New Folder Name Shot Noise & Random Forces

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concerning: h, shot noise and random forces

Definition of h

In the traceless gauge (one where the coordinates x and y are real lengths and t is the time kept by proper clocks) and for the + polarization for a wave propagating in the z direction the interval between events is written as

    ds^2 = - (c*dt)^2 + [(1 + h(t))]dx^2 + [(1 - h(t))]dy^2

Using the property that light has zero interval, ds^2 = 0, for the space time interval between emission and receipt of a light signal, the time variation of a beam of light traversing along the x direction in the presence of a gravitational wave of amplitude h(t) becomes

    delta t = 1/2 h(t) L/c

where L is the proper length of the end points of the path along the x direction. The change in time can be interpreted as a change of length due to the gravitational wave

    delta L = c delta t = 1/2 h(t)L

So for a one armed interferometer
the measured strain

    Delta L/L = 1/2 h(t)

The gravitational wave affects both arms but in opposite directions so the equivalent Delta L when both arms are considered as in a Michelson configuration is

    Delta L(equiv)/L = h(t)

This is the way Kip usually defines h(t).

Now in the Michelson configuration where the optical phase phi is measured

    Delta L(equiv) = (dL/dphi)(single arm)*Delta phi (differential)

Delta phi (differential) = delta ph1(arm1)-delta(phi(arm2) = 2* delta(phi) single arm.

The interferometer measures the differential phase at the antisymmetric port.

The relation then between the phase noise amplitude spectral density at the antisymmetric port and the inferred amplitude spectral density of h(f) becomes

h(f) = (dL/phi)(single arm) phi differential(f)/L

In the limiting case phi differential(f) becomes the shot noise.

There are still some further subtleties however. Since we modulate the light and convert with a mixer the equivalent noise bandwidth after the electronic recovery of the signal is twice the post detection integration bandwidth. This comes about because we use both the upper and lower sidebands. This causes the noise in h(f) to be sqrt(2) larger after detection electronics than the above formulation would give.

The relation for random forces on the masses

When the random forces are uncorrelated- each mirror gets driven randomly on its own - say by thermal noise- and we calculate a displacement amplitude spectrum for a single mirror of x(f) then if all four mirrors have the same noise one would get by adding powers and taking the square root

h(f) = 4 * x(f)/L