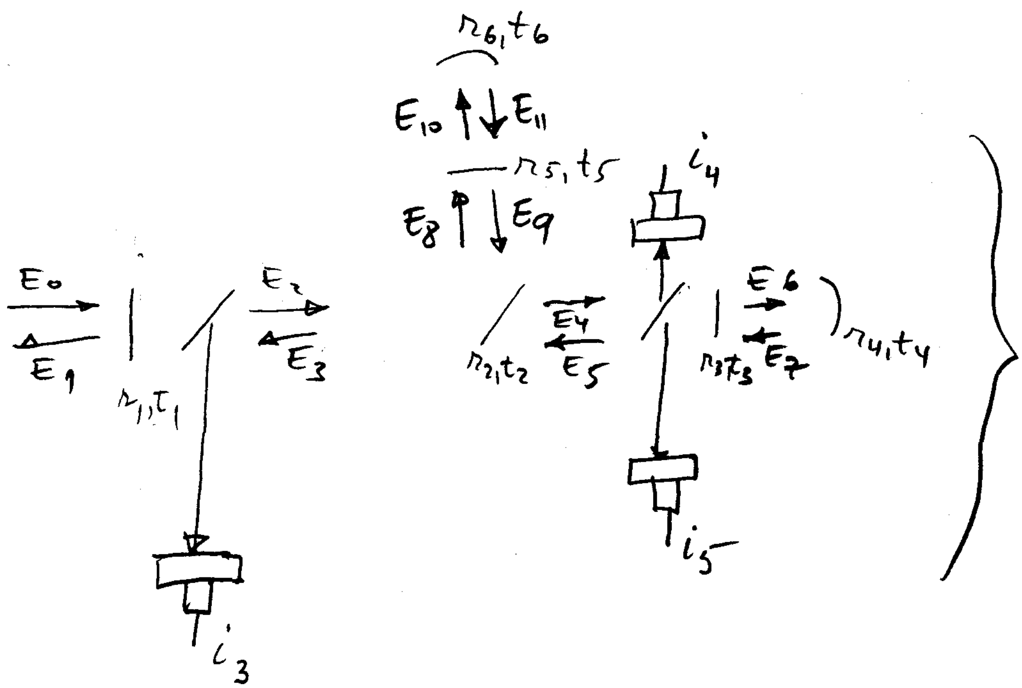
New Folder Name Demodulated Photocurrents

19-APRIL-91

Demodulated Photocurrents in a Recombined, Recycled, Interferometer with Fabry-Perot Arms.



← fields and r_i, t_i as in Stan's notes

Assumptions

- $r_3 = r_5$; $r_4 = r_6 = 1$; no losses.
  $t_3 = t_5$; $r_2 = t_2 = \frac{1}{\sqrt{2}}$

- All cavities close to resonance, Michelson close to dark fringe

- $L_i = L_i^{resonance} + y_i$
  $l_i = l_i^{resonance} + x_i$

$$E_5 = E_4 \frac{r_3 + r_4(r_3^2+t_3^2)e^{2ikL_1}}{1+r_3 r_4 e^{2ikL_1}} = r_{c1} E_4 \text{ at mirror\#3}$$

$$E_4 = t_2 E_{2,B} e^{ikl_1}$$

$$E_{2,B} = E_{0,I} \frac{t_1 e^{ikl_0}}{1 + r_1 e^{2ikl_0}\left(r_2^2 r_{c2} e^{2ikl_2} + t_2^2 r_{c1} e^{2ikl_1}\right)}$$

$$r_{c1} = \frac{r_3 + r_4(r_3^2+t_3^2)e^{2ikL_1}}{1+r_3 r_4 e^{2ikL_1}}$$

$$r_{c2} = \frac{r_5 + r_6(r_5^2+t_5^2)e^{2ikL_2}}{1+r_5 r_6 e^{2ikL_2}}$$

$$E_{A,B} = t_2 r_2 \left(r_{c2} e^{2ikl_2} - r_{c1} e^{2ikl_1}\right) E_{2,B}$$

Sidebands are obtained by using the above formulae, with the substitutions:

$$r_{c1} \rightarrow r_3$$

$$r_{c2} \rightarrow r_5$$

$$k \rightarrow \begin{cases} k + k_m & \text{right side band} \\ k - k_m & \text{left side band} \end{cases}$$

and by multiplying right sideband with $e^{-i\omega_m t}$

— " — ~~left~~ — " — $e^{i\omega_m t}$

At mirror 3: (without the factor $e^{i\omega_m t}$),

$$E_{5+} = r_3 E_{4+}$$

$$E_{4+} = t_2 E_{2,B+}\, e^{i(k+k_m)l_1}$$

$$E_{2,B+} = E_{0+} \frac{t_1\, e^{i(k+k_m)l_0}}{1+ r_1 e^{2i(k+k_m)l_0}\left[r_2^2 r_5\, e^{2i(k+k_m)l_2} + t_2^2 r_3\, e^{2i(k+k_m)l_1}\right]}$$

$$E_{4+} = \frac{E_{0+}\quad t_1 t_2\, e^{i(k+k_m)(l_0+l_1)}}{1+ r_1 e^{2i(k+k_m)l_0}\left[r_2^2 r_5\, e^{2i(k+k_m)l_2} + t_2^2 r_3\, e^{2i(k+k_m)l_1}\right]}$$

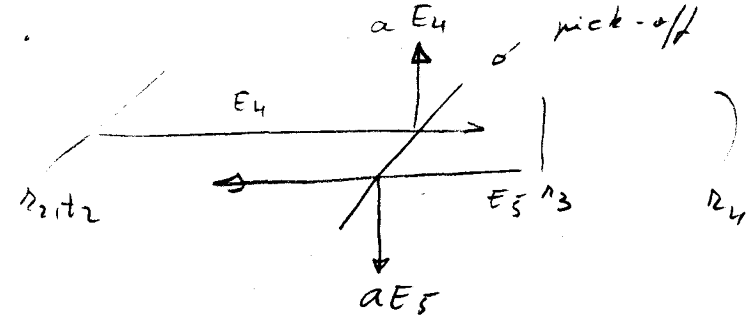
Take $r_3 = r_5$, $r_4 = r_6$, $t_2 = r_2 = \frac{1}{\sqrt{2}}$, $r_3^2 + t_3^2 = 1$

Then: $$r_{c_1} = \frac{r_3 + r_4\, e^{2ikL_1}}{1+ r_3 r_4\, e^{2ikL_1}} \; ; \quad r_{c_2} = \frac{r_3 + r_4\, e^{2ikL_2}}{1+ r_3 r_4\, e^{2ikL_2}}$$

$$E_{2,B} = E_0 \frac{t_1\, e^{ikl_0}}{1+ \frac{r_1}{2} e^{2ikl_0}\left(r_{c_2} e^{2ikl_2} + r_{c_1} e^{2ikl_1}\right)}$$

$$\begin{cases} E_4 = \frac{E_{0I}}{\sqrt{2}} \cdot \dfrac{t_1\, e^{ik(l_0+l_1)}}{1+\frac{r_1}{2}\left[r_{c2}\, e^{2ik(l_0+l_2)} + r_{c_1}\, e^{2ik(l_0+l_1)}\right]} \\ E_5 = r_{c_1} E_4 \end{cases}$$

$$\begin{cases} E_{4+} = \frac{E_{0+}}{\sqrt{2}} \cdot \dfrac{t_1\, e^{i(k+k_m)(l_0+l_1)}}{1+\frac{r_1 r_3}{2}\left[e^{2i(k+k_m)(l_0+l_2)} + e^{2i(k+k_m)(l_0+l_1)}\right]} \\ E_{5+} = r_3 E_{4+} \end{cases}$$



The photocurrent due to aE_{4+} is (dropping "a"):

$$i_{4+} = 2\mathrm{Re}\; E_4 E_{4+}^*$$

$$i_{4+} = E_{0I} E_{0+} t_1^2\, \mathrm{Re}\, \frac{e^{ik(l_0+l_1)}\, e^{-i(k+k_m)(l_0+l_1)}\, e^{+i\omega_m t}}{\left\{1+\frac{r_1}{2}\left[r_{c2} e^{2ik(l_0+l_2)} + r_{c1} e^{2ik(l_0+l_1)}\right]\right\}\left\{1+\frac{r_1 r_3}{2}\left[e^{-2i(k+k_m)(l_0+l_2)} + e^{-2i(k+k_m)(l_0+l_1)}\right.\right.}$$

$$i_{4+} = E_{0I} E_{0+} t_1^2\, \mathrm{Re}\, \frac{e^{-ik_m(l_0+l_1)}\, e^{+i\omega_m t}}{\left\{\;\cdots\;\right\}\left\{1+\frac{r_1 r_3}{2}\left[e^{-2i(k+k_m)(l_0+l_2)} + e^{-2i(k+k_m)(l_0+l_1)}\right]\right\}}$$

$$i_{4-} = -E_{0I} E_{0+} t_1^2\, \mathrm{Re}\, \frac{e^{ik_m(l_0+l_1)}\, e^{-i\omega_m t}}{\left\{\quad\right\}\left\{1+\frac{r_1 r_3}{2}\left[e^{-2i(k-k_m)(l_0+l_2)} + e^{-2i(k-k_m)(l_0+l_1)}\right]\right\}}$$

$E_{0-} = -E_{0+}$

$$i_4 = i_{4+} + i_{4-} = \mathrm{Re}\, A$$

For i_5, the carrier is multiplied by r_{c1}, while the sidebands are multiplied by r_3.

Thus, $i = i_4 - i_5 = \mathrm{Re}\left\{A(1 - r_3 r_{c1})\right\} = \mathrm{Re}\left\{A \frac{t_3^2}{1 + r_3 r_4 e^{2ikl_4}}\right.$

$$i_4 = \mathrm{Re}\, A = E_{0\,\bar{s}} E_{0+} t_1^2 \, \mathrm{Re}\Big\{ \underbrace{\cdots \quad - \cdots}_{B} \Big\}$$

$$B = \underbrace{\frac{1}{1 + \frac{r_1}{2}\left[r_{c2}\, e^{2ik(l_0+l_2)} + r_{c1}\, e^{2ik(l_0+l_1)} \right]}}_{B_1} \times \Bigg\{ \frac{e^{-ik_m(l_0+l_1)}\, e^{i\omega_m t}}{\underbrace{1 + \frac{r_1 r_3}{2}\left[e^{-2i(k+k_m)(l_0+l_2)} + e^{-2i(k+k_m)(l_0+l_1)} \right]}_{B_2}}$$

$$\underbrace{- \frac{e^{ik_m(l_0+l_1)}\, e^{-i\omega_m t}}{1 + \frac{r_1 r_3}{2}\left[e^{-2i(k-k_m)(l_0+l_2)} + e^{-2i(k-k_m)(l_0+l_1)} \right]} \Bigg\}}_{B_3}$$

Resonance Conditions

1. Arms : $e^{2ikL_i} = -1 \qquad L_i = n_i \frac{\lambda}{2} + \frac{\lambda}{4}$

Close to resonance : $L_i = n_i \frac{\lambda}{2} + \frac{\lambda}{4} + y_i$

$$e^{2ikL_i} = -(\cancel{1+}\, 2iky_i)$$

2. Recycling cavity (from condition to maximize $E_{2,B}$) ;

- assume arms at resonance :

$$r_{c1} = r_{c2} = (-r_c) < 0$$

- $e^{2ik(l_0+l_1)} = e^{2ik(l_0+l_2)} = 1$

- $l_0 + l_i = m_i \frac{\lambda}{2}$

- Close to resonance : $l_0 + l_i = m_i \frac{\lambda}{2} + x_i$

$$e^{2ik(l_0+l_i)} = 1 + 2ikx_i$$

3. Michelson (from dark fringe condition):

- $e^{2ik(l_2-l_1)} = 1$
- $l_2 - l_1 = p_i \frac{\lambda}{2}$
- close to a dark fringe: $l_2 - l_1 = p_i \frac{\lambda}{2} + (x_2 - x_1)$

$$e^{2ik(l_2-l_1)} = 1 + 2ik(x_2 - x_1)$$

Close to resonance, the cavity reflectivities are:

$$r_{ci} \cong \frac{r_3 - r_4(1+2iky_i)}{1 - r_3 r_4(1+2iky_i)} = \frac{(r_3 - r_4) - 2i r_4 k y_i}{1 - r_3 r_4 - 2i r_3 r_4 k y_i}$$

where $r_5 = r_3$, $r_6 = r_4$, $r_3^2 + t_3^2 = 1$ (lossless mirrors).

$$r_{ci} = \frac{r_3 - r_4}{1 - r_3 r_4} \cdot \frac{1 - 2i \frac{r_4}{r_3 - r_4} k y_i}{1 - 2i \frac{r_3 r_4}{1 - r_3 r_4} k y_i}$$

For $r_4 = 1$:

$$r_{ci} = - \frac{1 - 2i \frac{1}{r_3 - 1} k y_i}{1 - 2i \frac{r_3}{1 - r_3} k y_i}$$

For y_i sufficiently small:

$$r_{ci} = -\left[1 + 2iky_i\left(\frac{r_3}{1-r_3} + \frac{1}{1-r_3}\right)\right] \approx -\left[1 + \frac{4iky_i}{1-r_3}\right]$$

$$\boxed{r_{ci} \simeq -\left[1 + 2i \frac{y_i}{y_{1/2}}\right]}\,;$$ $y_{1/2}$: half width in terms of cavity length change.

$$y_{1/2}^{-1} = \frac{2kr_3}{1-r_3}$$

$$i_4 = \mathcal{R}_e A = E_{0}^{*} E_0 + t_1^2 \mathcal{R}_e\{B\}$$ p. 4, top

$$B = B_1 (B_2 - B_3)$$

$$B_1 = \frac{1}{1 - \frac{r_1}{2}\left[\left(1+2i\frac{y_1}{y_{1/2}}\right)(1+2ikx_1) + \left(1+2i\frac{y_2}{y_{1/2}}\right)(1+2ikx_2)\right]} =$$

$$= \frac{1}{1 - \frac{r_1}{2}\left[\cancel{2} + \cancel{2}i\left(\frac{y_1+y_2}{y_{1/2}} + kx_1 + kx_2\right)\right]}$$

$$B_1 = \frac{1}{1-r_1} \cdot \frac{1}{1 - i\frac{r_1}{1-r_1}\left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2)\right]}$$

$$\boxed{B_1 \cong \frac{1 + i\frac{r_1}{1-r_1}\left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2)\right]}{1-r_1}}$$

With the recycling cavity at resonance:

$$e^{-2i(k+k_m)(l_0+l_1)} = e^{-2ik(l_0+l_1)} e^{-2ik_m(l_0+l_1)} =$$

$$= (1-2ikx_1)(1-2ik_m x_1)\, e^{-i\varphi}, \text{ where } e^{-i\varphi} = e^{-2ik_m(l_0+l_1)}$$ is

the added phase, ~~due to an imbalance in the arms~~ the arms l_1, l_2.

Then:

$$e^{-2i(k+k_m)(l_0+l_2)} = (1-2ikx_2)(1-2ik_m x_2)\, e^{+i\varphi}$$

Since $k_m \ll k$:

$$e^{-2i(k+k_m)(l_0+l_1)} = (1-2ikx_1)\, e^{-i\varphi}$$

$$e^{-2i(k+k_m)(l_0+l_2)} = (1-2ikx_2)\, e^{i\varphi}$$

$$e^{-2i(k-k_m)(l_0+l_1)} = (1-2ikx_1)\, e^{i\varphi}$$

$$e^{-2i(k-k_m)(l_0+l_2)} = (1-2ikx_2)\, e^{-i\varphi}$$

Also: $e^{-ik_m(l_0+l_1)} = e^{-i\frac{\varphi}{2}}$

$e^{ik_m(l_0+l_1)} = e^{i\frac{\varphi}{2}}$

$$B_2 = \frac{e^{-i\frac{\varphi}{2}} e^{i\omega_m t}}{1 + \frac{r_1 r_3}{2}\left\{(1-2ikx_1)e^{-i\varphi} + (1-2ikx_2)e^{i\varphi}\right\}} =$$

$$= \frac{e^{-i\frac{\varphi}{2}} e^{i\omega_m t}}{1 + r_1 r_3\left[\cos\varphi - ik\left(x_1 e^{-i\varphi} + x_2 e^{+i\varphi}\right)\right]} =$$

$$= \frac{e^{-i\frac{\varphi}{2}} e^{i\omega_m t}}{(1 + r_1 r_3 \cos\varphi) - ik\, r_1 r_3 (x_1 e^{-i\varphi} + x_2 e^{+i\varphi})} =$$

$$= \frac{1}{1 + r_1 r_3 \cos\varphi} \cdot \frac{e^{-i\frac{\varphi}{2}} e^{i\omega_m t}}{1 - ik\frac{r_1 r_3}{1 + r_1 r_3\cos\varphi}(x_1 e^{-i\varphi} + x_2 e^{+i\varphi})}$$

$$\boxed{B_2 = \frac{1}{1 + r_1 r_3 \cos\varphi}\left[1 + ik\frac{r_1 r_3}{1 + r_1 r_3 \cos\varphi}\left(x_1 e^{-i\varphi} + x_2 e^{+i\varphi}\right)\right] e^{-\frac{i\varphi}{2}} e^{i\omega_m t}}$$

$$\boxed{B_3 = \frac{1}{1 + r_1 r_3 \cos\varphi} \cdot \left[1 + ik\frac{r_1 r_3}{1 + r_1 r_3 \cos\varphi}\left(x_1 e^{i\varphi} + x_2 e^{-i\varphi}\right)\right] e^{\frac{i\varphi}{2}} e^{-i\omega_m t}}$$

$$B_2 + B_3 = \frac{1}{1+r_1 r_3 \cos\varphi}\left\{2i\sin(\omega_m t - \tfrac{\varphi}{2}) + ik\frac{r_1 r_3}{1+r_1 r_3\cos\varphi}\left[\left(x_1 e^{-i\varphi} + x_2 e^{i\varphi}\right)e^{-\frac{i\varphi}{2}}e^{i\omega_m t} - \left(x_1 e^{i\varphi} + x_2 e^{-i\varphi}\right)e^{\frac{i\varphi}{2}}e^{-i\omega_m t}\right]\right\} =$$

$$= \frac{1}{1+r_1 r_3\cos\varphi}\left\{2i\sin(\omega_m t - \tfrac{\varphi}{2}) + ik\frac{r_1 r_3}{1+r_1 r_3\cos\varphi}\left[x_1\left(e^{-i\frac{3\varphi}{2}+i\omega_m t} - e^{i\frac{3\varphi}{2}-i\omega_m t}\right) + x_2\left(e^{i\frac{\varphi}{2}+i\omega_m t} - e^{-i\frac{\varphi}{2}-i\omega_m t}\right)\right]\right\}$$

$$= \frac{1}{1+r_1 r_3\cos\varphi}\left\{2i\sin(\omega_m t - \tfrac{\varphi}{2}) - 2k\frac{r_1 r_3}{1+r_1 r_3\cos\varphi}\times\left[x_1\sin(\omega_m t - \tfrac{3\varphi}{2}) + x_2\sin(\omega_m t + \tfrac{\varphi}{2})\right]\right\}$$

$$\mathrm{Re}\{B_1(B_2 - B_3)\} = \frac{-2}{(1-r_1)(1+r_1 r_3\cos\varphi)}\left\{\frac{r_1}{1-r_1}\left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2)\right]\sin(\omega_m t - \tfrac{\varphi}{2}) + k\frac{r_1 r_3}{1+r_1 r_3\cos\varphi}\left[x_1\sin(\omega_m t - \tfrac{3\varphi}{2}) + x_2\sin(\omega_m t + \varphi/2)\right]\right\}$$

and finally:

$$\boxed{i_4 = -\frac{2E_{0,I}\,E_{0+}\,t_1^2}{(1-r_1)(1+r_1r_3\cos\varphi)}\left\{\frac{r_1}{1-r_1}\left[\frac{y_1+y_2}{y_{1/2}}+k(x_1+x_2)\right]\sin\left(\omega_m t-\frac{\varphi}{2}\right) + \frac{k\,r_1r_3}{1+r_1r_3\cos\varphi}\left[x_1\sin\left(\omega_m t-\frac{3\varphi}{2}\right)+x_2\sin\left(\omega_m t+\frac{\varphi}{2}\right)\right]\right\}}$$

Summary of assumptions:
- no losses
- $r_2 = t_2 = \frac{1}{\sqrt{2}}$
- $r_4 = r_6 = 1$
- Everything very close to resonance

Demodulated signals:

$$\left\langle i_4\cos\left(\omega_m t-\frac{\varphi}{2}\right)\right\rangle = -\left[\frac{k\,E_{0,I}\,E_{0+}\,t_1^2\,r_1r_3}{(1-r_1)(1+r_1r_3\cos\varphi)^2}\sin\varphi\right]\cdot(x_2-x_1)$$

$$\left\langle i_4\sin\left(\omega_m t-\frac{\varphi}{2}\right)\right\rangle = \frac{E_{0,I}\,E_{0+}\,t_1^2}{(1-r_1)(1+r_1r_3\cos\varphi)}\left\{\frac{r_1}{1-r_1}\left[\frac{y_1+y_2}{y_{1/2}}+k(x_1+x_2)\right]+\frac{k\,r_1r_3}{1+r_1r_3\cos\varphi}\cos\varphi\,(x_1+x_2)\right\}$$

this term is smaller than the other x_1+x_2 term by a factor $\sim\frac{2r_1}{1-r_1}$

$$i_5 = E_{0,I}\, E_{0+}\, t_1^2\, \mathrm{Re}\left\{ B_1 (B_2 - B_3)\, r_3\, r_{c_1} \right\}$$

Close to resonance, $r_{c_1} = -\left[1 + 2i \dfrac{y_1}{y_{1/2}}\right]$ (p. 5)

$$i_5 = -2 E_{0,I}\, I_{0+}\, t_1^2\, r_3\, \mathrm{Re}\left\{ \frac{1 + i \frac{r_1}{1-r_1}\left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2)\right]}{1-r_1} \times \right.$$

$$\times \frac{i \sin\left(\omega_m t - \frac{\varphi}{2}\right) - k \frac{r_1 r_3}{1 + r_1 r_3 \cos\varphi}\left[x_1 \sin\left(\omega_m t - \frac{3\varphi}{2}\right) + x_2 \sin\left(\omega_m t + \frac{\varphi}{2}\right)\right]}{1 + r_1 r_3 \cos\varphi} \times$$

$$\left. \times \left(1 + 2i \frac{y_1}{y_{1/2}}\right) \right\}$$

$$i_5 = +2 E_{0,I}\, E_{0+} \frac{t_1^2 r_3}{(1-r_1)(1 + r_1 r_3 \cos\varphi)} \times \left\{ + \left[2\frac{y_1}{y_{1/2}} + \frac{r_1}{1-r_1}\left(\frac{y_1+y_2}{y_{1/2}} + kx_1 + kx_2\right)\right] \right.$$

$$\left. \times \sin\left(\omega_m t - \frac{\varphi}{2}\right) + k \frac{r_1 r_3}{1 + r_1 r_3 \cos\varphi}\left[x_1 \sin\left(\omega_m t - \frac{3\varphi}{2}\right) + x_2 \sin\left(\omega_m t + \frac{\varphi}{2}\right)\right] \right\}$$

$$i_4 + \frac{1}{r_3} i_5 = 2E_{0,I}\, E_{0+} \frac{t_1^2}{(1-r_1)(1+r_1 r_3 \cos\varphi)} \left(\frac{2y_1}{y_{1/2}}\right) \sin\left(\omega_m t - \frac{\varphi}{2}\right)$$

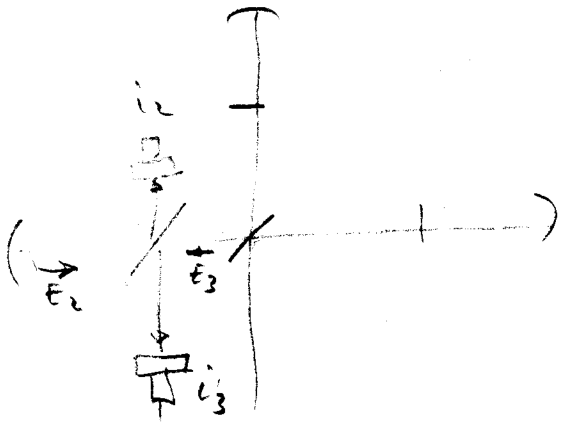
$$\boxed{\left\langle \left(i_4 + \frac{1}{r_3} i_5\right) \sin\left(\omega_m t - \frac{\varphi}{2}\right) \right\rangle = \frac{t_1^2\, E_{0I}\, E_{0+}}{(1-r_1)(1+r_1 r_3 \cos\varphi)} \left[\cdot \frac{2y_1}{y_{1/2}} + \text{Small terms in } (\right]}$$

Small term equals:

$$\frac{k\, r_1 r_3}{1 + r_1 r_3 \cos\varphi} \left(1 - \frac{1}{r_3}\right) (x_1 + x_2)$$

Compare $\dfrac{2}{y_{1/2}}$ with $\dfrac{k \cdot r_1 (r_3 - 1)}{1 + r_1 r_3 \cos\varphi}$.

Pick-off between Recycling Mirror and Beam Splitter



At the recycling mirror

$$\begin{cases} E_{3,M} = E_{2,B}\left(r_2^2 r_{c2} e^{2ikl_2} + t_2^2 r_{c1} e^{2ikl_1}\right) e^{ikl_0} \\ E_{3,M+} = E_{2,B+}\left[r_2^2 r_3 e^{2i(k+k_m)l_2} + t_2^2 r_3 e^{2i(k+k_m)l_1}\right] e^{i(k+k_m)l_0} \end{cases}$$

$$\begin{cases} E_{3,M} = \frac{1}{2} E_{2,B}\left(r_{c2} e^{2ikl_2} + r_{c1} e^{2ikl_1}\right) e^{ikl_0} \\ E_{3,M+} = \frac{r_3}{2} E_{2,B+}\left[e^{2i(k+k_m)l_2} + e^{2i(k+k_m)l_1}\right] e^{i(k+k_m)l_0} \\ E_{3,M-} = \frac{r_3}{2} E_{2,B-}\left[e^{2i(k-k_m)l_2} + e^{2i(k-k_m)l_1}\right] e^{i(k-k_m)l_0} \end{cases}$$

$$i_3 = 2\,\mathrm{Re}\left\{E_{3M} E_{3M+}^{*} e^{i\omega_m t} - E_{3M} E_{3M-}^{*} e^{-i\omega_m t}\right\}$$

$$E_{2B} = E_{0I} \frac{t_1 e^{ikl_0}}{1 + \frac{r_1}{2} e^{2ikl_0} \left(r_{c2} e^{2ikl_2} + r_{c1} e^{2ikl_1} \right)}$$

$$E_{2B+} = E_{0+} \frac{t_1 e^{i(k+k_m) l_0}}{1 + \frac{r_1 r_3}{2} \left(e^{2i(k+k_m)(l_0+l_2)} + e^{2i(k+k_m)(l_0+l_1)} \right)}$$

$$E_{2B-} = E_{0-} \frac{t_1 e^{i(k-k_m) l_0}}{1 + \frac{r_1 r_3}{2} \left[e^{2i(k-k_m)(l_0+l_2)} + e^{2i(k-k_m)(l_0+l_1)} \right]}$$

p 2, see E_4.

<u>At resonance</u> :

- $r_{ci} \cong - \left[1 + 2i \frac{y_i}{y_{1/2}} \right]$ p.5
- $e^{2ik(l_2 - l_1)} = 1 + 2ik(x_2 - x_1)$ p.5
- $e^{2ik(l_0 + l_i)} = 1 + 2ikx_i$ p.4.

$$\frac{E_{3,M}}{E_{0I}} = \frac{1}{2} t_1 \frac{r_{c2} e^{2ik(l_0+l_2)} + r_{c1} e^{2ik(l_0+l_1)}}{1 + \frac{r_1}{2}\left[r_{c2} e^{2ik(l_0+l_2)} + r_{c1} e^{2ik(l_0+l_1)} \right]}$$

$$\frac{E_{3M+}}{E_{0+}} = \frac{r_3 t_1}{2} \cdot \frac{e^{2i(k+k_m)(l_0+l_2)} + e^{2i(k+k_m)(l_0+l_1)}}{1 + \frac{r_1 r_3}{2}\left[e^{2i(k+k_m)(l_0+l_2)} + e^{2i(k+k_m)(l_0+l_1)} \right]}$$

$$\frac{E_{3M-}}{E_{0-}} = \frac{r_3 t_1}{2} \cdot \frac{e^{2i(k-k_m)(l_0+l_2)} + e^{2i(k+k_m)(l_0+l_1)}}{1 + \frac{r_1 r_3}{2}\left[e^{2i(k-k_m)(l_0+l_2)} + e^{2i(k-k_m)(l_0+l_1)} \right]}$$

Close to resonance:

$$E_{3M} = -E_{0I} \cdot \frac{1}{2} t_1 \frac{\left(1 + 2i \frac{y_2}{y_{1/2}}\right)(1 + 2ik x_2) + \left(1 + 2i \frac{y_1}{y_{1/2}}\right)(1 + 2ik x_1)}{1 - \frac{r_1}{2}\left\{ (\quad)(\quad) + (\quad)(\quad) \right.}$$

$$E_{3M} = -t_1 E_{0I} \cdot \frac{1 + i\left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2) \right]}{1 - r_1 \left\{ 1 + i\left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2) \right] \right\}}$$

$$E_{3M} = -\frac{t_1 E_{0I}}{1-r_1} \cdot \frac{1 + i\,[\quad]}{1 - i\frac{r_1}{1-r_1}[\quad]}$$

$$\boxed{E_{3M} = -\frac{t_1 E_{0I}}{1-r_1} \left\{ 1 + i \frac{1}{1-r_1} \left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2) \right] \right\}}$$

(14)

$$E_{3M+} = \frac{r_3 t_1}{2} E_{0+} \frac{(1+2ikx_2)e^{-i\varphi} + (1+2ikx_1)e^{i\varphi}}{1+\frac{r_1 r_3}{2}\left[\;\cdot\; - - - - -\;\right]}$$

See φ on pp. 6,7

$$E_{3M+} = r_3 t_1 E_{0+} \frac{\cos\varphi + ik(x_2 e^{-i\varphi} + x_1 e^{i\varphi})}{1 + r_1 r_3\left[\cos\varphi + ik(x_2 e^{-i\varphi} + x_1 e^{i\varphi})\right]}$$

$$E_{3M+} = \frac{r_3 t_1 E_{0+}}{1+r_1 r_3 \cos\varphi} \cdot \frac{\cos\varphi + ik(\quad\quad)}{1 + i\frac{r_1 r_3}{1+r_1 r_3\cos\varphi} k(\quad\quad)}$$

$$\boxed{E_{3M+} = \frac{r_3 t_1 E_{0+}}{1+r_1 r_3 \cos\varphi} \cdot \left\{\cos\varphi + ik \cdot \frac{1}{1+r_1 r_3\cos\varphi}\left(x_2 e^{-i\varphi} + x_1 e^{i\varphi}\right)\right\}}$$

$$E_{3M-} = r_3 t_1 E_{0-} \frac{\cos\varphi + ik(x_2 e^{i\varphi} + x_1 e^{-i\varphi})}{1 + r_1 r_3\left[\cos\varphi + ik(x_2 e^{i\varphi} + x_1 e^{-i\varphi})\right]}$$

$$E_{3M-} = \frac{r_3 t_1 E_{0-}}{1+r_1 r_3 \cos\varphi} \cdot \frac{\cos\varphi + ik(x_2 e^{i\varphi} + x_1 e^{-i\varphi})}{1 + ik\frac{r_1 r_3}{1+r_1 r_3\cos\varphi}(x_2 e^{i\varphi} + x_1 e^{-i\varphi})}$$

$$\boxed{E_{3M-} = \frac{r_3 t_1 E_{0-}}{1+r_1 r_3 \cos\varphi} \cdot \left\{\cos\varphi + ik \frac{1}{1+r_1 r_3\cos\varphi}\left(x_2 e^{i\varphi} + x_1 e^{-i\varphi}\right)\right\}}$$

$$i_3 = 2\,\mathrm{Re}\left[E_{3H}\left(E_{3n+}^{*}\,e^{i\omega_m t} - E_{3n-}^{*}\,e^{-i\omega_m t}\right)\right]$$

$$i_3 = 2\,\frac{r_3 t_1^2 E_{0I} E_{0+}}{(1-r_1)(1+r_1 r_3\cos\varphi)}\,\mathrm{Re}\left\{\left[1 + i\,\frac{1}{1-r_1}\left(\frac{y_1+y_2}{y_{1/2}} + kx_1 + kx_2\right)\right] \times\right.$$

$$\left.\left[2i\cos\varphi\sin\omega_m t + \frac{ik}{1+r_1 r_3\cos\varphi}\left(2i\,x_2\sin(\omega_m t-\varphi) + 2i\,x_1\sin(\omega_m t+\varphi)\right)\right]\right\}$$

$$\boxed{i_3 = -\frac{4 r_3 t_1^2 E_{0I} E_{0+}}{(1-r_1)^2(1+r_1 r_3\cos\varphi)}\left(\frac{y_1+y_2}{y_{1/2}} + kx_1 + kx_2\right)\cos\varphi\sin\omega_m t}$$

$$\boxed{\langle i_3 \sin\omega_m t\rangle = -\frac{2 r_3 t_1^2 E_{0I} E_{0+}\cos\varphi}{(1-r_1)^2(1+r_1 r_3\cos\varphi)}\left[\frac{y_1+y_2}{y_{1/2}} + k(x_1+x_2)\right]}$$