
New Folder Name Gravitational Radiation

Gravitational Radiation, Inspiring Binaries, and Cosmology

David F. Chernoff

Astronomy Department, Cornell University, Ithaca, NY 14853

Lee Samuel Finn

Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208

Received: _____; Accepted: _____

Abstract

We show how to measure cosmological parameters using observations of inspiring binary neutron star or black hole systems in one or more gravitational wave detectors. To illustrate, we focus on the case of fixed mass binary systems observed in a single Laser Interferometer Gravitational-wave Observatory (LIGO)-like detector. Using realistic detector noise estimates, we characterize the rate of detections as a function of a threshold signal-to-noise ratio ρ_0 , the Hubble constant H_0 , and the binary “chirp” mass. For $\rho_0 = 8$, $H_0 = 100$ km/s/Mpc, and $1.4M_\odot$ neutron star binaries, the sample has a median redshift of 0.22. Under the same assumptions but independent of H_0 , a conservative rate density of coalescing binaries ($8 \times 10^{-8} \text{ yr}^{-1} \text{ Mpc}^{-3}$) implies LIGO will observe $\sim 50 \text{ yr}^{-1}$ binary inspiral events.

The precision with which H_0 and the deceleration parameter q_0 may be determined depends on the number of observed inspirals. For fixed mass binary systems, ~ 100 observations with $\rho_0 = 10$ in the LIGO detector will give H_0 to 10% in an Einstein-DeSitter cosmology, and 3000 will give q_0 to 20%. For the conservative rate density of coalescing binaries, 100 detections with $\rho_0 = 10$ will require about 4 yrs.

Subject headings: gravitation — binaries: close — cosmology: theory — distance scale — methods: statistical

1 Introduction

When completed in the late 1990's, the Laser Interferometer Gravitational-wave Observatory (LIGO) (Abramovici *et al.* 1992) will consist of two interferometers. Concurrently, the VIRGO consortium (Bradaschia *et al.* 1990) will complete a single interferometer of comparable precision. In this letter we introduce a new and general class of cosmology tests based on the anticipated observation of the gravitational radiation from inspiraling binary neutron star and/or black hole systems.

Schutz (1986) and Krolak and Schutz (1987) noted that observation of binary inspiral in three independent interferometers will reveal the source's luminosity distance d_L . With an *independent* measure of the source redshift, such observations can determine cosmological parameters (e.g., Hubble's constant H_0). Our tests differ from theirs: the simplest can determine cosmological parameters with observations made in a *single* interferometer and without any other independent knowledge about individual binary systems. Extensions systematically exploit additional information available from multiple interferometers.

To assess the capabilities of these tests we consider observations in a single LIGO-like Interferometer (LLI); thus, our work applies directly to the individual interferometers in the LIGO and VIRGO projects. In addition, the two components making up the LIGO detector (LD) will be arranged in nearly the same plane and with nearly identical arm orientations (Whitcomb 1992) - a configuration which operates like a single LLI of increased sensitivity (Finn and Chernoff 1993, FC). We apply our recent results (FC) for the precision with which a binary's parameters can be determined in a single LLI.

Below we describe the physical basis and methodology of our tests. As a concrete application we specialize to neutron star binary systems of constant mass. We find the effective volumes sampled and the cosmological source rates. We derive the precision with which H_0 and q_0 may be determined in a Friedmann-Robertson-Walker (FRW) universe. We

demonstrate that source evolution may be handled in a straightforward manner.

2 Gravitational radiation from inspiraling binaries

Assume that quadrupole-formula gravitational radiation adiabatically damps a binary's Newtonian orbit.¹ Let the distance between the source and the LLI be d . The time-dependent response to the gravity-wave is ($G = c = 1$)

$$m(t) = \frac{\mathcal{M}}{d} (\pi \mathcal{M} f)^{2/3} \Theta(\hat{n}_S, \hat{n}_B) \cos\left(\int^t 2\pi f dt + \Psi\right), \quad (1)$$

where the “chirp” mass $\mathcal{M} = \mu^{3/5} M^{2/5}$ and μ and M are the reduced and total mass of the binary system. The frequency $f = f(\mathcal{M}, T - t)$ where T is the instant the binary separation $\rightarrow 0$ given the above approximations and Ψ is a constant phase. Θ is a known function of four angles specifying the orientation of the binary system with respect to the LLI ($0 \leq \Theta \leq 4$, FC): \hat{n}_B lies along the binary's orbital angular momentum and \hat{n}_S points from the detector to the binary. The wave frequency f is twice the binary's orbital frequency, and a LLI will be sensitive to the radiation during the last few minutes of inspiral as f sweeps from ~ 10 Hz to ~ 1 KHz. For binaries at cosmological distances the form of the response is the same, only now in place of d we have the *luminosity distance* d_L and in place of \mathcal{M} we have $\mathcal{M}' \equiv (1 + z)\mathcal{M}$, where z is the redshift. For more detail, see FC.

Schutz (1986) showed that three interferometers of different orientations are capable of determining d_L , \mathcal{M}' , Ψ , T , \hat{n}_S , and \hat{n}_B . In contrast, a single LLI can measure only $\mathcal{A} \equiv \Theta/d_L$, \mathcal{M}' , T , and Ψ (FC); thus, d_L cannot be determined directly since \hat{n}_S and \hat{n}_B are unknown. However, we know that \hat{n}_S and \hat{n}_B are each uniformly distributed on the sphere; consequently, the distribution of $\Theta(\hat{n}_S, \hat{n}_B)$ is also known. Thus, the observed distribution

¹Cutler *et al.* 1992 have shown that higher order relativistic contributions modify the orbital evolution. The tests described herein are entirely compatible with the use of a more accurate evolution, and we do not expect our conclusions to be altered significantly by the adoption of a refined waveform.

of \mathcal{A} is related to the number density of binary systems on spheres of constant d_L . Similarly, the observed distribution of \mathcal{M}' is connected to the number density on shells of constant z . With a physically plausible assumption regarding the evolution of the source distribution function with z , the joint distribution of observed inspiral events as a function of \mathcal{M}' and \mathcal{A} is sufficient to measure H_0 , q_0 , and otherwise test cosmological models.

3 Method

Let $\mathcal{N}d\mathcal{M}dV$ be the *intrinsic* binary coalescence rate (events per time), where \mathcal{M} is the intrinsic chirp mass and dV is the local volume element. The SNR ρ is a function of the parameters that describe the binary system and its orientation with respect to the detector. We give the form explicitly for a single interferometer in FC (eqs. 3.29, 3.31, and 4.10). The quantities dV and \mathcal{N} depend on the cosmology and the binary source evolution, and we denote the full set of model cosmology parameters as μ .

The observer counts as detections only those events for which the SNR exceeds a threshold ρ_0 , measuring \mathcal{M}' , ρ , and (for observations in more than one LLI) additional parameters ζ that are functions of the relative orientations. We wish to compare the observed distribution with that implied by a model μ . The expected event rate density corresponding to the model is

$$\dot{n}(\mathcal{M}', \hat{\rho}, \zeta) = \int d\mathcal{M}dV \frac{d\Omega_B}{4\pi} \frac{\mathcal{N}}{1+z} \delta[\hat{\rho} - \rho(d_L, \mathcal{M}', \hat{\mathbf{n}}_B, \hat{\mathbf{n}}_S)] \delta[\mathcal{M}' - (1+z)\mathcal{M}] \delta[\zeta - \zeta(\hat{\mathbf{n}}_B, \hat{\mathbf{n}}_S)]. \quad (2)$$

The integration over $d\Omega_B d\Omega_S$ effectively averages over $\hat{\mathbf{n}}_B$ and $\hat{\mathbf{n}}_S$ since, by assumption, \mathcal{N} depends on neither. The total rate $\dot{n}(> \rho_0)$ is $\dot{n}(\mathcal{M}', \hat{\rho}, \zeta)$ integrated over all \mathcal{M}' , ζ , and $\hat{\rho} > \rho_0$. The probability density for observations exceeding the threshold for a given set of

cosmological parameters is

$$p(\mathcal{M}', \hat{\rho}, \zeta | > \rho_0, \mu) = \frac{\dot{n}(\mathcal{M}', \hat{\rho}, \zeta)}{\dot{n}(> \rho_0)}. \quad (3)$$

Given a set of N observations we now apply a standard maximum likelihood analysis (Eadie *et al.* 1971). If the confidence volume $d\mathcal{M}' d\rho dV_\zeta$ is small compared to the scale on which the probability density p varies, then the total likelihood $\Lambda(\mu) \propto \prod_{j=1}^N (p d\mathcal{M}' d\rho dV_\zeta)_j$. The cosmological parameters μ that best fit the observed distribution are those that maximize Λ . Regions with $\Lambda > \Lambda_0$ (for fixed Λ_0) are confidence volumes for μ .

4 Application

As a concrete application, we show how observations in a single LLI can determine the parameters of a matter-dominated FRW model. Write the volume element dV as $f(z; H_0, q_0) dz d\Omega$, noting $f = \tilde{f}(z; q_0)(c/H_0)^3$ and $d_L = \tilde{d}_L(z; q_0)(c/H_0)$ (where \tilde{f} and \tilde{d}_L are dimensionless and independent of H_0).

Assume that \mathcal{N} can be written as $\mathcal{N} = \mathcal{N}_0(\mathcal{M})h(z)(1+z)^3$ where $\mathcal{N}_0(\mathcal{M})$ is the intrinsic rate density for coalescences in the *local* neighborhood. The factor $h(z)(1+z)^3$ expresses the time-dependence of the intrinsic rate and $h(z)$ encompasses all evolutionary effects for a *comoving* volume [$h(0) = 1$]. The factorization of \mathcal{N} implies that the chirp mass distribution is independent of the age of the universe, although the coalescence rate may vary. This assumption is justified if the character of the binary formation process (suitably averaged over many *galaxies*) is independent of z . The assumption is *very* plausible for neutron star binaries. Neutron stars form in the collapse of degenerate stellar cores in type II supernovae or by accretion induced collapse of white dwarfs. In either case the mechanism that triggers the collapse and determines the mass depends largely on the equation of state of degenerate matter. The resultant mass distribution should be relatively insensitive to the specific evolutionary pathway. The assumption is *moderately* plausible for binaries containing a black

hole. Current understanding suggests that the hole's upper mass is not tightly constrained by microscopic physical processes and, in fact, observations show a range of candidate masses (Casares, Charles and Naylor 1992; Remillard, McClintock and Bailyn 1992). The assumption that source evolution is a function of z *alone* is a phenomenological, not a physical, treatment. In contrast, a physical approach would introduce additional, non-cosmological timescales. Then h would depend, not only on z , but also q_0 and the timescales for gas cooling, star formation, and so forth. Here, $h(z)$ serves as a parameterization of the effects of evolution.

Now assume that neutron star binaries are the predominant sources. Noting that the measured range of pulsar masses is small ($\pm 0.1M_\odot$; Lamb 1991), while the derived range of masses in binary X-ray pulsars is somewhat larger ($\pm 0.4M_\odot$; Nagase 1989), take $\mathcal{N}_0(\mathcal{M}) = \widehat{\mathcal{N}}_0 \delta(\mathcal{M} - \mathcal{M}_0)$, *i.e.*, assume all binary systems have the same intrinsic chirp mass \mathcal{M}_0 (we relax this assumption in our more detailed report). Since z is a function of $\mathcal{M}'/\mathcal{M}_0$, we can express $\dot{n}(z, \rho)$ as $\dot{n}(\mathcal{M}', \rho)/\mathcal{M}_0$. Using FC (eq. 3.16) for ρ in eq. (2), we find

$$\dot{n}(z | > \rho_0) = \widehat{\mathcal{N}}_0 4\pi \left(\frac{c}{H_0} \right)^3 \tilde{f}(z; q_0) h(z) (1+z)^2 P \left[\delta \frac{\tilde{d}_L(z; q_0)}{(1+z)^{5/6}} \right], \quad (4)$$

where $P(x)$ is the fraction of randomly oriented binaries with Θ exceeding x [tabulated in FC, where it is called $P(\Theta > x)$]. The distribution is cutoff as the argument of P increases: $P(0) = 1$ and $P(x > 4) = 0$. Note the appearance of the dimensionless factor

$$\delta = 9.0 \left(\frac{\rho_0}{8} \right) \left(\frac{\mathcal{M}_0}{1.2M_\odot} \right)^{-5/6} \left(\frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}} \right)^{-1} \left(\frac{F_{7/3}}{\sqrt{2}} \right)^{-1}. \quad (5)$$

The interferometer's sensitivity is described by $F_{7/3}$ (for details see FC) which has a nominal value of 1 for a single LLI of advanced design and, as used here, $\sqrt{2}$ for the LD composed of two such LLI. We refer to the parameters in eq. (5) as fiducial parameters. As $\delta \rightarrow \infty$ the fraction of observable binaries goes to zero. The total observed rate above threshold is eq. (4) integrated over all z .

Assuming no source evolution [$h(z) = 1$], expand $\dot{n}(z | > \rho_0)$ to lowest order in z to obtain the Euclidean rate

$$\dot{n}_E(> \rho_0) = \widehat{\mathcal{N}}_0 4\pi \left(\frac{c}{H_0} \right)^3 \left(\frac{1.836}{\delta^3} \right), \quad (6)$$

independent of H_0 and $\propto 1/\rho^3$ above threshold. We summarize observation rates in terms of $Q(q_0, \delta, h) = \dot{n}(> \rho_0)/\dot{n}_E(> \rho_0)$. Note $\lim_{\delta \rightarrow \infty} Q = 1$.

5 Results

Consider first the characteristic depth of the observed sample. Let $z_{1/2}$ (z_{max}) be the redshift at which $P = 0.5$ (0.0) in eq. (4). To lowest order in redshift, $z_{1/2} = 0.145(9.0/\delta)$ and $z_{max} = 0.44(9.0/\delta)$. Note that we have scaled δ to the fiducial parameters of eq. (5). The observed sample is more precisely characterized by the cumulative distribution of *detected* binaries and by its *completeness*, displayed in figure 1a for $\delta = 4.5, 9.0, 13.5$ and 18.0 (SNR thresholds of 4.0, 8.0, 12.0 and 16.0 for fiducial parameters). Koo and Kron (1992, KK) model the distribution of faint galaxies without source evolution but with a low H_0 . To compare with KK adopt $H_0 = 50$ km/s/Mpc; then the median observed binary has $z = 0.10$ at $\rho_0 = 8$ ($\delta = 18.0$). In KK, the median of the cumulative galaxy distribution with $18 < B < 19$ occurs at roughly the same redshift. The depth is modest, but (as figure 1a shows) the extent is large. The cumulative distribution for different values of q_0 show discernible changes.

The detection rate $\dot{n}(> \rho_0)$ scales directly with the local coalescence rate. Phinney (1991) has discussed the observational and theoretical limits. Based on the several observed neutron star binaries, the time until coalescence, and the effective Galactic volumes sampled, he gives a “best guess” estimate for the coalescence rate density of 8×10^{-8} Mpc $^{-3}$ yr $^{-1}$. This is a *conservative* estimate because it counts only binaries like those observed to date; for a number of reasons the true value may exceed the estimate by as much as ~ 800 [$H_0/(100$

km/s/Mpc)]³. Scaling to the conservative value, the anticipated detection rate is

$$\dot{n}(> \rho_0) = 68 \cdot \frac{\widehat{N}_0}{8 \times 10^{-8} \text{Mpc}^{-3} \text{s}^{-1}} Q(q_0, \delta, h) \left(\frac{8}{\rho_0} \right)^3 \left(\frac{\mathcal{M}_0}{1.2 M_\odot} \right)^{5/2} \left(\frac{F_{7/3}}{\sqrt{2}} \right)^3. \quad (7)$$

Recall that the function Q is the ratio of the cosmological rate to the Euclidean rate; figure 1b gives Q for a range of δ for values of $q_0 = 0.25, 0.50,$ and 0.75 assuming no source evolution. For the fiducial parameters, $Q = 0.715$ for $q_0 = 0.5$, which corresponds to an observed rate of $\dot{n}(> 8) = 49. \text{ yr}^{-1}$ events. Notice Q is only weakly sensitive to q_0 : in an analytic expansion to lowest order in z it is independent of q_0 . The effects of source evolution may dominate the influence of q_0 on the value of Q .

Our main interest here is to assess our cosmology test when source evolution takes place; consequently we adopt an *ad hoc* model $h(z) = \exp[-(\alpha z + \beta z^2)]$ for $z < 10$ and $h(z) = 0$ otherwise. This model is useful for both strong and weak evolution. Broadhurst *et al.* (1988) consider a similar form for galaxy luminosity evolution; however, we make no attempt to apply their results. In adopting this model, we envision three qualitatively different possibilities: (1) The coalescence rate may be simply proportional to the star formation rate. Then, based on the faint galaxy studies, there should be little evolution for $z < 0.5$ (KK). We assume $\alpha = \beta = 0$ for *uniform production* (no evolution). (2) Neutron star binaries may be formed by collisional encounters in the dense cores of galaxies. The luminosity evolution of QSOs is a strong function of z (Boyle *et al.* 1987, 1988); hence, if the coalescence rate is proportional to the QSO luminosity, a substantial increase in the production rate with z is expected. We take $\alpha = -3/2$ and $\beta = 1/4$ [for which $h(z)$ peaks at $z = 3$, where it is $h \approx 9.5$] for this case of *early production*. (3) Binaries may be formed by collisional encounters in globular clusters after core collapse. The time to core collapse is of order $1/H_0$ for clusters in the Galaxy. For the case of *late production*, we take $\alpha = 0$ and $\beta = 1$, which cuts off strongly for $z > 1$. Figure 1b illustrates how early and late production alters Q in the $q_0 = 0.5$ model.

We used a Monte Carlo analysis to calculate the precision with which μ can be determined. We chose “true” model parameters $\tilde{\mu}$ and formed a realization of N observations. Since we have restricted our analysis to large ρ_0 the small observational uncertainties have not been included in the realization.² We applied a maximum likelihood procedure to this realization to deduce the most likely values $\hat{\mu}$, and found the difference $\delta\mu = \hat{\mu} - \tilde{\mu}$ as a function of N by repeating the procedure 100 times for each N . For each parameter μ_i , we found the bias $\langle\delta\mu_i\rangle$ and the rms dispersion $\sigma_{\mu_i} \equiv \langle\delta\mu_i^2\rangle^{1/2}$. In all cases we found the magnitude of the bias to be much less than the dispersion; so, we deal with the latter as an indication of the best performance that can be expected for measurement of μ .

Our model depends upon five physical quantities: Hubble’s constant, the deceleration parameter, the chirp mass, α and β . Denote the “true” values by $\{\tilde{H}_0, \tilde{q}_0, \tilde{\mathcal{M}}_0, \tilde{\alpha}, \tilde{\beta}\}$ and write the cosmological parameters as $\{\mu\} = \{h_0 \equiv H_0/\tilde{H}_0, q_0, m \equiv \mathcal{M}/\tilde{\mathcal{M}}_0, \alpha, \beta\}$. (In this dimensionless form $\{\tilde{\mu}\} = \{1, \tilde{q}_0, 1, \tilde{\alpha}, \tilde{\beta}\}$.) We carried out the Monte Carlo calculations of the rms dispersion of \hat{h}_0 and \hat{q}_0 for Einstein-DeSitter cosmologies (*i.e.*, $\tilde{q}_0 = 0.5$). Our results are unchanged by scaling \tilde{H}_0 , $\tilde{\mathcal{M}}_0$, ρ_0 and $F_{7/3}$ for fixed δ . We assumed uniform *a priori* probabilities for $h_0 \in (0.1, 2.0)$ and $q_0 \in (0.1, 2.0)$; the allowed ranges of m , α , and β were large and will be discussed in more detail elsewhere.

Table 1 summarizes results for σ_{h_0} and σ_{q_0} for two values of δ , three choices of α and β , and several different N . While the dispersion and bias in h_0 , q_0 , α and β are all strongly correlated, we defer a more detailed discussion to a forthcoming paper. The dispersions reported in Table 1 should only be taken as rough approximations: we have not attempted to determine the precision to which they are found by our Monte Carlo analysis. It is clear, however, that our samples (for fixed δ) extend into the asymptotic regime, where the dispersion is $\propto N^{-1/2}$.

²FC show that the fractional errors $d\mathcal{M}'/\mathcal{M}' \approx 10^{-4}/\rho$ and $d\rho/\rho \approx 1/\rho$.

Note how, at fixed N , the Hubble constant is not better determined by the deeper sample (smaller threshold ρ_0 , which implies smaller δ): thus, sample size is more important than sample depth to the accurate determination of h_0 . In contrast, the uncertainties in q_0 are significantly smaller in the deeper sample. More quantitatively, we find that with a threshold $\rho_0 \sim 10.7$ ($\delta = 12$), LIGO will determine H_0 to $\sim 10\%$ with ~ 100 observations. Similarly, q_0 may be determined to approximately $\sim 20\%$ with ~ 3000 observations.

6 Discussion and conclusions

Observations of the gravitational radiation from binary inspiral by the LIGO detector can be used to measure important cosmological parameters. These measurements rely on comparing the observed distribution of inspiral events, as a function of signal strength and chirp mass, with model predictions. It is necessary to adopt a model for the intrinsic coalescence rate of binaries as a function of the chirp mass and the redshift. We argue that a factorized form is plausible but that other forms are possible. We provide a simple demonstration of the power of the test by specializing to the case of a narrow, well-defined chirp mass range. We characterize the observational volumes by the sample's median redshift and by its completeness. We give the rate of detections as a function of the detector threshold signal-to-noise ratio using Phinney's (1991) conservative estimate of the local binary coalescence rate. We find that, with a modest number of detections, these observations can determine the Hubble constant to 10%. The conservative rate estimate implies that the observations **would** take ~ 4 years. We address the optimal choice of the SNR threshold, measurement errors at small ρ and a range of binary masses in a paper in preparation.

We thank Drs. Bildsten, Cutler, Flanagan, Loredo, Thorne, Wasserman, and Whitcomb for discussions. LSF acknowledges the Sloan Foundation and NASA NAGW-2936 (Northwestern); DFC the PYI program, NSF AST-8657467 and NASA NAGW-2224 (Cornell).

REFERENCES

- Abramovici, A. et al., 1992, *Science*, **256**, 325.
- Bradaschia, C. et al., 1990, *Nucl. Inst. A.*, **289**, 518.
- Broadhurst, T. J., Ellis, R. S., Shanks, T. 1988, *M.N.R.A.S.*, **235**, 827.
- Boyle, B. J., Fong, R., Shanks, T. and Peterson, B. A. 1987, *M.N.R.A.S.*, **227**, 717.
- Boyle, B. J., Shanks, T. and Peterson, B. A. 1988, *M.N.R.A.S.*, **235**, 935.
- Casares, J., Charles, P. A., and Naylor, T. 1992, *Nature*, **355**, 614.
- Cutler, C., Finn, L. S., Poisson, E. and Sussman, G. J. 1993, *Phys. Rev. D.*, **47**, 1511-1518.
- Eadie, W. T., Drijard, D., James, F. E., Roos, M. and Sadoulet, B. 1971, *Statistical Methods in Experimental Physics*, North Holland, 121.
- Finn, L. S. and Chernoff, D. F. 1993, *Phys. Rev. D.*, **47**, 2198-2219.
- Koo, D. C. and Kron, R. G. 1992, *Ann. Rev. Astron. Astrophys.*, **30**, 613.
- Krolak, A. and Schutz, B. F. 1987, *Gen. Rel. and Grav.*, **19**, 1163.
- Lamb, F. K. 1991, Neutron Stars and Black Holes in *Frontiers of Stellar Structure*, ed. D. L. Lambert (ASP), 299.
- Nagase, F. 1989, *Publ. Astr. Soc. Japan* **41**, 1.
- Phinney, E. S. 1991 *Astroph. J.*, **380**, L17.
- Remillard, R. A., McClintock, J. E., and Bailyn, C. D. 1992, *Astroph. J.*, **399**, L145.
- Schutz, B. F. 1986 *Nature*, **323**, 310.
- Whitcomb, S. 1992, private communication.

This manuscript was prepared with the AAS WGAS L^AT_EX macros v2.2

FIGURES

Figure 1a. The observational volume accessible to the LD is characterized by the cumulative distribution of *detected* binaries $S(z, \rho_0) = \int_0^z dz' \dot{n}(z' | > \rho_0) / \dot{n}(> \rho_0)$ (the rising set of curves) and by the sample's *completeness* $T(z, \rho_0) = \int_0^z dz' \dot{n}(z' | > \rho_0) / \int_0^z dz' \dot{n}(z' | > 0)$ (the falling curves) for the case of no source evolution. The widely spaced families of curves represent $\delta = 4.5, 9.0, 13.5$ and 18.0 (eq. 5) from right to left; these correspond to thresholds of $\rho_0 = 4, 8, 12$ and 16 for fiducial parameters. Each family consists of three closely spaced curves corresponding to different values of q_0 (dashed 0.25; solid 0.5; dotted 0.75). The curves describing the sample's completeness terminate at the maximum z at which an inspiraling binary can be observed.

Figure 1b. The ratio of the cosmological to the Euclidean rate of detections, Q , is given as a function of δ (eq. 5). For the case of no source evolution, three different values of q_0 are displayed, as in Figure 1a; these correspond to the three closely spaced lines. Two additional models discussed in the text are shown for the $q_0 = 0.5$ case. The long dashes illustrate a coalescence rate lower in the past than in the present. The chain dashes illustrate one with a higher rate in the past.

Table 1: The results of applying the maximum-likelihood analysis to finding h_0 , q_0 , m_0 , α and β .

δ	N	No Evolution ($\alpha = \beta = 0$)		Early Production ($\alpha = 0, \beta = 1$)		Late Production ($\alpha = -1.5, \beta = 0.25$)	
		σ_{h_0}	σ_{q_0}	σ_{h_0}	σ_{q_0}	σ_{h_0}	σ_{q_0}
		12.0	30	0.144	0.844	0.143	0.838
	100	0.099	0.681	0.093	0.674	0.104	0.666
	300	0.071	0.479	0.063	0.425	0.075	0.425
	1000	0.035	0.197	0.039	0.226	0.036	0.193
	3000	0.019	0.110	0.017	0.102	0.020	0.110
	10000	-	-	0.009	0.055	0.010	0.054
20.0	30	0.097	0.890	0.112	0.865	0.109	0.896
	100	0.075	0.780	0.076	0.742	0.067	0.696
	300	0.054	0.590	0.049	0.515	0.054	0.558
	1000	0.029	0.311	0.026	0.255	0.027	0.273
	3000	0.013	0.141	0.015	0.157	-	-

