(n-1)$, it results from Eq. (2) that $n - 1 \propto A$ and:

$$C = \frac{A_{H_2}^2 M_{H_2}^{\frac{1}{2}}}{A_G^2 M_G^{\frac{1}{2}}} \qquad (4)$$

where, according to Eq. (3) and Table 1, $A_{H_2} = 2.056$.

Values of C, calculated by using Eqs. (1,4), are compared in Table 2, for a few gasses with known $n - 1$. The agreement is quite good, supporting the use of the additivity of atomic refractivities for estimating the effect of heavier molecules on interferometer sensitivity.

Table 2: Comparison between C-numbers calculated by using Eq. (4) (subscript A) and Eq. (1) (subscript $n-1$)

Gas	A	C_A	C_{n-1}
N_2	4.752	0.050	0.058
O_2	4.244	0.059	0.062
CO_2	6.835	0.019	0.021
CO	4.713	0.051	0.045
H_2O	3.699	0.103	0.104