New Folder Name Review of Work

Review of Work Toward Development of an Optical Topology for LIGO

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1. Introduction

The selection of an optical topology for an interferometer involves the definition of the configuration of mirrors, beamsplitters and detectors which form the interferometer, and identification of the signal extraction and control servomechanisms that make the interferometer work. Here I will describe work that I have pursued with M. Regehr and T. Lyons toward development of a specific optical topology for LIGO interferometers. The selection of an optical topology for LIGO was a major enabling step in the broader interferometer design effort because the interfaces between many of the interferometer subsytems are related to the topology chosen. Over the years, a number of possible topologies had been suggested, but few schemes had been investigated in detail. Because of the importance of the topology decision, the LIGO Project commissioned two demonstration experiments that would be developed in parallel. Martin Regehr and I would develop a topology called the asymmetry scheme at Caltech; J. Giaime, D. Shoemaker and R. Weiss would pursue an external modulation scheme at MIT.

In broad strokes, the topology of the initial LIGO interferometers will incorporate optical recombination and power recycling. Optical recombination refers to allowing light from the two interferometer arms to interfere at the beamsplitter. The simplest case of this is the Michelson interferometer, shown in Figure 1a. If the phases and amplitudes of the light returning from the two arms are matched, the returning power is directed back toward the light source. A photodiode viewing the antisymmetric port of the beamsplitter sees darkness, sometimes referred to as a "dark fringe". If one of the mirrors is moved slightly, then light will appear at the antisymmetric port. Also, since most photons returning to the beamsplitter in a recombined system are preserved in the interference process, they can be reused in subsequent measurements. This is referred to as power recycling, which effectively amplifies the available light power incident on the arms by the recycling factor. Figure 1b shows optical recombination and power recycling incorporated into an interferometer with Fabry-Perot cavities in the arms. This is the optical configuration for the initital LIGO interferometer.

There are significant differences between the initial LIGO configuration and the configuration of the current 40–meter interferometer, shown in Figure 1c. In its current configuration, light from the two interferometer arms is detected independently; the 40–meter interferometer essentially compares each arm separately to the wavelength of the light. This has the advantage that the interferometer arms can be locked in steps and it is easy to perform diagnostics on the system. This configuration is quite adequate for studying backgrounds due to seismic and thermal noise and for studying sensing noise caused by fluctuations of the input light. However this topology uses light inefficiently (it does not permit recycling) and it offers no common mode rejection of laser frequency noise. Recombination would allow the arms to be compared directly against each other by interfering the electric fields returning from each arm. This more direct process can boost the signal to noise ratio by more than a factor of three for a given amount of light.

After providing some historical perspective for this work, I will briefly present the basic requirements for signal extraction and control of auxiliary lengths in a power recycled interferometer. I will then describe the interferometer topology we have been developing, which we refer to as the asymmetry scheme. I will discuss key aspects of the work and what has been learned. This topology is striking in terms of its extreme optical simplicity, but feedback signals needed to control the interferometer are more strongly coupled than in some other schemes. This ill-conditioned aspect is remedied by a certain (natural) hierarchy of servo control gains and bandwidths, as was discovered in the process of this work. Modeling efforts and experiments with a table-top prototype interferometer have successfully demonstrated the utility of this scheme, and the asymmetry technique for extracting the gravitational-wave signals has been adopted for the initial LIGO interferometers. I will end by discussing issues related to implementing power recycling in the 40–meter interferometer.

2. Historical Perspective

The basic idea of recycling an interferometer was developed by R. Drever[1]. Since then, a number of newer recycling configurations were derived, the most significant of which were dual recycling[2], and its derivative, resonant sideband extraction[3]. Table-top experiments have demonstrated the expected optical gain and signal to noise ratio enhancements expected in a power recycled Fabry-Perot interferometer[4] and a dual recycled Michelson interferometer[5] configuration.

However the techniques used in these experiments to sense differences in arm lengths and to hold the interferometer at its correct operating point were not applicable to LIGO-scale interferometers.

As of the summer of 1991, there had been no demonstration of a recycled interferometer with a signal extraction scheme that was scalable to real gravitational-wave detectors like LIGO or VIRGO. Martin Regehr and I had agreed that his thesis work would focus on developing a scheme that would be suitable for LIGO interferometers and could be tested on the 40–meter interferometer. After Martin passed his oral candidacy exams we proposed development of an extension of a scheme originally conceived by L. Schnupp[6] of the Garching gravitational-wave group, the "asymmetry scheme." An alternative external modulation scheme was pursued in a parallel effort by Joe Giaime, David Shoemaker and Rai Weiss.

3. Basic Requirements for Signal Extraction and Length Control in a Power Recycled Interferometer

A successful optical topology for an interferometer must be able to extract any gravitational-wave signal with a signal to noise ratio determined by shot noise. A schematic optical layout for a power recycled interferometer is shown in figure 2. The two Fabry-Perot arm cavities have lengths L_1 and L_2. Light reflected from each of the cavities returns to the beamsplitter where interference occurs. If the interferometer arms are perfectly matched, interference directs this light back toward the laser and the recycling mirror. Any differential variation between the two arms (e.g., the strain induced by a gravitational wave) will cause light to be directed toward the antisymmetric port.

The phase of the light returning toward the beamsplitter from an arm cavity is very sensitive to the length of the cavity, if the cavity is near resonance. To see this, let E_i be the field amplitude incident on one of the cavities. The reflected light amplitude is a combination of the field $E_R = r_1 E_i$ that is reflected from the front mirror of the cavity, and the field E_C, corresponding to light stored in the cavity which subsequently is transmitted back through the front mirror. This gives

$$E_r = E_R - E_C = \left[r_1 - \frac{r_2 t_1^2 e^{2ikL}}{1 - r_1 r_2 e^{2ikL}} \right] E_i \tag{1}$$

where r and t refer to the amplitude reflectivity and transmission of the mirrors (subscripts 1 and 2 refer to the front and rear mirror of the cavity, respectively,

$k = 2\pi/\lambda$ is the wavenumber of the light and L is the length of the cavity. When the cavity is very close to resonance, the field amplitude is nearly constant but the phase varies with L,

$$E_C \simeq |E_C| e^{i\theta(L)}. \tag{2}$$

In interferometers planned for LIGO, the transmission of the cavity input mirror is much larger than the transmission of the output mirror and the reflection losses caused by scattering and absorption[1]. In this case we have

$$|E_r| \simeq |E_R| \simeq \frac{1}{2}|E_C| \simeq |E_i| \tag{3}$$

and the phase is given by

$$\theta(L) = \frac{4\pi}{1 - r_1 r_2} \cdot \left(\frac{\delta L}{\lambda}\right) \tag{4}$$

where δL is the deviation from the resonant length of the cavity. The small denominator $(1 - r_1 r_2 \approx 1/60)$ causes the large phase change for a small change in the length of the Fabry-Perot cavity.

For greatest sensitivity, the interferometer lengths are set so that the field

$$E_A = \frac{1}{\sqrt{2}}(E_{r_1} - E_{r_2}) \simeq \frac{-|E_C|}{\sqrt{2}}\left(e^{i\theta(L_1)} - e^{i\theta(L_2)}\right) \tag{5}$$

has a minimum magnitude when no gravitational wave is present. (By energy conservation this allows maximization of the light stored in the interferometer.) A gravitational wave of amplitude h, incident on an interferometer with optimal alignment, will cause a length change $\delta L = hL/2$ of opposite sign in each of the two Fabry-Perot cavities. This causes a field amplitude to appear at the antisymmetric port, given by

$$E_A = 2iE_0 sin\theta \approx iE_0 \cdot \frac{4\pi}{1 - r_1 r_2} \cdot \frac{hL}{\lambda}. \tag{6}$$

Unfortunately, a simple measurements of the intensity, $I \propto |E_A|^2$ is only sensitive to h in second order.

[1] The input mirror transmission is chosen so that the storage time τ for light in the cavity is approximately half the period of the lowest frequency gravitational wave that the interferometer is designed to detect. For the initial LIGO interferometers, this sets $r_1^2 = 0.97$, $t_1^2 = 0.03$, and $r_2^2 \approx 0.9999$

One method to derive a first order signal in h is to interfere the light at the antisymmetric port with another light beam, that was originally of the same frequency but which has been phase modulated. The electric field amplitude of the second beam is then

$$E_{PM} = E_1 e^{i\Gamma sin\Omega t} = E_1 \left[J_0(\Gamma) + J_1(\Gamma)\left\{ e^{i\Omega t} - e^{-i\Omega t} \right\} + \ldots \right] \tag{7}$$

where J_0 and J_1 are Bessel functions. The detected intensity will then have a component which oscillates at the phase modulation frequency Ω, of magnitude

$$P(\Omega) \propto 8|E_0||E_1|J_1(\Gamma)sin\theta \tag{8}$$

and the interferometer has a first order sensitivity to gravitational waves. One of the key challenges in the design of an optical topology is to generate the phase modulated light and to get it to interfere properly with the light at the antisymmetric port.

In addition to the needs of signal extraction, there are important control requirements to be satisfied. First, the Fabry-Perot cavities must be maintained on resonance. Furthermore, in the example above, the dependence of the interference on the lengths l_1 and l_2 was ignored. A complete analysis shows that E_A also depends on the quantity $l_1 - l_2$, although less strongly than on $L_1 - L_2$. Thus all four lengths l_1, l_2, L_1 and L_2 must be controlled very accurately, while at the same time the mirrors must be suspended from vibration isolated platforms to reduce seismic noise transmission. This requires an auxiliary length control system which, in turn, requires more information than is available at the signal port. In any optical topology, provision must be made to sample the light at other points in the interferometer to acquire the additional auxiliary length control information.

4. The Asymmetry Scheme

There are at least three methods of supplying modulation at the signal port. In an internal modulation scheme, as shown schematically in Figure 3a, phase modulators (pockels cells) are inserted into the arms between the beamsplitter and the Fabry-Perot cavities to supply the modulation. This scheme works well for a small apparatus, but pockels cells with the large apertures ($\approx$ 20 cm) and low loss ($\approx$ 100) ppm required for kilometer-scale interferometers are far beyond the state of the art. External sidearm modulation[7] can be accomplished by picking off some light before the beamsplitter, applying phase modulation to it, and then recombining it with light from the signal output port of the beamsplitter, as shown in Figure 3b.This scheme can be scaled up to large systems and high power; however it introduces additional complexity, requiring construction of an additional interferometer (typically a Mach-Zender configuration) to interfere the modulation and signal beams. The third scheme is the asymmetry scheme described here.

In 1986, L. Schnupp realized that modulation for a recycled Michelson interferometer could be done with a phase modulator between the laser and the recycling mirror, if the Michelson interferometer were asymmetric. This is shown schematically in figure 4. If $l_1 = l_2$, both the carrier and the sidebands interfere destructively at the beamsplitter. The absence of light on the signal photodetector in this interference condition is often referred to as a "white light fringe." However if l_1 and l_2 differ by a nonzero half-integral number of carrier wavelengths, then the carrier and the sidebands cannot both interfere destructively. Thus, in the absence of a signal, the carrier light can interfere destructively (required for efficient recycling and low shot noise) while some sideband light leaks out (required for recovering the any signal and for controlling the interferometer). In the absence of recycling, the optimum transmission of sidebands from the input to the signal output occurs when the asymmetry

$$\delta \equiv l_1 - l_2 = \frac{c}{4\nu_M} \tag{9}$$

where $\nu_M = \Omega/2\pi$ is the modulation frequency. Recycling gain in the sidebands lowers the value of δ that optimizes sideband transmission.

The addition of Fabry-Perot cavities to the interferometer severely complicates the simple picture that was analyzed by Schnupp. In the asymmetry scheme, as depicted in figure 5, a phase modulation is also inserted between the laser and the recycling mirror. The interferometer lengths and the modulation frequency are chosen so that the both the carrier light and the modulation sidebands can resonate in the recycling cavity, but only the carrier and not the sidebands can resonate in the arms. A gravitational wave will principally affect the light in the Fabry-Perot cavities and thus will appear as a phase shift on the carrier light, without affecting the sidebands. The asymmetry, $\delta = l_1 - l_2$, allows transmission of sideband light to the signal port, even while the carrier is held on a dark fringe, provided

$$\delta = l_1 - l_2 = \frac{n\lambda_0}{2} \tag{10}$$

where λ_0 is the wavelength of the carrier (514.5 nm nominal) and n is an integer.

Other lengths and frequencies in the interferometer are set as follows. The condition for the carrier to be resonant in the Fabry-Perot cavities while the sidebands are in antiresonance gives

$$L_1 \approx L_2 \approx \frac{c}{2\nu_M} \cdot (m + 1/2) \tag{11}$$

where m is an integer. To resonate both the sidebands and the carrier in the recycling cavity, requires

$$\frac{l_1 + l_2}{2} \approx \frac{c}{2\nu_M} \cdot (p + 1/2) \tag{12}$$

where p is an integer.

5. Control of an Ill-Conditioned Plant and the Hierarchy of Servo Loops

The control system for a recycled interferometer is an example of a multiple–input/multiple-output (or MIMO) servo system. In the language of control theory, the system to be controlled (in this case, the interferometer) is known as the plant. Additional pick-offs are used to sample light at strategic positions as shown in figure 5. Appropriate demodulation of this light (I and Q in figure 5 refer to the in-phase and quadrature phases of the demodulator) provides the additional information required for controlling the four lengths in the power recycled interferometer. In fact there are five parameters if the laser wavelength is included, but the proper operating point of the interferometer is specified by four constraints. Here we will assume that the laser wavelength λ_0 will be locked to the average of the Fabry-Perot cavity lengths, $(L_1 + L_2)/2$, as the last step of the laser frequency stabilization. Then the average of the cavity lengths is a free parameter. In this case, the normalized demodulator outputs are given by

$$V_1 = \quad \delta\lambda \quad + \varepsilon_1(\delta l_1 + \delta l_2) \tag{13}$$

$$V_2 = \quad \delta\lambda \quad + \varepsilon_2(\delta l_1 + \delta l_2) \tag{14}$$

$$V_3 = (\delta l_1 - \delta l_2) + \varepsilon_3(\delta L_1 - \delta L_2) \tag{15}$$

$$V_4 = (\delta L_1 - \delta L_2) + \varepsilon_4(\delta l_1 - \delta l_2) \tag{16}$$

where we have absorbed an arbitrary gain factor into the definition of the V_i. The quantity $\delta\lambda$ denotes deviations of the laser wavelength from the exact resonance condition for the arms, and the quantities δL_i and δl_i denote small deviations from the respective desired positions. Each of the demodulator outputs is principally sensitive to a particular quantity with some slight dependence on another parameter. (For the parameters typical of LIGO interferometers the ε's are of order 10^{-2}. The output V_4 depends principally on the difference between arm lengths which represents the gravitational-wave signal. The output V_3 can be used to control the beamsplitter position. A potential difficulty in designing the control system comes from the fact that the light picked off anywhere before the beamsplitter is much more sensitive to the deviation of the laser from resonance in the Fabry-Perot cavities ($\delta\lambda$) than to any deviation from resonance in the recycling cavity ($\delta l_1 + \delta l_2$). A plant which exhibits this rather weak dependence of the error signals on one of the parameters is referred to as being "ill-conditioned".

We initially intended to address this problem by designing electronic decoders to separate the signals. This can be done provided the electronics and the interferometer operating parameters are sufficiently stable, but it places stringent demands on the interferometer hardware. In the course of this work we discovered that the need for electronic decoding can be eliminated if one of the control loops has sufficiently higher gain than the others. To understand this point it is useful to consider a simpler system of a two coupled Fabry-Perot cavities as shown in figure 6. The equations for this plant are

$$V_1 = \delta\lambda + \varepsilon_1 \delta l \tag{17}$$

$$V_2 = \delta\lambda + \varepsilon_2 \delta l \tag{18}$$

If we feed back the signal V_1 to the laser, we can suppress the error in laser wavelength. If the fluctuations in V_1 are made sufficiently small by using a high loop gain G_1, then the fluctuations appearing in V_2 need no longer be dominated by errors in the laser wavelength. Clearly this also depends on the suppression of V_2 by the gain G_2 of loop 2 (which feeds back to δl). A rigorous treatment of the loop equations shows that if the condition $\varepsilon_2 G_1 > G_2$ is met, then the control of δl is not compromised by the ill-conditioned nature of the plant, and efforts to decode the signals are not necessary. This has also been shown to be the case in numerical examples worked out for the full asymmetry scheme.

This requirement on the gains can be met quite naturally since the laser frequency stabilization servo typically has much higher gain than the other servos whose actuators push on the test masses.

6. Status of Experimental and Modeling Efforts

A significant effort has been invested in modeling the behavior of the asymmetry topology and comparing the results to experimental data. A number of optical configurations were studied this way, starting with a simple Fabry-Perot cavity, moving on to a system of two coupled cavities and eventually leading to a full power recycled interferometer using the asymmetry scheme. In addition to confirming the validity of the modeling which so far only treats the linear aspects of control, the experimental prototype allowed us to show that such a system could acquire lock, and investigate its sensitivity to perturbations.

The modeling of the interferometer falls into two classes. Steady state models are valid in the regime where all motions of the mirrors are slow compared to a typical storage time τ in the interferometer cavities. For example, equations (1) through (8) are the result of a steady state model of a single Fabry-Perot cavity. It predicts a lineshape and strength for the light when optical fields within the cavity are in equilibrium. If one of the mirrors of this cavity is moving at a frequency above the cavity pole frequency ($\nu_C = 1/4\pi\tau$), a different treatment is needed. A high frequency model is then used which treats the moving mirror as a source of optical sidebands injected into the cavity at the mirror. For an electric field E_i incident on a mirror of reflectivity r which is at position $x(t) = x_0 sin\omega_m t$, the reflected wave is

$$E_{refl} = rE_i e^{2ikx_0 sin\omega_m t}. \tag{19}$$

For $2kx_0 \ll 1$ this reduces to

$$E_{refl} \simeq rE_i\left[1 + J_1(2kx_0)\cdot\left(e^{i\omega_m t} - e^{-i\omega_m t}\right)\right]. \tag{20}$$

Effectively, the mirror is a source of optical sidebands, which must be propagated throughout the interferometer. The high frequency model involves a much more extensive set of equations than the steady state model, where the cavity lengths are parameters, but there is only one source.

We have constructed a table-top interferometer with the asymmetry topology, using mirrors in fixed mounts with piezoelectric actuators. Such a fixed-mass interferometer does not use a vacuum system or vibration isolation, which makes it easier to make changes and do diagnostic testing. Such an interferometer cannot demonstrate low noise performance, but can be used to test signal strengths and couplings. Following equations (9) through (12), and using a phase modulation frequency of 12 MHz, the smallest possible interferometer had approximately 6–meter recycling and arm cavities, with an asymmetry of about 30 cm. This was accomodated by using extra mirrors to fold the cavities onto a 5'x12' optical table. The physical layout is shown in figure 7. The recycling factor was 4 and the power gain of the Fabry-Perot arm cavities was about 40. The differences between these values and the LIGO values (a recycling factor of 30 and a power gain of 130) were concessions to the higher optical losses and acoustic noise in a lab as opposed to a pristine vacuum system with seismic isolation. The increasing circulating power along the optical paths in the interferometer is denoted by increasingly bold lines in figure 7. The servo topology for the prototype is shown in figure 8. Here I and Q refer to the in-phase and quadrature components of the demodulated photocurrents at each photodetector. Lock was acquired by sweeping the beamsplitter while the other three servo loops were connected. When the system passed through a point where all cavities were resonant, the beamsplitter's sweep was disabled and its loop was closed. The interferometer would hold lock reliably, provided ventilation fans were turned off in the lab. (The fringe width for the Fabry-Perot cavities was about 2 Angstroms wide.) Banging on the table would throw the interferometer out of lock, but the automated circuitry would relock the interferometer typically in about 10 seconds.

The experimental response of the interferometer to motion of the mirrors is being compared to predictions from the steady state model. (The mirrors cannot be reliably driven above the cavity poles for this interferometer because of mechanical resonances in the mirror mounts). The transfer functions measured so far agree with the model to within experimental uncertainties. The high frequency model has been checked for consistency with the steady state model and with analytical formulae that can be obtained when certain approximations are made. More stringent testing will be done when the 40–meter interferometer is recycled.

7. Adoption of an Optical Topology for LIGO

In October 1993 the progress of the development programs for the asymmetry scheme and external modulation scheme were reviewed with the intent on choosing a scheme for LIGO and then concentrating effort on that single scheme. J. Giaime, D. Shoemaker, M. Regehr, L. Sievers and I were charged with presenting a recommendation. Based on experience gained in the work at Caltech and MIT, we recommended that the asymmetry scheme be adopted for extraction of the gravitational-wave signals in LIGO instead of the external modulation scheme, which was optically more complicated and required extra control hardware. For auxiliary length sensing we recommended that a technique developed at MIT, which used additional phase modulations to sense the auxiliary lengths, should be adopted because it allowed a stepwise lock acquisition process and was useful for diagnosing lock acquisition problems. Although the asymmetry scheme prototype had locked quite readily, the loops acquire nearly simultaneously, making lock acquisition problems more difficult to diagnose. In the absence of a sophisticated nonlinear model for lock acquisition, it was hard to ensure that locking acquisition on suspended mass interferometers would work as well as in the table-top model. For this reason, a more conservative approach was recommended. After review of the data, the recommendations were adopted.

8. Preparing to Recycle a Suspended-Mass Interferometer

The next steps in the program are to model the LIGO optics and servo loops, and to develop an experimental demonstration of power recycling on interferometers with suspended masses. Optical modeling work in progress is addressing issues related to coupling of higher order spatial modes by mirror distortions and misalignments. This work is principally being done at MIT, where there is considerable expertise in wavefront propagation codes. Work has begun here on specifying the optics for converting the 40–meter interferometer to a power recycling configuration.

A small interferometer cannot obtain the same storage times and recycling factors as a well designed interferometer of larger size. (For a given storage time, the smaller interferometer needs to have the light bounce between the mirrors more times resulting in higher losses and a lower recycling factor.) Torrey Lyons has been studying the design of a recycled configuration for the 40–meter interferometer. This entails predicting the losses that can be expected in the arm cavity mirrors on a 40–meter baseline, which are different in principle from the losses measured in our usual 1–meter test setup. Using these estimates, we have calculated a family of curves which trade shot-noise sensitivity at low frequencies against the cavity pole frequency (figure 9 shows the curves for 1 watt of input power). Each curve represents a possible value of mirror loss L. One moves along the curve by varying the transmission of the input mirror to the Fabry-Perot arm cavity and then choosing a recycling mirror transmission which optimizes the recycling factor. Each pair of mirror choices determines a DC sensitivity and a corner frequency, which completely characterize the shot noise. The straight lines of positive slope are lines of constant recycling factor ρ. From optical measurements of losses on a 1–meter baseline apparatus and analysis of excess scattering due to estimated figure errors, Torrey has estimated an upper bound on mirror losses of 40 ppm per mirror (dashed line in the figure). This shows that a recycling factor of 5 with a corner frequency of 500 Hz should be achievable in the 40–meter interferometer, and would give the shot noise shown as a dashed line in figure 9. This compares to a recycling factor of 30 with a corner frequency of 90 Hz for the initial LIGO interferometers, and is a reasonable compromise.

The recycling of the 40–meter interferometer will test predictions of the high frequency modeling and will help clarify many other servo issues. The system is far more complex than the fixed-mass prototypes because there is more low frequency motion in the suspended components, including more degrees of freedom. The nature of lock acquisition will more closely resemble the situation in LIGO. While the fixed-mass interferometer development was a necessary precursor to implementing recycling, the 40–meter system with its suspended masses will provide a better simulation of the performance issues relevant to LIGO.

Notes

[1] R.W.P. Drever, in **Gravitational Radiation**, ed. N. Deruelle and T. Piran, 321, North Holland, (1983); apparently similar results were derived independently by members of the Garching collaboration.

[2] B.J. Meers, *Phys. Rev. D*, **38**, 2317, (1988); a variant of this technique, called doubly resonant dual recycling has been proposed by Meers and Drever (unpublished).

[3] J. Mizuno, K.A. Strain, P.G. Nelson, J.M. Chen, R. Schilling, A. Rudiger, W. Winkler and K. Danzmann, *Phys. Lett. A*, **175**, 273, (1993).

[4] P. Fritschel, D. Shoemaker, and R. Weiss, *Appl. Opt.*, **31**, 1412, (1992).

[5] K.A. Strain and B.J. Meers, *Phys. Rev. Lett.*, **66**, 1391, (1991).

[6] L. Schnupp, 1986, unpublished.

[7] This technique was used in the optical study of power recycling described in reference 4, above.