New Folder Name Mode Matching

Mode Matching for Mk II

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VERSION 3.1
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file: ~robert/source/modematch/modematch3.tex

Abstract

A method of analyzing multi-lens mode matching design is developed and applied to 4-lens matching between the 93-cm mode cleaner and the beamsplitter for the 40 m interferometer, Mk II.

1. REQUIREMENTS AND SPECIFICATIONS

Mode matching between the mode cleaner and the 40 m arms is effected by a set of lenses satisfying the following requirements:

1. The beams incident on the vertex test masses match the mode defined by the 40 m arms.
2. A segment of the beam that passes through transmissive components (Faraday isolater and Pockels cell) has a diameter consistent with the component apertures (not too big) and intensity damage threshold (not too small), over a length determined by the length and spacing of the components.
3. Matching is not affected significantly by drift in lens positions or focal lengths.
4. There is sufficient range of motorized adjustment to optimize matching in vacuum, as measured by 40 m cavity visibilities.
5. The arrangement of lenses allows the option of non-iterative ("orthogonal knobs") adjustment of beam properties.
6. Spherical aberration does not limit the 40 m cavity visibility.

These requirements must be satisfied within the constraints indicated in Table 1. Some of the symbols are defined in Figure 1.

The requirements and constraints lead to the specifications in Table 2.

Symbol	Description	Value	Notes
R	Mode cleaner mirror curvature	50 cm	
s	Mode cleaner length	92.5 cm	
w_0	Mode cleaner waist radius	0.10 mm	Eq. (1) with $n = 1.5$.
	Mode cleaner output-end flange to first mirror	52 cm	
t	Mode cleaner (imaged) waist to output mirror	32.5 cm	Eq. (2)
d_0	Spacing from w_0 to L1	1.18 m	
$s23 + s34$	L2 to L4	65–125 cm	
w_4	Cavity waist	2.2 mm	
d_4	L4 lens to cavity waist	3.0 m	Solution insensitive to precise value
	Faraday isolator (index, length)	(1.68, 25 mm)	Hoya FR-5
	Pockels cell (index, length)	(1.5, 28 mm)	Gsänger PM-25
	Adjustment range for lens positions	± 0.6 cm	
$\Delta f/f$	Focal length tolerance	1%	[redacted]
$\frac{\Delta f}{f}(\Delta T)$	Focal length tempco	-8×10^{-6}/deg	$df/f = -dn/(n-1)$; for fused silica, dominated by $dn/dT = 1.2 \times 10^{-5}$/deg

Table 1: Input parameters for Mk II mode matching.

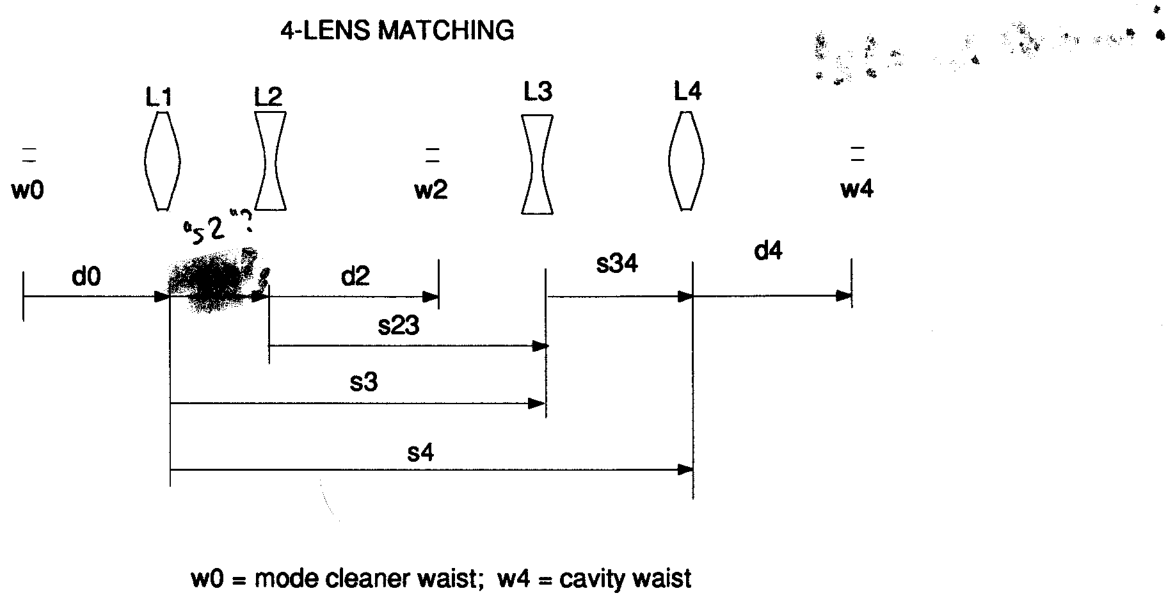

Figure 1: Arrangement of lenses for Mk II mode matching. The (L1–L2 pair) is a collimator for small-aperture components between L2 and L3. A folding mirror (not shown) precedes L3. Motorized adjustment of position along beam is required on three lenses.

Symbol	Description	Specification	Driving parameter, or explanation
w_{max}	Maximum beam size through transmissive components	0.8 mm	Apertures = 5 mm
w_{min}	Minimum beam size through transmissive components	0.36 mm	$I_{\mathrm{peak}} = 2P/\pi w^2$; $P = 2$ W, $I_{\mathrm{peak}} = 1$ kw/cm^2
	Minimum beam length between L2 and L3 with $w_{\mathrm{min}} < w < w_{\mathrm{max}}$	30 cm	Allows for 2 Faraday isolators, 1 Pockels cell.
Δw_4	Adjustment range of cavity waist size	± 0.1 mm	Corresponds to 4 m change in 62 m curvature of end mirror.
Δd_4	Adjustment range of cavity waist position	4 m	5% of unfolded cavity length.
$f_n/w(\mathrm{L}_n)$	"$f\#$", (focal length)/(beam radius) at lens L_n	> 100	For reduced spherical aberration.

Table 2: Mode matching specifications. The maximum allowed intensity for Pockels cell and Faraday isolators, I_{peak} is a guess, as are the required adjustment ranges for the cavity waist size and position. Spherical aberrations have not been calculated; the intention is to achieve lower spherical aberration than in Mk I.

Solution	f1	f2	f3	f4	d0	s12	$s23_v$	s34	$s3_v$	$s4_v$
4f	22	-10	-20	50	118	16.82	70.18	30.96	87	117.96
4g	22	-10	-25	50	118	19.43	69.57	34.80	89	123.80

Table 3: Lens focal lengths and nominal positions for two 4-lens configurations. These solutions apply to vacuum mode matching, and must be adjusted for the introduction of Pockels cell and isolators as follows: $s23 = s23_v + (n-1)t, n =$ index of refraction, $t =$ length of component; the same correction is required for $s3_v$ and $s4_v$. For one Hoya FR–25 isolator and one Gsänger PM–25 Pockels cell, the total correction is $s23 \approx s23_v + 3.1$ cm. All quantities are in units of cm.

2. IMPLEMENTATION

Two solutions—labeled 4f and 4g— are presented in Table 3. Table 4 indicates sensitivities to changes in lens focal length or position. Changes on the order of 0.1 mm—larger than expected drifts—will change visibility by less than 10^{-3} (the fractional change in power matching is $\Delta P/P = (\Delta w/w)^2 + (\Delta d_4/l_0)^2; l_0 = 2\pi w_4^2/\lambda$).

Table 5 shows the change in positions of L2 and L3 required to adjust waist position and size independently. The results are similar if the L2–L4 pair is adjusted instead (except L4 moves in the opposite direction). Adjustment of s12 gives approximately the same result whether accomplished by motion of L1 or L2. The matching parameters change only over a small range in response to motion of the (s3, s4) pair (results not shown).

Neither 4f nor 4g allows independent adjustment of waist size or position by motion of any single lens. Fig. 2 shows that the range of matching parameters spanned by available motion of L2 and L3 meets the requirements for $\Delta w_4, \Delta d_4$, Table 2. It also shows how s12 and s3 must move in concert for "orthogonal knobs" adjustment of waist size and position. Solution 4f gives slightly more range in waist size adjustment (Δs_3 smaller than 4g Δs_3, for $\Delta w_4 = .1$ mm; Table 5), and is slightly farther from the tricky cusp in Fig. 2(c). 4f is the recommended configuration.

Other details of the solutions are shown in the spreadsheet, Table 6.

A. FORMULAE

The mode cleaner waist size (radius) is given by

$$w_o = \left[\frac{2\bar{\lambda}R\sqrt{d(2R-s)}}{2R+s(n^2-1)}\right]^{1/2} \tag{1}$$

where $\bar{\lambda} = \lambda/2\pi$, R is the radius curvature of both mirrors, s is their separation, and n is their index of refraction. The waist position is given by

$$t = \frac{nsR}{2R+s(n^2-1)} \tag{2}$$

where t is measured from the output mirror, towards the center of the cavity.

The $\approx$ 38 m cavities have their waists at the input mirror, and the waist size is given by Equation (1) with $n = 1$.

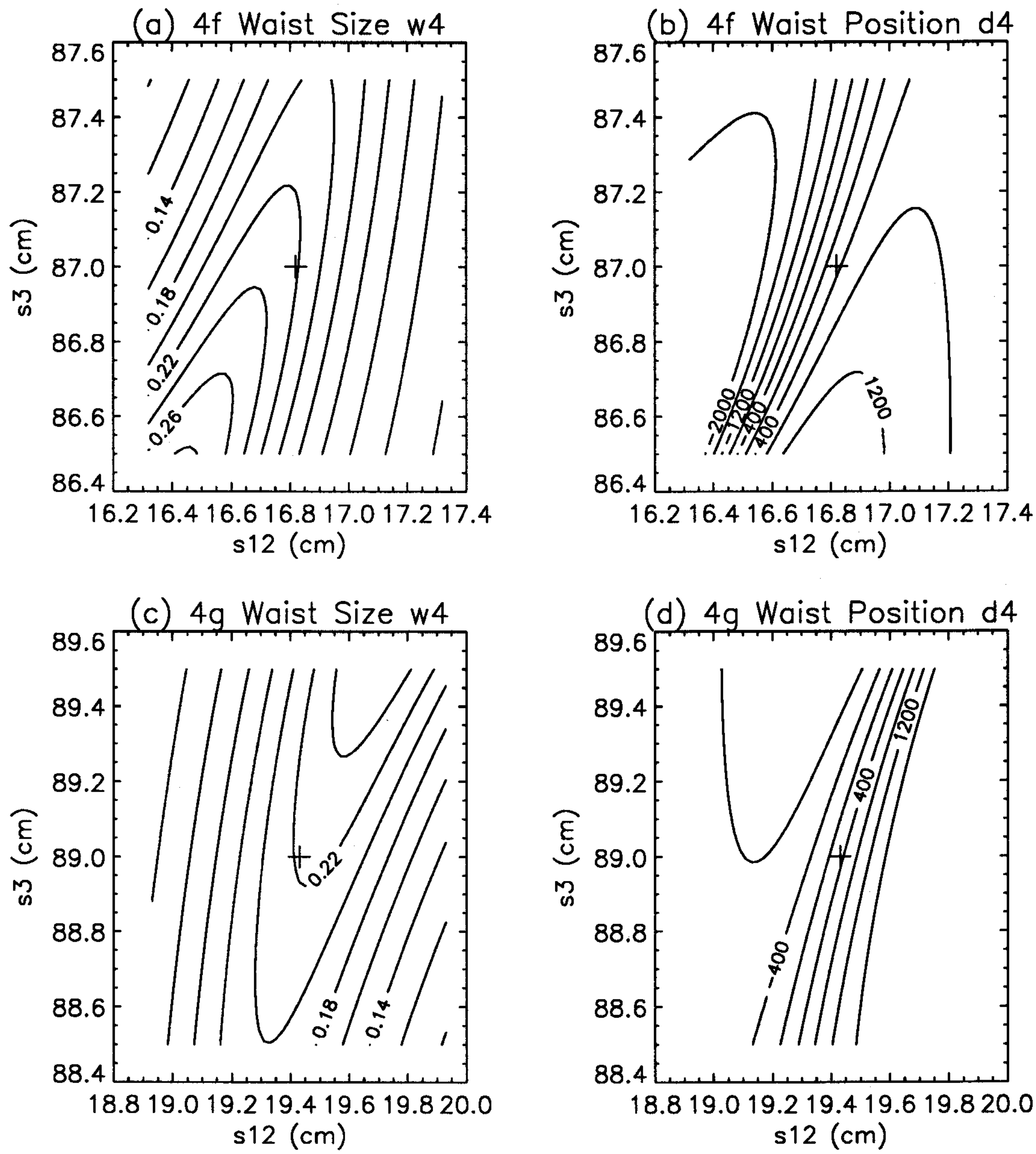

Figure 2: Contours of constant waist size [(a) and (c)] and position [(b) and (d)], as lens positions are adjusted. The + marks the nominal parameters for configurations 4f (upper plots) and 4g (lower plots).

η	4f $\frac{\Delta d_2}{d_2}$	4f $\frac{\Delta w_4}{w_4}$	4f $\frac{\Delta d_4}{40\text{m}}$	4g $\frac{\Delta d_2}{d_2}$	4g $\frac{\Delta w_4}{w_4}$	4g $\frac{\Delta d_4}{40\text{m}}$
f1	+.023	+.0137	−.030	+.0080	−.0078	−.033
f2	+.0149	+.0091	−.0208	+.0030	−.0033	−.0126
s12	−.0192	−.0101	+.0178	−.0053	+.0037	+.0229
s23	0	+.000022	+9.1E−06	0	+2.0E−06	+.00137
f3	0	+.00132	−.0082	0	+.00061	−.0033
s34	0	−.00107	+.0091	0	−.00107	+.0089
f4	0	+.00113	−.0091	0	+.00113	−.0090

Table 4: Sensitivity table for configurations 4f, 4g. Fractional change in the position of the intermediate waist, d_2, and the main cavity waist parameters, w_4 and d_4 for a change of 0.1 mm in each of the input parameters, η, in the first column. The resulting change in visibility is approximately the square of the largest of the $(\Delta w_4/w_4, \Delta d_4/40\text{ m})$ entries.

	Δw_4=0.1 mm		Δd_4=+4 m	
	$(\Delta s_{12}, \Delta s_3)$	arctan $(\frac{\Delta s_3}{\Delta s_{12}})$	$(\Delta s_{12}, \Delta s_3)$	arctan $(\frac{\Delta s_3}{\Delta s_{12}})$
4f	(-.057, -.116)	63.7 deg	(-.0145,-.141)	84.1 deg
4g	(.065,.182)	70.4	(.019,-.078)	-76.3

Table 5: Required adjustment of lens positions (Δs_{12} and Δs_3), in cm, for specified change in output cavity waist parameters (0.1 mm and 4 m for cavity waist size and position, respectively). The angles are the slopes of the trajectories in Fig. 2 that leave the waist position (left block) or size (right block) fixed.

Each lens propagates an input waist $w_{\rm in}$ to an output waist $w_{\rm out}$ as follows:

$$w_{\rm out} = \frac{w_{\rm in}}{\sqrt{D}}$$
$$d_{\rm out} = f\left[1 - \frac{1 - d_{\rm in}/f}{D}\right] \tag{3}$$

where

$$D = (1 - d_{\rm in}/f)^2 + \left(\frac{w_{\rm in}^2/\lambda}{2f}\right)^2. \tag{4}$$

Here $d_{\rm in}$ and $d_{\rm out}$ are the distances from the input waist to the lens, and from the lens to the output waist, respectively; f is the focal length.

B. METHOD

The nominal lens spacings, read from Karl's drawing and indicated in Table 1, were taken as the starting point, and the focal lengths were adjusted without constraint, to provide a roughly collimated beam between L2 and L3, and matching to w_0 and w_4.

Then the focal lengths were set to standard values, and the lens spacing parameters were adjusted slightly to restore matching.

The following software tools were used:

1. `beam`, a 4-lens propagation program written by Harry Ward in TurboBasic, slightly modified and run under QuickBasic. This program is well-suited for "experimenting" with parameters, and rapidly iterating by hand for solutions. It is based on the "ABCD" transformation (see, e.g., Yariv, *Introduction to Optical Electronics*, 2nd ed. p. 36 ff.).

2. `mode10`, the 2-lens matching program, written in Fortran, that takes as input the separation between the input and output waists.

3. `4lensa`, a spreadsheet that propagates Eqs. (3) through multiple lenses. Originally written in Excel 3.0, translated by machine to Lotus 1-2-3. Useful for obtaining intermediate beam properties, and sensitivity analysis.

4. Several `Mathematica` scripts that solve for lens positions, provide sensitivity matrices, and symbolically differentiate for the inverse solution: the required change in lens positions for a given change in waist size or position.

5. `PV-WAVE` Command line version, to create the contour plots from Mathematica output.

TABLE 6

Solt'n	f1	f2	s12	w0	d0	w(l1)	dif1	bf1	Df1
4g	22	-10	19.43044	0.01	118	0.19332	-7.59087	12.2241	19.118506
4e	40	-10	44.85657	0.01	160	0.261969	-8.44227	12.2241	9.0233482
4f	22	-10	16.8192	0.01	118	0.19332	-10.2021	12.2241	19.118506
4f-delta	22	-10	16.8192	w1	d1		-10.2021	12.2241	19.118506
				0.002287	27.02131		diff2	bf2	Df2
				0.003329	53.29883		38.77902	0.639386	0.0590609
				0.002287	27.02131		37.5533	1.354718	0.0288535
				0.002287	27.02131		221.4666	0.639386	0.0014305
			s23	w2	d2	w(l2)	221.4666	0.639386	0.0014305
4g			69.56956	0.009411	30.79054	0.054352	diff3	bf3	Df3
4e			81.541	0.019598	43.9877	0.041625	50.06897	10.82587	6.5553014
4f			70.1808	0.060468	-151.286	0.07302	50.09312	46.95163	28.124306
4f-delta			70.1808	0.060468	-151.286	0.07302	50.06823	446.9545	270.6205
							50.06823	446.9545	270.6205
		f3	s34	w3	d3	w(l3)	bf4	Df4	f1/w1
4g		-25	34.79835	0.003676	-15.2706	0.068073	1.651468	0.000275	113.80088
4e		-10	41.78394	0.003696	-8.30917	0.036972	1.669433	0.000282	152.68983
4f		-20	30.9605	0.003676	-19.1077	0.08513	1.651591	0.000275	113.80088
4f-delta		-20	30.9605	0.003676	-19.1077	0.08513	1.651591	0.000275	113.80088
		f4		w4	d4	w(l4)	f2/w2	f3/w3	f4/w4
4g		50		0.221793	301.126	0.222902	-183.985	-367.251	224.3137
4e		50		0.219999	380	0.221807	-240.243	-270.473	225.42107
4f		50		0.221801	298.4503	0.222891	-136.948	-234.934	224.32535
4f-delta		50		0.221801	298.4503	0.222891	-136.948	-234.934	224.32535
delta d2, w4, d4 =		0	0	0					
4-lens mode matching;		res;	26 Sept 92				1/2p =	8.18E-06	

(f1,f2,...): focal lengths of lenses l1, l2... (all dimensions cm)
sptnl-2: distance from l1 to l2
w0: input waist radius
d0: spacing from input waist to l1
(w1,d1) first waist size, position (relative to l1)
w(l1) = size of beam (radius) at l1
4f solution for no side chamber

Table 6: Spreadsheet output, showing details of three distinct mode matching configurations.