New Folder Name Intensity Noise

Intensity Noise in Asymmetric Interferometer

FVR
9/30/93

1. Consider a Symmetric Michelson with audio AM @ Ω with fractional amplitude (peak) a, and with phase modulation (@ ω) with Γ radians amplitude but with spurious AM of σ.

Let E_0 be the field before AM & PM are applied. Let $\bar{l}$ be the Michelson length. We may write the field contributed to the antisymmetric port by either arm as:

$$E_{\bar{l}\pm} = \pm \frac{E_0}{2}\left[\frac{a}{4}e^{i\Omega t} + 1 + \frac{a}{4}e^{-i\Omega t}\right]$$

$$\cdot\left[\left(1+\frac{\sigma}{4}\right)J_1(\Gamma)\,e^{i\omega t} + 1 - \left(1-\frac{\sigma}{4}\right)J_1(\Gamma)\,e^{-i\omega t}\right]$$

$$= \pm\frac{E_0}{2}\Bigg[\overset{\text{audio AM}}{\frac{a}{2}\cos\Omega t} + \overset{\text{carrier}}{1} + \overset{\text{RF-modulation}}{2iJ_1\sin\omega t + \frac{\sigma}{2}J_1\cos\omega t}$$

$$+ \frac{iaJ_1}{2}\left\{\sin(\omega+\Omega)t + \sin(\omega-\Omega)t\right\}$$

$$+ \frac{\sigma a J_1}{8}\left\{\cos(\omega+\Omega)t + \cos(\omega-\Omega)t\right\}\Bigg]$$

LIGO-T930036-00-R

2. Replace Michelson mirrors by identical Fabry-Perot cavities on resonance with carrier.

$$E_{I,\pm} = \pm \frac{E_0}{2}\Bigg[\overbrace{\frac{\gamma a}{2}\cos(\Omega t+\phi)}^{\text{audio phase shift and loss}} - \overset{\text{carrier phase reversal and loss}}{\beta} + 2iJ_1 \sin\omega t + \frac{\sigma}{2}J_1\cos\omega t$$

$$+ \frac{iaJ_1}{2}\left\{\sin(\omega+\Omega)t + \sin(\omega-\Omega)t\right\}$$

$$+ \frac{a\sigma J_1}{8}\left\{\cos(\omega+\Omega)t + \cos(\omega-\Omega)t\right\}\Bigg]$$

3. Now introduce asymmetry of $\pm\delta$ into near mirror to beamsplitter differences $\ni$ the arm which gets the (+) phase has a longer round trip time by $\tau = \frac{2\delta}{c}$ and the (−) phase arm has a shorter round trip time by τ. Summing the two contributions and using

$$\cos\omega(t+\tau) - \cos\omega(t-\tau) = -2\sin(\omega t)\cdot\sin(\omega\tau)$$

$$\sin\omega(t+\tau) - \sin\omega(t-\tau) = 2\cos(\omega t)\cdot\sin(\omega\tau)$$

we get

$$E_A = \frac{E_0}{2}\Big[-\gamma a \sin(\Omega t+\phi)\cdot\sin\Omega\tau + 4iJ_1\cos\omega t\cdot\sin\omega\tau$$

$$-\sigma_1 J_1 \sin\omega t\cdot\sin\omega\tau$$

$$+iaJ_1\left\{\cos(\omega+\Omega)t\cdot\sin(\omega+\Omega)\tau + \cos(\omega-\Omega)t\cdot\sin(\omega-\Omega)\tau\right\}$$

$$-\frac{\sigma a J_1}{4}\left\{\sin(\omega+\Omega)t\cdot\sin(\omega+\Omega)\tau + \sin(\omega-\Omega)t\cdot\sin(\omega-\Omega)\tau\right\}\Big]$$

The power at the antisymmetric port in a frequency band near ω is

$$P_A(\omega\text{-band}) = \frac{P_0}{4}\left[\frac{\sigma\gamma a^2 J_1}{4}\left\{\sin(\Omega t+\phi)\cdot\sin(\omega+\Omega)t\cdot\sin\Omega\tau\cdot\sin(\omega+\Omega)\tau \right.\right.$$

$$\left. + \sin(\Omega t+\phi)\cdot\sin(\omega-\Omega)t\cdot\sin\Omega\tau\cdot\sin(\omega-\Omega)\tau\right\}$$

$$\left. + \sigma\gamma a J_1\left\{\sin(\Omega t+\phi)\cdot\sin\omega t\cdot\sin\Omega\tau\cdot\sin\omega\tau\right\}\right]$$

Frequency Components which give audio noise upon demodulation

frequency	amplitude	phase
$\omega \pm 2\Omega$	$\mp\frac{\sigma\gamma a^2 J_1}{8}\cdot\sin\Omega\tau\cdot\sin(\omega+\Omega)\tau\cdot P_0$	$\pm\phi$
$\omega+\Omega$	$-\frac{\sigma\gamma a J_1}{2}\cdot\sin\Omega\tau\cdot\sin\omega\tau\cdot P_0$	$\pm\phi$

4. For case of no AM, with asymmetry, but cavities $\pm\frac{\Delta x}{2}$ from resonance, $\left(\frac{\phi}{2} = \frac{8\pi c\tau_s}{L}\cdot\frac{\Delta x}{2\lambda}\right)$:

$$E_A = \frac{E_0}{2}\left[4iJ_1\cos\omega t\cdot\sin\omega\tau - (1+\beta)(e^{i\phi/2} - e^{-i\phi/2})\right]$$

$$= \frac{E_0}{2}\left[4iJ_1\cos\omega t\cdot\sin\omega\tau - 2i(1+\beta)\sin(\phi/2)\right]$$

$$P_A = P_0\left[J_1\cos\omega t\cdot\sin\omega\tau - \frac{(1+\beta)}{2}\sin(\phi/2)\right]^2$$

For $\Delta x(t) = \Delta x \cdot \sin \Omega t$ or $\phi(t) = \phi \sin \Omega t$

we have

$$P_A(\omega\text{-band}) = -\frac{\phi(1+\beta)J_1}{2} \cdot \cos \omega t \cdot \sin \Omega t \cdot \sin \omega \tau \cdot P_0$$

For over coupling, $\frac{1+\beta}{2} \simeq 1$

$$P_A(\omega \pm \Omega) \simeq \frac{\phi J_1 \sin \omega \tau}{2} \cdot P_0$$

$$\boxed{|P_A(\omega \pm \Omega)| \simeq \frac{4\pi c \tau_s}{L} \frac{\Delta x(\Omega)}{\lambda} \cdot J_1 |\sin \omega \tau| \cdot P_0}$$

5. For intensity noise to be negligible, require

$$\frac{\sigma \gamma a}{2} \cdot \sin \Omega \tau \ll \frac{4\pi c \tau_s}{L \lambda} \Delta x(\Omega)$$

$$\sigma \gamma a \ll \frac{8\pi c \tau_s}{L \lambda \Omega \tau} \Delta x(\Omega)$$

For $\Delta x(100\,Hz) = 10^{-19}\, m/\sqrt{Hz}$, require

$$\boxed{\sigma \gamma a \ll 3 \times 10^{-4}\ Hz^{-1/2} \cdot rad^{-1}}$$